Radial electric field generation due to edge plasma turbulence


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I. Introduction

It is generally accepted that tokamak edge plasma turbulence causes anomalous diffusion. Potential structures, formed as a result of plasma turbulence, are observed in the poloidal plane of tokamaks (e.g., [1], [2]), with typical poloidal correlation lengths \(\lambda \approx 10 \rightarrow 20 \text{ mm}\), lifetimes \(\tau \approx 10 \rightarrow 20 \mu\text{s}\), and amplitudes \(U < 100 \text{ V}\). Theoretical studies addressing the anomalous diffusion in these fields are usually based on the test-particle drift approximation and on the electrostatic field resulting from the Hasegawa-Mima model (see, e.g. [3]) or the Hasegawa-Wakatani model [4].

In our preceding short comments (e.g. [5]), we discussed the effect of the anomalous ion diffusion in question on radial electric field generation. There we used a very simplified model of the turbulent potential structures, namely a spatially periodic and time-independent potential. Using a Hamiltonian approach (which also takes into account the cyclotron motion), we have found, for impurity ions \(C^+\) and usual potential amplitudes, a substantial increase in the diffusion of these ions (both of the Gaussian and Lévy-walk forms) [6], resulting the generation of a radial electric field [5]. Using a drift approximation for this case, no diffusion and no electric field is observed. Hence, there is a major difference between these two approaches and there can be some uncertainty in using the drift approximation.

Radial electric fields play an important role in the formation of transport barriers in tokamaks (see, e.g., [7]). They give rise to velocity shear, together with strongly reduced turbulence and transport. Several mechanisms have been proposed for the generation of radial electric fields (see, e.g., [8]). It seems that in this context our mechanism, resulting from anomalous diffusion of impurities (and not mentioned in [8]), can play an interesting role. Of course, since the turbulence mentioned here appears in the edge plasma, the transport barrier possibly generated can be located only in this region. For the core plasma, we presented in [5] another mechanism, which may also be able to generate conditions for anomalous diffusion of ions.

II. Anomalous diffusion of impurities and generation of radial electric field in the Hasegawa-Wakatani potential

In the work described above, the anomalous diffusion in question was studied in a rather simple approximation of the turbulent potential appearing in the edge plasma. In this section,
a more realistic form of this potential, namely the Hasegawa-Wakatani potential, will be considered. The Hasegawa-Wakatani (HW) model [9] is based on the numerical solution of mode-coupling equations for the resistive drift-wave instability, modelling in this way the turbulent processes in the edge plasma. Recently, we have followed their approach, using for the numerical simulation the parameters of the Czech CASTOR tokamak [10] \((R = 0.4 \text{ m}, a = 0.1 \text{ m}, B = 1 \text{ T}, \text{ the density } n = 10^{19} \text{ m}^{-3} \text{ and the temperature } T_e = 200 \text{ eV})\). A typical instantaneous potential relief inside a space sample with dimensions 4 cm radially \(\times\) 8 cm poloidally, taken from the fluid simulation results for the entire poloidal cross-section of the tokamak scrape-off layer (SOL), is presented in Fig. 2. In comparison with the periodic relief of Fig. 1, the potential amplitudes can be considered as very close. It is therefore of some interest to compare the dynamics in both models.

For this purpose, we use the 2-dimensional Particle-in-Cell (PIC) code BIT2 to follow approximately \(10^6\) particles (\(\text{C}^+\) ions). These particles were initially distributed uniformly in a small rectangular subregion located in the centre of the above-mentioned space sample (first simulation) or over the whole sample (second and third simulations). We assume periodic boundary conditions in the poloidal direction. In the radial direction, the boundary conditions are periodic for the calculation of the variance and diffusion coefficient (test-particle case, first and second simulations), and non-periodic for the self-consistent radial electric field generation with particles diffusing in from the plasma outside the simulation region according to Eq. (2) below (third simulation). The initial temperature of the impurities is \(T = 10\) eV. The electrons and plasma ions are treated as a fixed background and the HW potential is taken from a fluid simulation.

As in the case of the periodic relief, we calculate (in the first and second simulations) the time histories of the average variance \(X^2(t)\) and the running diffusion coefficient \(D_x(t)\) [4] for the corresponding dynamics (test-particle solution), defined as

\[
X^2(t) = \left\langle (x_j(t) - x_j(t = 0))^2 \right\rangle = \frac{1}{N} \sum_{j=1}^{N} \left( x_j(t) - x_j(t = 0) \right)^2, \quad D_x(t) = \frac{X^2}{2t}, \quad (1)
\]

(and similarly for the \(y\) direction), where \(x_j, y_j\) are the radial and poloidal co-ordinates of the \(j\)th particle, respectively, \(t\) is time, and \(N\) is the number of particles.

The results are presented in Fig. 3. Here, the time traces of the variances and running diffusion coefficients are presented separately for the radial and poloidal directions. We see...
that the diffusion is larger in the poloidal direction than in the radial one. This effect has already been noted in [4]. Due to the large number of particles, the variance time traces are practically continuous. Contrary to the results presented in [4], the diffusion in the radial direction is rather Gaussian, and only the diffusion in the poloidal direction has a parabolic-type variance time trace, thus clearly indicating Lévy-walk dynamics. Comparing this with the diffusion coefficient for the dynamics in the periodic potential relief [6], the enlargement of the diffusion coefficient is much enhanced in the poloidal direction, whereas in radial direction the diffusion is analogous to [6].

Typical time sequences of the spatial diffusion are shown in Fig. 4. Due to the larger diffusivity in the poloidal direction, the form of the initially well defined sample is elongated in the poloidal direction.

As already proposed in our recent papers [5] and [11], this anomalous diffusion must (in the self-consistent approach) necessarily result in a positive-charge deficit and, consequently, in the generation of a radial electric field. The profile of this radial electric field, following from our third simulation after 30 μs, is shown in Fig. 5, which indicates that the amplitude is of the order of $10^4$ V/m. As in the case of the periodic potential relief, such a field results in a shear of...
the poloidal plasma rotation.

Moreover, we have found that not only heavier impurities, but also plasma ions (here, we consider protons) diffuse significantly faster than if collisions were the only mechanism acting. Using the well-known expression for the ion-ion collision time (see, e.g. [12])

$$\tau_i = \frac{12 m_i^{1/2} e_0^2 (\pi T_i)^{1/2}}{\ln \Lambda e^4 Z^4 n_i},$$

we obtain for $T_e = 10$ eV and $n = 10^{18}$ m$^{-3}$ the diffusion coefficient of the protons to be of the order $D_p \approx 6.4 \times 10^{-3}$ m$^2$s$^{-1}$. The radial variance calculated for a sample of $10^6$ protons (see Fig. 6) yields a running radial diffusion coefficient of about 2 m$^2$s$^{-1}$ (again in the test-particle approach). This latter diffusion will have a stronger effect than the diffusion of impurities, because the proton density is two orders of magnitude larger than the density of impurities. The direct implication of this result is not quite correct because the HW equations were derived just from the fluid equations for these ions. Nevertheless, this striking result shows that since the potential relief, obtained from the HW equations, cannot take into account the mentioned stochastic diffusion, the real potential will be different. Whether this effect tends to increase HW potential or diminish it, will be proved by new simulations combining the PIC code with the fluid code and also by a comparison of HW potential with the real potential, measured by the appropriate diagnostics. In the paper [13] an alternative mechanism of the charge separation is discussed.

### III. Summary

The paper discusses the anomalous ion diffusion in the Hasegawa-Wakatani model of the low-frequency potential. It has been found that for parameters of the CASTOR tokamak, carbon impurities diffusion is anomalous (in comparison with the collisional diffusion). In the radial direction, the diffusion has Gaussian form, whereas in the poloidal direction it exhibits Lévy-walk character. Due to this diffusion, a radial electric field appears in the turbulence region. Moreover, the same effect can be expected during the injection of non-intrinsic ions from outside. These effects can be of interest in the discussion of thermal barriers. Nevertheless, together with the diffusion of impurities, also the diffusion of plasma ions can appear. This has to be considered in the discussion of the generation of the turbulent potential itself.

### Acknowledgements

This work was supported by Academy of Sciences of the Czech Republic Grant IAA1043201, Grant Agency of the Czech Republic Grants 202/03/P062, 202/03/0786 and Austrian Science Fund (FWF) Project P15013-N08. Its content is the sole responsibility of the authors and does not necessarily represent the view of the EU Commission or its Services.

### References: