Isotope Effect in Hydrogen Ion Distribution Function

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1. Introduction

The neutral particles charge-exchange diagnostics is routinely used for the plasma hydrogen isotope composition measurement in high temperature plasmas. It is also considered as a candidate for the fuel control in deuterium-tritium plasma in ITER. To recalculate the isotope density ratio in plasma from neutral fluxes measured by neutral particle diagnostics the information on local ion distribution function (IDF) is required. Usually it is supposed to be maxwellian in the ohmically heated discharges \cite{1}. For the small plasma machines with low ion temperature and in absence of auxiliary heating the assumption is sufficiently accurate. However for the large tokamaks additional analysis has to be performed to define the energy range where the distortion of IDF would not lead to the errors in isotope ratio measurements. In the this paper such an analysis based on the experimental results of particle charge-exchange diagnostics on ASDEX-Upgrade is presented.

2. Experimental results

To find the relation between ion isotope ratio (IR) in plasma and charge-exchange fluxes such plasma parameters as electron and impurity profiles and the temperatures are required. In the case of equal isotope temperatures the IR is directly proportional to the corresponding flux ratio corrected for plasma opacity and may be used for the plasma composition control \cite{2}.

However in ASDEX-Upgrade experiments in ohmic low density discharges with $T_i \sim 1$ keV this proportionality has been found to exist only up to the energies 4-5 keV. Above this range the flux ratio as a function of energy depends on $E$ and shows significant deflection from the expected behavior. The fluxes for deuterium and hydrogen are shown in fig.1. There are also shown the effective temperatures $T_{\text{eff}}(E) = -\left(\frac{\partial \ln f}{\partial E}\right)^{-1}$ for the isotopes and IR derived from the spectra. At low energies $T_{\text{eff}}(E)$ practically coincide in value and increase with energy. This behavior corresponds to the registration of atoms originated from inner plasma radii with higher ion temperature. The IR derived from energies
E < 4 keV does not depend on E and may be interpreted as its constancy over radius. For the energy above 4 keV \( T_{\text{eff}}(E) \) for hydrogen (H) and deuterium (D) approach the constant values, which correspond to the maximal temperatures on the analyzer view line, however they differ by 20-25%. In standard interpretation that means the IR depends on E and it looks obviously contradictory. Several possible explanations of the effect have been considered already. It has been shown that neither a difference in the absolute temperatures of H and D nor the spatial variation of ion temperature and density can give consistent account for the effect [3]. Estimates of energy balance also indicate that the effect cannot result from a difference in H and D temperatures. Firstly because ion-ion collision frequency is significantly higher than electron-ion one. Secondly the difference in H and D temperatures must depend on the IR, what were not observed in the experiments.

Here we consider another approach to the problem. NPA diagnostics is usually placed near the minimum of toroidal magnetic field so it is necessary to take into account the effects related to the drift of locally trapped particles. It has been shown earlier on ASDEX-Upgrade [4] that IDF varies during L- to H-mode transition. Theoretical consideration of the fast particles behavior in these modes has been given in [5].

The similar approach may be developed for the ohmic plasma. In this report the ion convective transport in the local magnetic field ripple as a possible explanation of the effect is considered. The model is based on a qualitative description of the process. Neither true variation of the of the ripple depth along drift trajectories nor the radial electric field which is supposed to be small in ohmic plasma discharges were taken into account. Nevertheless it allows to describe experimentally observed effect as the result of the convective losses which lead to a depletion of ion distribution function at high energies.

3. Numerical model

In our work we will follow the qualitative model developed in [6]. It does not allow to calculate IDF accurately but it defines the energy range where kinetic convective transport may be essential for IDF formation. The characteristic energy at which the drift length of the locally trapped ions becomes comparable with minor plasma radius may be estimated as:

\[
E_{dr} \approx 2 \left( aBR_o \left( \frac{N}{10\Delta} \right) \frac{1}{\sqrt{A_i}} \right)^{2/5} \text{ (keV; m, T, } 10^{19}\text{ m}^{-3})
\]

where \( a \)- minor, \( R_o \) – major plasma radii, \( B \)- magnetic field, \( N \) - plasma density, \( A_i \) – ion atomic number, \( \Delta \)- depth of magnetic field ripple. For typical ASDEX-Upgrade ohmic
plasmas it gives the value $E_{d,r} \approx 5$–7 keV which is close to the energy range of interest.

The kinetic equation for IDF, provided that the condition of the adiabatic trapping is fulfilled, looks as follows [6]:

$$\frac{\partial^2 f}{\partial \xi^2} + \frac{\partial f}{\partial \xi} - \lambda H(E - E_{d,r}) \xi^{3/2} f = 0,$$

where $H(E - E_{d,r})$ - is Heaviside function, the rest of parameters are defined by formulae:

$$\xi = \frac{E}{T_i}; \lambda = \sqrt{2} \frac{\nu_{ad}^i(E)}{T_i} \approx \left( \frac{T_i}{E_{ad}} \right)^{3/2} \frac{1}{30 \sqrt{\varepsilon}}; \varepsilon = r/R_o$$

The solution in WKB approximation for IDF normalized to unity at zero energy may be expressed analytically as:

$$f(E) = \begin{cases} \exp \left( -\frac{E}{T_i} \left( \frac{Q_o^* - 1}{Q_o^* + 1} \right) \right) & \text{for } E \leq E_{ad} \\ \left( \frac{Q_o^*}{Q_o^*} \right)^{1/2} \frac{2}{1 + Q_o^*} \exp \left[ -\frac{1}{2} \left( \frac{E + E_{ad}}{T_i} + \int_{E_{ad}}^E \frac{dE}{T_i} \right) \right] & \text{for } E \geq E_{ad} \end{cases}$$

$$Q_o^* (E) = \left[ 1 + 4\lambda \left( \frac{E}{T_i} \right)^{3/2} \right]^{1/2} \approx \left[ 1 + \frac{0.1}{\sqrt{\varepsilon \Delta}} \left( \frac{E}{E_{ad}} \right)^{3/2} \right]^{1/2},$$

where $E_{ad}$ - the energy of adiabatic trapping, $Q_o^* = Q_o^* (E_{ad})$. It can be seen that for $E \geq E_{ad}$ due to additional integral term in the exponent the energy dependence of IDF substantially differs from maxwellian and is a function of atomic number.

The local IDF was calculated according to the formulae for the ohmic discharge of ASDEX-Upgrade with plasma parameters and radial ripple depth approximation shown in fig.2. Afterwards the neutral density profile was calculated in the standard way. To obtain the fluxes as measured by NPA IDF was integrated along the view line taking into account the probability of ion charge-exchange and ionization of outgoing atoms. The computed results for $T_{eff} (E)$ for H and D and their flux ratio as a function of E are shown in fig.2.

The steps on the flux ratio arise from Heaviside function in the kinetic equation.
4. Conclusions

1. Toroidal field ripple and radial electric field substantially affect IDF.

2. Drift model for IDF gives satisfactory agreement with the experimental results for ohmic shots taking into account simplifying assumption made in our calculation.

3. Extrapolation of NPA diagnostics results for ITER scale tokamaks requires detailed analysis of IDF and is impossible without accurate numerical modeling similar to that made in [5].

References


