Ion-acoustic waves in a multicomponent complex plasma with negative ions

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Negative ions and negatively charged micro- to nano-meter sized dust grains are ubiquitous in astrophysical as well as industrial processing plasmas [1,2]. The presence of dusts and negative ions can affect the characteristics of most collective processes of the plasma since the charge balance in both the steady and dynamic states can be significantly altered. In the negative-ion free plasmas, the effect of variation of the dust charge on electron plasma and ion acoustic waves in a low-temperature plasma was investigated [3,4] with self-consistent particle balance taken into consideration. It was shown that dust-charge relaxation is significantly affected by ionization and recombination. These processes self-consistently maintain the equilibrium state of the discharge as well as the particle number densities during the perturbations. It was also shown that dissipation due to inter-plasma particle collisions as well as elastic Coulomb and the inelastic dust-charging collisions can lead to strong damping of the waves in a typical laboratory plasma [3,4]. When the plasma contains negative ions, the magnitudes of many characteristic parameters, such as the plasma frequencies can become quite different from that of a normal dusty plasma. These modifications alone can in certain situations already radically affect the properties of the natural modes, the near-wall sheaths, as well as the dust charging process [5]. Here, we consider wave propagation in a complex dusty plasma with negative ions taking into account self-consistently ionization, electron attachment, diffusion, positive-negative ion recombination, plasma particle collisions, as well as elastic Coulomb and inelastic dust-charging collisions. It is found that the equilibrium of the plasma as well as the propagation of ion waves are modified by these effects.

Consider linear wave propagation in a multi-component weakly ionized plasma (1D planar geometry) with finite effective electron (Tₑ), ion (Tᵢ), and negative-ion (T₋) temperatures. The charge of a dust grain varies because of the microscopic electron and ion currents flowing into it according to the potential difference between its surface
and the adjacent plasma. For simplicity, all heavy particle collisions, except that for \( n_e - n_- \) recombination (since it is the major sink of negative ions), are neglected. The negative ions are assumed to be due to electron attachment, although other processes can also be easily included. Accordingly, the conservation equations are

\[
\partial_t n_j + \nabla \cdot (n_j v_j) = S_j, \tag{1}
\]

\[
\partial_t v_j + \nu_{\text{eff}}^j v_j + \frac{T_j}{m_j} \partial_z n_j = \frac{q_j}{m_j} E, \tag{2}
\]

\[
dt q_d + \nu_d^{ch} q_d = -|I_e| \bar{n}_e/n_{e0} + |I_d| \bar{n}_i/n_{d0} - |I_-| \bar{n}_-/n_{-0}, \tag{3}
\]

\[
\nabla^2 \varphi = -4\pi e(n_i - n_- - n_e - |Z_d| n_d), \tag{4}
\]

where \( E = -\nabla \varphi \) is the electric field of the electrostatic waves, \( \varphi \) is the electrostatic potential, \( m_j, n_j = n_{j0} + \bar{n}_j \), and \( v_j \) are the mass, density, and fluid velocity of the plasma species \( j = e, i, - \) for electron, ion, and negative ion, respectively. The mass, density, and average charge of the dust particles are \( m_d, n_d, \) and \( q_d = -|Z_d| e = q_{d0} + \bar{q}_d \), respectively. The effective dust charging rate \( \nu_d^{ch} \), collision rates, and microscopic currents of the electrons and positive and negative ions at a dust grain \( |I_j| \) are standard and can be found elsewhere [3-5].

The source terms \( S_{(e,i,-)} \) for the electrons, and the positive and negative ions are

\[
S_e = \nu_{\text{ion}} n_e - \nu_{\text{ed}} n_e - \nu_{\text{att}} n_e - \nu_{\text{wall}}^e n_e, \tag{5}
\]

\[
S_i = \nu_{\text{ion}} n_i - \nu_{\text{id}} n_i - \nu_{\text{rec}} n_i n_- - \nu_{\text{wall}}^i n_i, \tag{6}
\]

\[
S_- = \nu_{\text{att}} n_e - \nu_{\text{d}} n_- - \nu_{\text{rec}} n_i n_- - \nu_{\text{wall}}^- n_-, \tag{7}
\]

where \( \nu_{\text{ion}} \) is the rate of electron impact ionization of the neutral particles, \( \nu_{(e,i,-)d} \) are the rates of collection of plasma species by the dusts, \( \nu_{\text{att}} \) is the rate of the electron attachment to the neutrals resulting in negative ion production, and \( \nu_{\text{rec}} \) is the rate of recombination of the positive and negative ions in the plasma bulk. The last terms in Eqs. (5) – (7) represent particle loss at the walls of the discharge, namely \( \nu_{\text{wall}}^e \propto S_{\text{surf}} V_T e n_e S \exp(-\phi_{\text{wall}}/T_e), \nu_{\text{wall}}^i \propto S_{\text{surf}} V_T - n_e S \exp(-\phi_{\text{wall}}/T_0), \) and \( \nu_{\text{wall}}^- \propto S_{\text{surf}} V_B n_i S. \)

Here, \( S_{\text{surf}} \) is the effective surface area of particle loss at the walls, \( V_B, V_T, \) and \( V_T^- \) are the Bohm, electron, and negative-ion thermal velocities, respectively. Furthermore, \( \phi_{\text{wall}} \) is the potential difference between the wall and the plasma bulk, and \( n_{(e,i,-)S} \) are the densities of the plasma species near the wall.

We shall first obtain the self-consistent equilibrium state. In equilibrium, the system is charge neutral, so that \( n_{i0} = n_{e0} + n_{-0} + |Z_{d0}| n_{d0} \). The stationary dust charge is obtained by setting equal the lowest order microscopic electron and ion currents flowing onto the dust particles, or \( I_{i0} = |I_{e0}| + |I_{-0}| \). The equilibrium number densities of the
plasma particles can be obtained from \( S_{j0} = 0 \). From \( S_e = 0 \), one finds that the stationary state can exist only if the creation of electrons due to ionization of neutrals balances or exceeds their loss due to collection by the dusts, negative ion formation due to electron attachment, and flow to the chamber walls. That is, the condition \( \nu_{\text{ion}} > \nu_{\text{ed}} + \nu_{\text{att}} + \nu_{\text{wall}}^e \) is a prerequisite for the existence of a stationary state of the 1D electro-negative dust-contaminated discharge.

In addition, for low-pressure diffusion equilibrium, the flux of positive ions to the discharge walls is to be balanced by the electron and negative ion fluxes, or

\[
\nu_{\text{wall}}^e n_{e0} + \nu_{\text{wall}}^e n_{e0} = \nu_{\text{wall}}^e n_{e0},
\]

where we have assumed that the plasma particle densities are spatially uniform. From the above equations it follows that

\[
\nu_{\text{ion}} > \nu_{\text{ed}} + \nu_{\text{att}} + \nu_{\text{wall}}^e \]

which is the basic relation between the equilibrium densities of the plasma particles and the rates of electron and ion collection by the grains.

Using the overall charge neutrality condition, one can calculate the stationary values of the number densities of the electrons

\[
n_{e0} = n_{i0} \frac{\nu_{\text{ed}}/\nu_{\text{ed}}}{1 - \nu_{\text{ed}}/\nu_{\text{ed}}} \left[ 1 - \frac{\nu_{\text{ed}}}{\nu_{\text{ed}}} (1 - \kappa_0) \right],
\]

and negative ions

\[
n_{-0} = \frac{\nu_{\text{ed}}}{1 - \nu_{\text{ed}}/\nu_{\text{ed}}} \left[ 1 - \frac{\nu_{\text{ed}}}{\nu_{\text{ed}}} - \kappa_0 \right],
\]

where \( \kappa_0 = |Z_{d0}| n_{d0}/n_{i0} \) denotes the proportion of negative charge residing on the dust particles. The expressions (8) and (9) are also consistent with the case when the negatively charged ions are absent. Indeed, in that case we have \( n_{-0} = 0 \) and \( n_{e0} = n_{i0} \nu_{\text{ed}}/\nu_{\text{ed}} \) [4]. We also obtain

\[
n_{i0} = \frac{\nu_{\text{ion}} \left( \nu_{\text{ed}} - \nu_{\text{ed}}(1 - \kappa_0) \right) - (\nu_{\text{ed}} - \nu_{\text{wall}}^i)(\nu_{\text{ed}} - \nu_{\text{ed}})}{\nu_{\text{rec}} \nu_{\text{ed}}},
\]

for the equilibrium value of the ion number density in 1D dust-contaminated electro-negative discharge in the low-pressure diffusion equilibrium regime. Eq. (10) imposes the following restriction on the minimum ionization rate

\[
\nu_{\text{ion}} > \nu_{\text{ion}}^{\text{min}} = (\nu_{\text{ed}} - \nu_{\text{wall}}^i)(\nu_{\text{ed}} - \nu_{\text{ed}})/[\nu_{\text{ed}} - \nu_{\text{ed}}(1 - \kappa_0)]
\]

for the existence of the discharge in question. Otherwise, the rate of new particle production will not be sufficient to compensate the losses at the discharge walls and to the dust grains.

Here, we are interested in waves on the ion time scale. Assuming that perturbations are of the form \( \exp[i(kz - \omega t)] \), where \( k \) is the wavenumber, we linearize the basic equations, and assuming \( k^2 V_T^2 \gg [\omega^2, \omega \nu_{\text{eff}}, \omega(\nu_{\text{ion}} - \nu_{\text{ed}} - \nu_{\text{att}} - \nu_{\text{wall}}^e)], \omega \sim \nu_{\text{diss}}, \) where
\( \nu_i^{\text{diss}} \) is any of the ion dissipation terms, and \( \omega^2 \gg k^2 V_{Ti}^2 \) [4], where \( V_{Ti} \) is the positive ion thermal velocity, we obtain

\[
D_{\text{ch}}(\omega)D_{\text{IAW}}(\omega, k) = i \beta_{\text{coupl}}(\omega),
\] (11)

where \( D_{\text{IAW}} = -\left(\omega_{pe}^2/kV_{Te}\right)^2 + \omega_{pi}^2/w_{\text{eff}}^2 w_i^* + \omega_{pi}^2/w_{\text{eff}} w_i^* \), \( \beta_{\text{coupl}}(\omega) = -\tilde{\nu}_{\text{ch}}(\omega_{pe}/kV_{Te})^2 + \tilde{\nu}_{\text{d}}(\omega_{pi}/kV_{Te})^2 \), \( \omega_{\text{eff}} = \omega + i \nu_{\text{eff}} \), \( \nu_i^* = \nu_{\text{id}} + \nu_{\text{rec}n_0} + \nu_{\text{wall}} \), and \( \nu_i^* = \nu_{-d} + \nu_{\text{rec}n_0} + \nu_{\text{wall}} \).

Eq. (11) couples the ion acoustic waves given by the dispersion \( D_{\text{IAW}}(\omega, k) = 0 \) with the dust charge relaxation mode given by the relation \( D_{\text{ch}}(\omega) = 0 \). In the derivation of the dispersion relation (11), recombination effects have been neglected. A detailed analysis of Eq. (11) shows that the coupling between the ion acoustic and the charge relaxation modes leads to the down-shift of the ion wave frequency and an increase of the damping decrement. This is a typical consequence of the coexistence of ion acoustic waves and dust charge relaxation in dust-contaminated plasmas [4]. The dust charge relaxation rate diminishes because of coupling to the ion acoustic waves and can be attributed to additional dissipative wave energy transfer to the purely damped dust charge relaxation mode followed by the self-organization of the complex plasma system to minimize the dissipative loss.

The dispersion relation (11) has also been investigated numerically for real wave frequency and complex wave number, and three different regimes of dissipative loss (low, intermediate, and strong). The computations were carried out for the representative parameters of 10\% C\textsubscript{4}F\textsubscript{8} - 90\% Ar fluorocarbon plasmas often used for ultra-fine and highly selective etching of poly-silicons. One can show that in the absence of all dissipative effects, the wave number is real definite and the frequency approaches asymptotically to the upper limiting frequency \( \omega_{\text{IAW}}^{\text{lim}} \). The low-dissipation case also reflects this tendency. We have shown that the effect of the negative ions is to increase the upper limit of the ion acoustic wave frequency (as compared to \( \omega_{\text{pi}} \) for simple electron-ion plasmas).

**Acknowledgments.** Work of SVV was partially supported by the Max-Planck Society (Germany) and the Australian Research Council.

**References**