Screening of Dust Grains in a Weakly Ionized Gas. 
Effets of Charging by Plasma Currents

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The aim of this work is to study the screening of a dust grain charged by plasma currents in a weakly ionized high pressure gas. In contrast to the systems with a fixed grain charge (e.g., charged colloids), the charge of a dust grain immersed in a plasma background is maintained by ion and electron currents to the grain surface. This makes the conventional thermodynamically equilibrium approaches, such as Poisson-Boltzmann or linear Debye-Hückel theory, inapplicable.

The problem of grainscreening in dusty plasmas (DP) with allowance for charging by plasma currents attracted considerable interest of researchers in recent years. In most of the literature, the collisionless plasma background is considered, which is typical for laboratory experiments on DP and astrophysical observations. The opposite limit of strongly collisional background is much less examined. In contrast to the work [1], were a complicated semirealistic system is considered, we are going to examine the simplest case of a single grain with the emphasis on the basic features of this problem.

Thus, we consider a single spherical grain of a radius \( a \) imbedded in a weakly ionized high pressure gas. In this case, it is natural to use the drift-diffusion (DD) approach. Assuming two types of plasma particles (ions and electrons) only, we write the general time-dependent equations for the unknown ion/electron densities \( n_{i,e} \) and self-consistent potential \( \phi \) in the form

\[
\frac{\partial n_{i,e}}{\partial t} = -\text{div}\, \mathbf{j}_{i,e} + I_0 - \alpha n_i n_e, \quad (1)
\]

\[
\Delta \phi = -4\pi e(n_i - n_e). \quad (2)
\]

Here \( e \) is the absolute value of the electron charge, \( \alpha \) is the coefficient of recombination, \( I_0 \) is the intensity of plasma ionization (we examine the case of uniformly distributed plasma sources). The expression for the current densities \( \mathbf{j}_{i,e} \) has the form

\[
\mathbf{j}_{i,e} = -\mu_{i,e} n_{i,e} \nabla \phi - D_{i,e} \nabla n_{i,e},
\]

where \( \mu_{i,e} \) and \( D_{i,e} \) are the ionic/electronic mobility and diffusivity, respectively. These latter are assumed to be related due to the Einstein’s equation \( \mu_{i,e} = z_{i,e} e c_{i,e} D_{i,e}/k_B T \) (here \( z_{i,e} = \pm 1 \) is the ion/electron charge number). We consider below only the case that \( T_i = T_e = T \). The grain charge emerges as a result of plasma currents due to the difference in electron and ion diffusivities. With regard for spherical symmetry, the relevant equation for the grain charge number \( Z = Q_{\text{grain}}/e \) reads

\[
\frac{dZ}{dt} = -4\pi a^2(j_{r,i} - j_{r,e}), \quad (3)
\]
where the subscript \((r)\) denotes the radial component of a current.

In order to formulate the BC, we admit that the system is confined in a spherical volume of sufficiently large radius \(R \approx 50 - 500r_D\) (where \(r_D\) is the Debye screening length) with the grain placed at the center. The boundary conditions (BC) are specified at the surface of this sphere and at the surface of the grain. In our simulations, we consider the two basic cases and two types of BC, respectively. In the first case (I), the sources of plasma ionization, which compensate the loss of plasma particles due to the absorption on the grain surface, are assumed to be far from the grain (outside the spherical volume). The action of these sources is modeled by maintaining constant electron and ion densities on the surface of the sphere, \(n_i = n_e = n_0\). According to this, we write the BC for the densities \(n_{i,e}\)

\[ n_{i,e} = n_0 \quad \text{at} \quad r = R, \]

and assume the rates of plasma ionization and recombination over the volume \(I_0\) and \(\alpha\) to be equal to zero. In the second case (II), we examine the problem with uniformly distributed plasma sources \((I_0 \neq 0)\) with allowance for the plasma recombination over the volume \((\alpha \neq 0)\). Note, that in this case the quantities \(I_0\) and \(\alpha\) are related to the unperturbed bulk plasma density \(n_0\) by the equation \(I_0 = \alpha n_0^2\) valid in the absence of the grain. The relevant BC read

\[ \frac{\partial n_e}{\partial r} = \frac{\partial n_i}{\partial r} = 0 \quad \text{at} \quad r = R, \]

The BC for the potential at the grain surface have the form

\[ \frac{\partial \phi}{\partial r} = -\frac{Z(t)e}{a^2} \quad \text{at} \quad r = a, \]

and for the densities \(n_{i,e}\) we use the BC \(n_{i,e} = 0 \quad \text{at} \quad r = a\) appropriate for the case of strongly collisional background.

We solved the above system of equations by means of the method of lines and Gear’s method. Also, we performed a limited number of Brownian dynamics (BD) simulations based on particle-in-cell (PIC) method [2] with spherically symmetric concentric cells and the BC corresponding to the case (I). In these simulations, the plasma background is modeled by finite numbers of particles of two sorts representing the ion and electron components. The dynamics of the system is governed by the reduced Langevin equations of overdamped motion

\[ h \frac{d\mathbf{x}_k}{dt} = -\nabla_k U + \mathbf{F}_k(t). \]

Here \(\mathbf{x}_k\) is the radius vector of the \(k\)-th particle, and \(U\) is the potential energy of the configuration. The friction coefficient \(h\) and the random force \(\mathbf{F}_k(t)\) are determined by the properties of the heatbath (in our case the role of the heatbath plays the high
Fig. 1. Comparison of charge distributions for different types of BC for the same stationary bulk plasma parameters. Left: Relative charge distributions for the Debye length $r_D/a = 10$ for (1) and (1a), and $r_D/a = 2$ for (2) and (2a). Dashed and solid lines relate to the cases (I) and (II), respectively. Right: Relative charge distributions for BC (II) as dependent on the ionization rates at fixed bulk plasma density. The dimensionless intensity of plasma sources over the volume $i_0 = I_0 e^5 / D_i$ is (1) $1.25 \cdot 10^{-2}$; (2) $2.5 \cdot 10^{-3}$; (3) $5 \cdot 10^{-4}$; (4) $10^{-4}$. The bold line relates to the linear Debye-Hückel theory; dashed line is DD approach for BC (I). The bulk plasma parameters are fixed; the grain radius $a/r_D$ is 0.158.

pressure neutral gas). A detailed presentation of the issues concerning BD and its relation to the continuous probabilistic approaches, such as Fokker-Planck and Smoluchovsky equations, can be found in Ref [3]. Note that the overdamped BD represents the direct microscopic analogue to the DD approach, since the latter can be derived from the Smoluchovsky equations for one-particle distributions (i.e., within the additional mean field approximation).

The equations (1-3) were solved numerically, with regard for the spherical symmetry, for the range of parameters characteristic of the DP experiments in high pressure weakly ionized noble gases like Ne or Ar. The typical values are: plasma background coupling $\Gamma \simeq 10^{-3}$; plasma density $n_0 \simeq 10^{10} \text{ cm}^{-3}$; the density of the neutrals $n \simeq 10^{18} \text{ cm}^{-3}$; radius of the grain $a \simeq 10^{-3} \text{ cm}$; electron-ion recombination coefficient $\alpha \simeq 10^{-7} \text{ cm}^3/\text{sec}$; the ratio of the Debye length to the grain radius $r_D/a \simeq 0.1-50$. The ratio of diffusivities in all computations was fixed, $A = D_e / D_i = 10^3$ (with exception for the BD simulations).

In the problem of grain screening in spherically symmetric case, it is convenient to introduce the charge distribution function $Q(r)$ defined as the total charge residing within a sphere of a radius $r$. Note that in the case of the unscreened Coulomb field this function is constant due to the Ostrogradsky-Gauss theorem.

The results of simulations are given in the figures. The computations evidence that the account of grain charging results in a distinct qualitative change in the asymptotical behavior of the screened field as compared to the thermodynamically equilibrium Debye-
Hückel theory for a grain with constant charge. In the case that the plasma sources are placed at infinity (case I), we observe at long distances the Coulomb field with a certain effective charge (Fig. 1). The effect of screening manifests itself in the decrease of this effective charge as compared to the stationary grain charge. The smaller the ratio of the Debye length to the grain size, the smaller effective charge is observed. In the case that the plasma sources are distributed uniformly over the volume (case II), there exist a finite screening length depending on the rate of ionization. Typically, this screening length in the presence of plasma currents and strong collisions considerably exceeds the Debye radius. The stationary grain charge as well as the field within the sheath around the grain (\(\approx 10r_D\)) does not depend on the type of BC and the ionization rate provided that this latter is relatively low. At higher ionization rates, the properties of screening approach the predictions of Debye-Hückel theory. The tests based on the microscopic Brownian dynamics simulations correlate well with the continuous drift-diffusion approach (Fig2). The microscopicity of the plasma background results in some decrease in the steady charge acquired by the grain.

This work was supported by the State Fund for Fundamental Research of Ukraine, Project 02.07/00049.

References