Inverse Bremsstrahlung Absorption of Intense Laser Pulse in Inertial-Fusion Plasma Corona

A. Sid, A. Bendib

1Laboratoire des Sciences Nucléaires, Faculté de Physique, USTHB, Alger BP 32, El Alia, Bab Ezzouar, Alger, Algérie. E-mail: sid_abdelaziz@mail.univ-tebessa.dz
2Laboratoire d'Electronique Quantique, Faculté de Physique, USTHB, Alger BP 32, El Alia, Bab Ezzouar, Alger, Algérie.

This paper deals with the study of the nonlinear inverse bremsstrahlung absorption (IBA) in the case of inhomogeneous hot plasma, especially laser-produced plasma. The plasma is characterized by its electrons distribution function, given by the Fokker-Planck (FP) equation [1,2,3] in the presence of a high frequency (hf) laser wave. The laser wave is described by a nonlinear wave equation. The equation of the \( l^{th} \) component, \( f_l \), of electronic distribution function, \( f \), expended on the spherical harmonics and a nonlinear wave equation have been derived. These results are applied to establish the nonlinear IBA as function of the scaling parameter, \( Zv_0^2(x)/\nu^2 \).

Distribution function

Considering an inhomogeneous unmagnetized plasma in the presence of a (hf) laser wave, spreading into the direction \( x \) of the plasma inhomogeneity. Following the Braginski notations, the electrons Fokker-Planck (FP) equation can be represented as:

\[
\frac{\partial f}{\partial t} + \nu (x) \frac{\partial f}{\partial x} - e \hat{E}_h \frac{\partial f}{\partial \nu} = C_{ei}(f) + C_{ee}(f),
\]

where \( \hat{E}_h = \text{Re}(\hat{E}_0(x) \exp(\text{i} \omega t)) \) denotes the (hf) laser electric field and \( E_0(x) \) is its magnitude. \( C_{ei}(f) \) and \( C_{ee}(f) \) are the (e-i) and the (e-e) collision operators respectively [1,2],

\[
C_{ei} = \alpha \frac{\partial}{\partial \nu} \left( \frac{v^2 I - \nu^2}{v^2} \right) \frac{\partial f}{\partial \nu},
\]

where: \( \alpha = (Z_n e^4)/(8 \pi \varepsilon_0 m_e) \ln \Lambda \), and \( \ln \Lambda \) is the Coulomb logarithm.
In order to take into account the (hf) response of plasma electrons to the laser field excitation. We assume that \( f \) contains a slow frequency (sf) part, \( f^{(s)} \), and a (hf) part, \( \text{Re}(f^{(h)} \exp(i \omega t)) \), which oscillates at the laser frequency, \( \omega \), so: \( f(v,t) = f^{(s)}(v,t) + \text{Re}(f^{(h)}(v,t)) \)

Substituting this expression in equation (1) and separating the time scale orders. Then a system of two coupled equations is obtained, one is of the \( f^{(s)} \), obtained by the averaging of eq.(1) on the laser cycle time and the other is of \( f^{(h)} = \exp(i \omega t) \).

Considering a circular-polarized laser wave, as \( \vec{E} \) oscillates in the \( (y-z) \) plane, perpendicularly to the plasma density gradient direction, \( \vec{v}_n(x) \):
\[
\vec{E}_h(x,t) = \text{Re}(\vec{E}_o(x) / \sqrt{2}) \exp(i \omega t)(\vec{u}_y + i \vec{u}_z). \]

This choice of a circular polarization facilitates the analytical treatment of FP equation without effecting the physical aspect of the problem treated in this paper.

After some algebra, using recurrence relations between the Legendre polynomials, \( P_l(\mu) \) and the associated Legendre polynomials, \( P_l^\prime(\mu) \), of orders \( l-1, l \) and \( l+1 \), the equation of the \( l^{th} \) anisotropy (sf) distribution function is obtained as follows:
\[
\frac{\partial f_l^{(s)}}{\partial t} + \frac{l}{2l-1} \frac{\partial f_l^{(s)}}{\partial x} - \frac{\alpha}{4} \vec{I}(x) v_t^2 \times \left\{ -\frac{l^2(l-1)}{2l-1} \frac{\partial}{\partial v} \left[ \frac{l-1}{2l-3} \frac{1}{v} \frac{\partial}{\partial v} \left( f_{l+2}^{(s)} \right) \right] - \frac{1}{2l+1} \frac{1}{v^{l+3}} \frac{\partial}{\partial v} \left( v^{l+1} f_l^{(s)} \right) \right\} + \frac{1}{2l+3} \frac{1}{v^{l+1}} \frac{\partial}{\partial v} \left[ \frac{1}{2l+1} \frac{1}{v} \frac{\partial}{\partial v} \left( f_l^{(s)} \right) \right] = -\frac{\alpha}{v^3} f_l^{(s)} + C_{nl}(f_l^{(s)}), \tag{2}
\]

where \( \vec{I}(x) = v_0^2(x) / v_t^2 \) is a normalized laser intensity, where \( v_0(x) = e|E_o(x)| / m_e \omega \) is the peak oscillating electron velocity in the (hf) laser electric field.

The equation (2) gives more information compared to (eq. 5) of reference [1] by including the anisotropies distribution function \( f_{nl} \), \( (l>0) \). Furthermore, it gives more information than eq. (23) of reference [3] by including the transport term \( \frac{\partial f}{\partial x} \), which takes into account both the plasma and field inhomogeneity.

The solution of equation (2), for \( l = 0 \), has been simulated as a function:
\[
f_0 = \exp(-b(\frac{v^2}{2v_t^2})^{\beta/2}), \]

which takes into account the Langdon’ operator term, \( \sim \vec{I} \), and the
(e-e) collisions\textsuperscript{1,2}, where $\beta = 2 + 3/(1 + 1.66 (Z \bar{t})^{-0.72})$ is a laser-plasma parameter depends to the ion charge number, $Z$, and to the normalized laser intensity, $\bar{t}$.

The calculation of the electrons density and thermal velocity, using this solution, $n_e = \int f_0 \, d^3\nu$ and $n_e v_t^2 = \int v^2 f_0 \, d^3\nu$, leads to express $b$ and the normalization coefficient of $f_0$ as functions of the parameter $\beta$, so:

$$f_0^{(s)} = \frac{\beta}{4 \pi \Gamma(\frac{3}{\beta})} \left[ \frac{1}{3} \frac{\Gamma(\frac{5}{\beta})}{\Gamma(\frac{3}{\beta})} \right]^3 \frac{n_e(x)}{v_t^3} \exp\left( -b \frac{v^2}{2v_t^2} \right)^{\beta/2},$$

(3)

where $b = \left[ \frac{2}{3} \frac{\Gamma(\frac{5}{\beta})}{\Gamma(\frac{3}{\beta})} \right]^{\beta}$ and $\Gamma$ is the Euler function.

We denote that the expression (3) gives the solution of the equation (2), for $l = 0$, with a good precision, less then 2%. The numerical analysis of eq. (3) shows that the distribution becomes independent of the $Z \bar{t}$ for sufficiently large values of $Z \bar{t}$ ($Z \bar{t} \geq 4$). Which corresponds to the saturation of the plasma corona in the relatively hot electrons. It has been shown that the distortion of $f_0$ is important as $Z \bar{t}$ increases. Where the number of relatively hot electrons, having high values of $Y = v^2 / 2v_t^2$, increases; however the number of relatively cold electrons, having low values of $Y$, decreases. In the linear case where $Z \bar{t} \approx 0$, $f_0$ corresponds to the Maxwell’s function, $f_0 \sim \exp( -v^2 / 2v_t^2 )$. Though, for large values of $Z \bar{t}$ ($Z \bar{t} > 4$), the plasma becomes saturated in hot electrons; where the electrons distribution is described by $f_0$, mostly distorted.

**Nonlinear wave equation**

The determination of scaling parameter, $Z \bar{t}$, requires a non linear wave equation taking into account the NLBA.

The investigation is started from local Maxwell’s equations. After some algebra the nonlinear wave equation is obtained as follow:

$$\frac{\partial^2 E_0 (x)}{\partial x^2} + \frac{\omega^2}{c^2} \left( 1 - \frac{\omega_p^2(x)}{\omega^2} \right) E_0 (x) = -i \frac{\omega_p^2(x)}{c^2} \frac{v_t(x, \bar{t})}{\omega} E_0 (x).$$

(4)

The right hand side of eq. (4) is a nonlinear attenuation term by $IBA, \sim v_t(x, \bar{t}(x))$, where
\( \nu_c(x, \tilde{T}(x)) \) denotes the nonlinear (e-i) collision frequency. We obtained its expression by averaging the dependent velocity (e-i) collision frequency, \( \nu_c(v, x) = \alpha / v^3 \), using the distribution given by eq. (3), \( \nu_c(x, \tilde{T}(x)) = 4\pi \int_0^\infty f_0 \cdot \alpha / v^3 \cdot dv \), so:

\[
\nu_c(x, \tilde{T}(x)) = \frac{n_e(x)e^4Zm_e^2v_t^4}{24\pi^6e_n^2v_t^4} \cdot \frac{\beta^2}{\Gamma(\frac{5}{\beta})} \cdot \left[ \frac{1}{\Gamma(\frac{3}{\beta})} \right]^3 \ln \Lambda. \tag{5}
\]

The above expression (5) gives the nonlinear collision frequency in the plasma section determined by the \( x \) abscise as function of \( x \) and of the laser-plasma parameter, \( \beta \).

**Inverse bremsstrahlung absorption**

The results of the two above sections are enough to study the nonlinear IBA in the laser-created plasma corona. In the classical limit of a hot plasma (\( \hbar \omega \ll T_e \)), where there is a good agreement between the quantum and the classical description of laser-plasma interaction, the absorption, \( A = \langle \tilde{E}, \tilde{J} \rangle \), can be expressed as:

\[
A = \frac{4\pi n_e m_e v_t \tilde{T}(x)}{3} \cdot \frac{\beta}{\Gamma(\frac{3}{\beta})} \cdot \left[ \frac{1}{\Gamma(\frac{5}{\beta})} \right]^3 \cdot \frac{\partial f_0}{\partial v} dv, \tag{eq. 4 of ref. 1}
\]

Using the \( f_0 \) given by eq. (3), we then obtain:

\[
A = \frac{2n_i}{3} m_e \tilde{T}(x) \cdot \frac{\beta}{\Gamma(\frac{3}{\beta})} \cdot \left[ \frac{1}{\Gamma(\frac{5}{\beta})} \right]^3. \tag{6}
\]

It is reduced compared to the linear absorption, characterized by the Maxwell’s distribution, \( f_M \). The reduction can be interpreted as the flatness of the isotropic distribution function, \( f_0 \), by the non-linear effect compared to \( f_M \), such the plasma, heated by laser field, became depleted in slow electrons. The rate of this reduction is given by:

\[
R_{IB}(\beta) = 1 - f_0(\beta, v = 0)/f_M(0) \]

Numerical analysis of this eq. (6), shows that IBA is reduced compared to the linear case. For example the reduction rate is about 32% for \( Z\tilde{T} = 0.4 \).

**References**