ELECTRON CYCLOTRON RESONANCE BROADENING DUE TO STRONG FOCUSSING OF WAVE BEAMS
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Introduction
Strongly focused wave beams are being used in current tokamak experiments with the goal to obtain the smallest possible region of power deposition. Standard ray-tracing techniques as embodied in commonly used ray-tracing codes like TORAY [1] are inapplicable for the description of such experiments. Alternative methods are required that include effects of diffraction. Such methods can be regarded as generalisations of geometric ray-tracing to a form of Gaussian beam tracing [2,3]. One example of a numerical code that solves the ensuing beam tracing equations, is TORBEAM [4]. Apart from the trajectory of the central ray of the beam, TORBEAM evaluates the local width of the beam and the curvature of the local phase front. Power absorption and current drive are calculated on the basis of the properties of the central ray only, under the assumption that these properties and the consequent absorption vary only weakly over the beam.

This assumption in the power absorption calculation comes down to the neglect of the effects of finite beam width and curvature of the phase front on the resonant absorption. In this paper, the neglect of these effects is put to the test. The curvature of the phase front parallel to the magnetic field represents a finite spread of the parallel wave vector, \( \Delta N^\parallel \), over the beam. Although this spread will be small, electron cyclotron resonant absorption can vary strongly with \( N^\parallel \), thus violating the underlying assumption in the TORBEAM calculations. A finite beam width, \( w \), results in a finite interaction time between wave beam and particles and, consequently, a broadening of the wave particle resonance. In fact, the effects of both the spread in parallel wave vector as well as the finite beam width are incorporated in a single broadened resonance function, which is to replace the delta function representing the resonant wave-particle interaction in the power absorption calculations [5]:

\[
\frac{\pi}{2\Delta Q} \exp \left[ -\frac{1}{2\Delta Q} \left( \gamma - n\omega_{ce} / \omega - N^\parallel p^\parallel / m_e c \right) \right] \text{ with } \Delta Q = \left( \frac{\gamma N^\parallel |p^\parallel|}{\omega w} \right)^2 + \left( \frac{\gamma N^\parallel |p^\parallel|}{2c} \right)^2 .
\]

Here, \( \omega \) is the wave frequency, \( \omega_{ce} \) the electron cyclotron frequency, \( n \) the resonant harmonic number, and \( p^\parallel = \gamma m_e \omega^\parallel \) is the electron momentum parallel to the magnetic field. Note, that the beam width \( w \) and phase front curvature \( R \) and, consequently, \( \Delta N^\parallel = w/R \) are related over the trajectory of the beam. In vacuum this relation is given by

\[
w = \sqrt{\frac{w_0^2 + \frac{4x^2}{k_0^2 w_0^4}}{\frac{k_0^2 w_0^2}{k_0^2 w_0^2}}}, \quad \text{and} \quad R = \frac{x^2 + k_0^2 w_0^4 / 4}{x},
\]

where \( x \) is the distance to the waist. Obviously, at the position of the waist the phase front is flat and \( \Delta N^\parallel = 0 \), such that the total resonance broadening equals \( \Delta Q = (\gamma N^\parallel / \omega w_0)^2 \). In the opposite, far field limit \( w \) becomes very large and the resonance broadening is determined solely by the spread in the parallel wave vector, which in this limit becomes \( \Delta N^\parallel = 2/k_0 w_0 \). It
is easily verified then that in the far field limit the resonance broadening equals that at the position of the waist, i.e. \( \Delta Q = (\gamma_0/\omega_0) - 2 \).

The same broadened resonance function (1) appears in the quasi-linear wave diffusion operator of the bounce averaged Fokker-Planck model [5]. It is implemented in the bounce averaged quasi-linear Fokker-Planck code RELAX [6], which thus can conveniently be used to evaluate the consequences of this resonance broadening.

**Results of combined beam tracing and Fokker-Planck code calculations**

This section presents a numerical case study of the effects of resonance broadening on the power deposition of strongly focused beams for parameters typical of ECRH experiments in the TEXTOR tokamak [7]: major radius \( R_0 = 1.75 \text{ m} \), minor radius \( a = 0.465 \text{ m} \), temperature \( T_e(r) = 2 (1 - (r/a)^2)^2 \text{ keV} \), and density \( n_e(r) = 4 \times 10^{19} (1 - (r/a)^2) \text{ m}^{-3} \). The wave frequency is 110 GHz and power 300 kW. The magnetic field is chosen such that the resonance is close to mid radius (\( B_T = 2.274 \text{ T} \)), and the remaining parameters are chosen such that in vacuum the beam waist would coincide with the resonance. Only perpendicular injection in the mid plane has been considered. In this particular geometry the deposition profile is determined almost entirely by the resonance width. The TORBEAM code is then used to obtain the relevant beam parameters, including \( \Delta N_{||} \) and \( w_0 \), on a selected set of magnetic surfaces covering the entire power deposition region. These data are then used as input to RELAX in order to calculate the power deposition profile with the appropriately broadened resonance function (1).

![Figure 1](image-url)

**Figure 1** Power deposition profiles for beams of varying toroidal width in case of a central temperature \( T_{e0} \) of 2 (a), or 4 keV (b). The coloured curves, labelled 1 through 8, present results of the RELAX calculations including resonance broadening effects for toroidal beam widths of 0.5 to 2.0 cm, respectively. The vertical beam width, which has negligible effect on the deposition profile, in all cases is 2.5 cm. The full black line gives the result of the TORBEAM calculation, which neglects effects of resonance broadening.

Figure 1 shows the results of such calculations for varying toroidal widths of the beam waist both in case of a central temperature of 2 and 4 keV (yielding a temperature at the position of the resonance of \( \sim 1 \) and 2 keV, respectively). For beam widths \( w_0 \leq 1 \text{ cm} \), the resonance broadening effects become large and the power deposition is seen to shift outward to larger major radii and to broaden significantly. For beam widths \( w_0 \geq 2 \text{ cm} \), the results from
RELAX correspond well with those from TORBEAM, and the effect of the resonance broadening is negligible. Results of further calculations for a wider range of (hypothetical) temperatures and for a hypothetical major radius of $R_0 = 3.5$ m are summarised in Fig.s 2 and 3. In dependent of both $T_e$ and $R_0$ significant deviations in both location and width of the power deposition are found to appear in all circumstances for $w_0 \leq 1$ cm.

**Figure 2** The deposition profile width (a) and location of the maximum deposition (b) as a function of the width of the toroidal waist of the beam. Full curves represent the results from RELAX calculations including resonance broadening, and the dashed curves give the result from TORBEAM calculations, which neglect resonance broadening. The labels 1 through 4 refer to the different central temperatures (2, 4, 8, and 16 keV, resp.) used.

**Figure 3** As figure 2, but in case of a major radius of $R_0 = 350$ cm (the minor radius is kept equal at $a = 46.5$ cm).
Conclusion and discussion

In conclusion, for beam widths below 1 cm, a significant broadening and outward shift of the deposition profile is found independent of either plasma temperature or major radius. This width is to be compared with the actual width of the waist of the ECRH beam applied in TEXTOR of 1.4 cm [8]. The TEXTOR beam thus is about optimum: it provides close to the best possible geometric localisation without adverse effects from resonance broadening.

In order to explain these results, it is important to realise that for the parameters chosen 2nd harmonic X-mode absorption is very effective and the optical depth is much larger than unity. Consequently, not the entire profile of the absorption coefficient plays a role in determining the deposition profile. For close to perpendicular propagation and coming from the low field side, all power is already absorbed along the first steeply rising part of this profile. Due to the relativistic increase of the particle mass, \[ \frac{n \omega_{ce}}{\omega} = \sqrt{1 - N^2_{||}} \approx 1 - \frac{1}{2} N^2_{||} \tag{3} \]
The resonance broadening by either finite beam width or spread in \( N_{||} \) has the same effect of shifting the low field side edge of the absorption profile towards slightly lower fields. In this context the beam width can be seen as an “effective spread in \( N_{||} \)”. When combined with the broadening through the spread in \( N_{||} \) this results over the entire beam path in a total \( \Delta N_{||,\text{eff}} = 2/k_0 w_0 \). In terms of the major radius, this shifts the foot of the absorption coefficient profile to lower magnetic fields by \( \Delta R/R_0 = 0.5 \Delta N_{||,\text{eff}} \). For a 110 GHz beam with a width of 1 cm, one finds \( \Delta N_{||,\text{eff}} = 0.09 \). This corresponds to a shift in the position of the power absorption by \( \Delta R/R_0 = 0.004 \) or \( \Delta r/a = 0.015 \). This shift scales as \( \Delta R/R_0 \sim \Delta r/a \sim 1/w_0^2 \). The shift and its scaling compare well with the results presented in Figs. 2b and 3b. Moreover, the effect will be independent of \( T_e \) as long as the optical depth is sufficiently high and \( \Delta N_{||,\text{eff}} < v_t/c \), where \( v_t \) is the thermal velocity. Note that, in case of a beam width of 1 cm, the shift is very similar to the width of the deposition profile without the effect of resonance broadening. Consequently, the shift of the deposition profile goes together with a broadening of the profile, which is consistent with the results shown in Figs. 2 and 3.

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