On Cyclotron Emission from Toroidal Plasmas near the ECRH Frequency

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1. Introduction. Modification of an electron distribution function (EDF) due to interaction with strong microwave radiation under ECRH or ECCD conditions is well pronounced for high-field side (HFS) injection or oblique launch (typical ECCD scheme), when rf power is deposited to energetic electron populations. In ECRH scheme with nearly transverse low-field side (LFS) injection and optically thick plasma, heating radiation is absorbed presumably by low energetic electrons, therefore ECRH-driven perturbations in the EDF are strongly suppressed by Coulomb collisions and no substantial tail of energetic electrons is expected. Under these circumstances modification of the EDF may only result in some increase of EC emission (ECE) level with respect to its thermal value in a close vicinity of the heating frequency or its harmonics. A careful study of ECE spectrum near the heating frequency may provide an evidence in favor of the distorted shape of the EDF in the low energy range.

A number of papers have been devoted to modification of ECE spectra due to a distorted bulk in EDF [1-4]. Theoretical models implemented in these works were based on Fokker-Planck simulations of the EDF, which provided the modified non-thermal absorption and emissivity profiles used for the ECE calculation. However, only the results of numerical calculations have been reported for the emission of a distorted bulk unlike, e.g., the case of emission of energetic tails for which various analytical solutions are available. The aim of the present work is to propose a simplified (one-dimensional) analytical model, which inherits the main physics of the "complete" Fokker-Planck model. Use of the simplified model gives an opportunity to perform a qualitative analysis. The obtained results may be applied for testing of sophisticated quasi-linear Fokker-Planck/ECE codes.

2. ECRH modeling. We start with the standard Fokker-Planck kinetic equation, which on each magnetic surface describe evolution of the bounce-averaged EDF in two-dimensional momentum space under the ECR conditions. General form of such equation including the quasi-linear diffusion and Coulomb collision operators may be found, e.g., in [1].

This equation can be simplified under the following assumptions. (I) A steady-state plasma with the fixed injected rf power is investigated with temperature profile established under combined action of power deposition and transport processes. In this case the linear collision term may be used with the fixed "background" Maxwellian electron distribution. (II) Quasi-transverse propagation of the heating radiation with respect to the magnetic field is considered, when Doppler term may be neglected and the ECR condition at the s-th harmonics is defined by relativistic dependence of electron mass: \( \omega_r(l, \nu) = s \omega_{ce} / \gamma \), where
ω_{ce}(l) is the cyclotron frequency varying along the heating beam direction, and γ is the relativistic factor. As a result of the relativistic effect, the resonance position of high-energy electrons is shifted towards to the HFS as compared to the position for low-energy electrons. (III) Bounce averaging results in effective broadening of the heating radiation spectrum. Following [4, 5] we use a slab model in which the EDF is constant on the magnetic surfaces, but the heating radiation is launched with a noise spectrum in a finite frequency band defined by variation of the cyclotron frequency within the rf beam aperture. (IV) The isotropic EDF in a velocity space can be considered if the region of resonant velocities is narrow [6]. Under these assumptions the following kinetic equation may be obtained for the distribution function \( f_e(l, v) \) over velocity modulus \( v \) and co-ordinate \( l \) along the heating ray direction:

\[
\langle D_{ql} \rangle \frac{\partial f_e}{\partial \omega} + \langle D_c \rangle \left( \frac{\partial f_e}{\partial \omega} + \frac{2v}{v_c^2} f_e \right) = 0,
\]

where \( \langle D_{ql} \rangle \) and \( \langle D_c \rangle \) represent the quasi-linear and collisional diffusion coefficients averaged over the pitch-angle. The quasi-linear diffusion coefficient is defined by the spectral density of the heating wave energy \( W_\omega \), which is governed by the radiation transfer equation with the absorption coefficient \( \mu_\omega \) calculated with taking into account the quasi-linear modification of the EDF. Using the formal solution of this equation \( W_\omega = W_\omega^0 \exp(-\tau) \), where \( \tau \) is the optical depth corresponding to the given ray co-ordinate, one can obtain that

\[
\langle D_{ql} \rangle = D_{ql}^* \cdot \left\{ W_\omega^0 \exp(-\tau) \right\}_{v_0=v_r}, \quad \tau = \int_{-\infty}^{l} \mu_\omega \, dl, \quad \mu_\omega = 4\pi \left( \frac{m_e c}{\omega_0^2} D_{ql}^* \frac{\partial f_e}{\partial \omega} \right)_{v_0=v_r}.
\]

Here \( D_{ql}^* \) is some velocity dependent factor, and \( v_r \) is the resonant velocity defined by the condition \( \omega_r(l, v_r) = \omega \). The coefficient of collisional diffusion is presented, e.g., in [7].

For the case under investigation, ECE is modified mainly due to quasi-linear plateau formation in resonant region and resulting degradation of re-absorption, rather than due to modification of spontaneous emissivity: the emission level increases around the resonant frequencies due to ECRH-driven transparency window. This means that absolute number of the emitting electrons is practically the same as for the Maxwellian distribution \( f_{eM} \), while the slope of the EDF (which defines the absorption coefficient) may be changed significantly in the resonant velocity region. The value of this derivative may be estimated substituting \( f_e \rightarrow f_{eM} \) in the last term of Eq. (1). Finally, this results in the following ordinary differential equation for the optical depth:

\[
\frac{d\tau}{dl} \approx \left( 1 + W_\omega^0 \left\{ D_{ql}^*/\langle D_c \rangle \right\}_{v_0=v_r} \cdot \exp(-\tau) \right)^{-1} \mu_{\omega, \text{Maxw}}\, \mu_{\omega, \text{Maxw}}, \quad \tau(-\infty) = 0,
\]
where $\mu_{\text{Maxw}}^{\text{iso}}$ is the absorption coefficient in a Maxwellian plasma. Two parameters govern the solution of this equation: the ratio of averaged quasi-linear diffusion to thermal collision rates $Q \propto P_{\text{inj}} T_e^{3/2} / N_e$ and the total optical depth $\tau_0$ of the Maxwellian plasma layer. Explicit expressions for these quantities may be found in [4, 6] for specific cases of O1- and X2-mode propagation. Equation obtained allows estimation of the self-consistent ECRH power deposition profile together with the spatial variation of the EDF perturbations. A further step will be done in the next section where the characteristic ECE level is calculated for the obtained EDF over velocity and space co-ordinates.

3. ECE modeling. A particular case is considered below of emission at the same harmonic, polarization and ray trajectory as the heating radiation. In this case, at each point along the ray the resonance condition is the same for the ECRH and registered ECE beams, and the same absorption coefficient corresponds to the both beams. Hence, the main deviations from the thermal ECE level are located in the frequency range of the effective heating spectrum.

Within the accuracy of Eq. (4) non-thermal modification of spontaneous emissivity can be neglected, thus the emissivity is defined by Kirchhoff’s law. The effective radiation temperature [1] of the emitted radiation may be calculated from the radiation transfer equation as

$$\frac{dT_r}{dl} = T_e \mu_{\text{Maxw}}^{\text{iso}} - T_r \frac{d\tau}{dl} \Rightarrow \left[ \frac{T_r^+}{T_r^-} \right] = T_e \int_{-\infty}^{\infty} \mu_{\text{Maxw}}^{\text{iso}}(l) \exp \left\{ \frac{-\tau(l) - \tau(x)}{-\tau(l)} \right\} dl. \quad (4)$$

The upper (bottom) line corresponds to the emission in the direction of the heating beam propagation (in the opposite direction). Typical dependences of the disturbed ECE level on the thermal optical depth are shown in Fig. 1 for the emission registered from low- and high-field side in both cases of the low- and high-field side launch of the heating radiation.

The maximum limit for the effective radiation temperature increase may be estimated assuming that in the whole plasma layer resonant re-absorption is zero due to quasi-linear plateau formation. This results in the equation, which is usually obtained for optically thin layer of thermal plasma: $T_r^\pm \approx T_e \tau_0$. However, this equation is valid for any value of the non-perturbed optical depth $\tau_0$ until the heating power is so high that collisions cannot prevent the quasi-linear plateau formation. Thus, the non-thermal component in ECE spectra may be essential if the non-perturbed optical depth is high enough.
Let us concentrate on ECRH with the LFS launch. For relatively small values of unper-
turbed optical depth $\tau_0$ the ECE level is linearly increasing: $T^\pm_r \approx T^\pm_0 \tau_0$. With the optical depth increase, the role of Coulomb collisions increases because ECRH power deposition profile becomes more narrow, and the power is deposited into less energetic electrons population being stronger subjected to the collisions. As a result, $T^\pm_r(\tau_0)$ is increasing more slowly and reaches the maximal value at some $\tau_0$ dependent on the heating intensity. For the higher values of $\tau_0$, the dependence $T^\pm_r(\tau_0)$ changes to a decreasing one limiting with $\tau_0 \to \infty$ to the thermal ECE level (collision-dominated case). There is an asymmetry in the absorption profile resulting in different dependences of the LFS and HFS emission on the optical depth at the fixed heating intensity: the level of emission to the HFS is always characterized by a greater maximum distortion, but it falls much faster to the thermal value with increase of the optical depth. More detailed study of such optimal conditions as well as some analytical solutions for the radiation temperatures may be found in [6].

4. Conclusions. A self-consistent ECRH power deposition profile and the characteristic ECE level around the heating frequency are found analytically with taking into account the quasi-linear flattening of the resonant electron distribution. The particular case is investigated of emission in the same mode and along the same ray trajectory as for the heating radiation, which is characterized by the most pronounced non-thermal effects.

The interesting result is obtained for the optically thick plasma and LFS launch of the heating radiation. In this case, ECE in the LFS direction may be much greater than the emission in the opposite direction. Moreover, for realistic plasma parameters non-thermal emission may be only observed from the LFS. This does not meet conventional expectations that the emission of suprathermal electrons in the LFS direction at relativistically down-shifted frequencies should be screened by re-absorption in the thermal plasma.

The mechanism of ECE modification under discussion does not rely on generation of the energetic electron tail, but it is sensitively related to deformations in the main body of the electron distribution function. Such distortions reveal in ECE as an intensive, but extremely narrow spectral peak (at least for a quasi-transverse propagation). Registration of such peculiar distortions is possible in principle for ECRH experiments at low-density plasma provided that a special ECE detection system with well enough spectral resolution is used.

References