Scaling of the marginal $\beta_p$ of neoclassical tearing modes during power ramp-down experiments in ASDEX Upgrade

M. Maraschek$^1$, O. Sauter$^2$, S. Günter$^1$, H. Zohm$^1$, ASDEX Upgrade Team

$^1$MPI für Plasmaphysik, EURATOM Ass., Boltzmannstr. 2, D-85748 Garching, Germany, $^2$Centre de Recherches en Physique des Plasmas, CRPP - EPFL, EURATOM Association, CH-1015 Lausanne, Switzerland

Introduction

In terms of overall performance, neoclassical tearing modes (NTMs) are limiting the maximum achievable normalised $\beta_N = \beta_p/(I_p/a B_i)$. From the theory based on the generalised Rutherford equation the driving force is proportional to the local $\beta_p(r_{res})$ at the resonant surface $r_{res}$ of the mode [1,2]. A minimum local $\beta_{p,marg}$ value is required for the growth of an NTM after being triggered by a finite seed-island. Present scalings of the onset of NTMs are based on the assumption of a comparable and relatively large seed-island, i.e. assuming $\beta_{p,onset} \sim \beta_{p,marg}$. This is one of the main uncertainties for the separation of the relevance of the different terms governing the NTM behaviour. The island decays away independently of the value of $\beta_p$ itself, as soon as $\beta_p(r_{res}) < \beta_{p,marg}$ holds [3]. In the present paper we concentrate on the fit of the point where the mode starts decaying and the use of local parameters.

Generalised Rutherford equation and predictions

The generalised Rutherford equation with additional terms due to the neoclassical driving term, the polarisation current term and the $\chi_\perp/\chi_\parallel$-term provides a widely accepted description of neoclassically driven tearing modes [4,3] $^1$

$$
\frac{\Delta W}{dt} = r_{res} \Delta (W) + \frac{r_{res}}{l_p} \beta_p \left( a_2 \sqrt{\epsilon \frac{L_p}{l_p}} \frac{W}{w_0} - a_3 \frac{L_p}{l_p} \frac{l_p^2}{w_0} \frac{1}{\sqrt{w_0^2 + 0.2 w_d^2}} - a_4 g(\epsilon, \nu_{ii}, \omega^*_e, m) \left( \rho_{pi} \frac{l_p}{\rho_{pi}} \right)^2 \frac{1}{w_0} \right).
$$

The driving term for the NTM is the lack of the bootstrap current within the island, as the pressure profile is flattened and therefore the bootstrap current is reduced. For the unperturbed bootstrap current $j_{BS} \sim \nabla P/\langle B_p \rangle$ holds. The driving term is therefore proportional to $j_{BS}.L_q/\langle B_p \rangle \sim \beta_p . L_q/L_p$.

Under the assumption that the polarisation current term is the only stabilising term, one arrives at the following expression for the minimal $\beta_p$ required for the mode’s onset

$$
\beta_{p,marg}^{pol,curl} = \frac{3\sqrt{3}}{2} \frac{L_p}{L_q} a_2 \sqrt{\frac{w_0}{\epsilon}} \frac{a_4 g(\epsilon, \nu_{ii}, \omega^*_e, m)}{a_2^2} \cdot \left(-\Delta\right)^{1/2} \rho_{pi}.
$$

The stabilising effect from the $\chi_\perp/\chi_\parallel$-model is ignored here by setting $W_0 = W_{d,e} = 0$. The function $g(\epsilon, \nu_{ii}, \omega^*_e, m) = \epsilon^{3/2}$ or 1 governs the dependence on collisionality and is known from theory only in the two extreme cases of $\nu_{ii} = \nu_{ii}/m e \omega^*_e \ll C$ and $\nu_{ii} \gg C$ respectively, with $C$ of the order of unity. $^2$

Considering the $\chi_\perp/\chi_\parallel$-model alone ($a_3 = 0$, $a_4 = 0$) as a stabilising term for small islands one finds for the marginal $\beta_{p,marg}$ (Eq. (3a))

$$
\beta_{p,marg}^\chi = \frac{2W_0 \Delta \chi}{a_2 \sqrt{\epsilon}} \frac{L_p}{L_q} (a) \rightarrow \beta_{p,marg}^\chi \sim \nu_{ii}^{0.5} \frac{\rho_{pi}}{\epsilon} (b).
$$

At the onset of the island and especially at the point where the island reaches its marginal size the condition of a small island is by definition fulfilled. Taking $W_0 = 5.1(\chi_\perp/\chi_\parallel)^{1/4}(r_{res}L_qq_i/mc)^{1/2}$ [6], assuming a gyro-Bohm scaling ($\chi_\perp \sim T^{3/2}/B^2$) and taking the Spitzer formula $\chi_{Spitzer}$ for the parallel transport, one obtains Eq. (3b). Within the island, the parallel transport $\chi_\parallel$ is reduced due to the heat flux limit [7]. This leads to $\beta_{p,marg}^\chi \sim \rho_{pi}^*$, removing the dependence on the collisionality $\nu_{ii}$ [8,9].

$^1$r_{res}: minor radius of the resonant surface of the considered mode (distance from the magnetic axis to the intersection of a horizontal line from the axis to the low field side with the flux surface), $\nu_{res}$: resistive time scale on resonant surface; $L_p, L_q$: pressure gradient and q-gradient scale length; $L_a = \alpha / V_a = \alpha / \partial \alpha / \partial \alpha$; $\alpha$: numerical constants of the order of unity; $w$: island width; $\epsilon = r_{res}/R_0$: inverse aspect ratio of the resonant surface; $R_0$: geometric axis of the resonant surface; $\rho_{pi} = \sqrt{2m_i k T_i(r_{res})/e(B_p(r_{res}))}$: poloidal ion gyro radius at the resonant surface, which is normalised as $\rho_{pi}^* = \rho_{pi}/a$; $\nu_{ii}$ as used in [5]; $\omega^*_e = (dp_e/dr)_{res}/(n_e eB)$ electron diamagnetic drift frequency; $m$: poloidal mode number.
In Eq. 1 a small ion banana width \( w_b \) is assumed, i.e. \( w_b < < W \). This is questionable for a small initial seed-island at the mode onset and especially for the decay at the lower \( b_p, \text{marg} \). It is shown in [10] that for \( W \approx w_b \) a significant amount of the ion bootstrap current still survives inside the island even if the temperature and density profile are completely flattened. With this stabilising effect for small islands alone a marginal \( b_p \)-value results below which the NTM is stable. The scaling law for the marginal \( b_p, \text{marg} \) due to the remaining ion bootstrap current again gives \( b_p, \text{marg} \sim \rho_{p_i} \). The finite ion orbit width also modifies the polarisation current, and its increase for small islands (\( \sim 1/W^3 \)) is weakened [11].

In order to fit \( b_p, \text{onset} \) for the onset or the marginal \( b_p, \text{marg} \), a simple power-law ansatz has been used. This ansatz is strictly only appropriate for fitting the data according to the \( \chi_\perp/\chi_{||} \)-model as well as for the model by Poli [10], as from both models a power-law scaling is predicted.

**Discharge scenarios to determine the marginal \( b_p \) scaling**

H-mode discharges at low triangularity \( \langle \delta \rangle = (\delta_{\text{top}} + \delta_{\text{bottom}})/2 \approx 0.17 - 0.23, \kappa \approx 1.7 \) have been considered. A step-wise fast ramp-down of the NBI heating power (2.5 MW steps) or a pre-programmed slow power ramp after the mode’s onset has been used. For the (3/2)-NTM also pure hydrogen plasmas have been analysed, whereas for the (2/1)-NTM the \( \beta \)-limit could not be reached in hydrogen (reduced confinement and heating power).

With increasing heating power, decreasing \( q_{95} \) or decreasing feedback controlled density and hence collisionality \( \nu_{ii} \) additionally a (2/1)-NTM gets excited. Still at high heating power the (2/1)-NTM locks to the vessel and stops the entire toroidal plasma rotation. During the ramp-down phase the mode amplitude gets reduced, such that the mode can unlock again. In the following phase a (2/1) and (3/2)-NTM can coexist over several hundreds of milliseconds only during these slow power ramps.

**Extension of the parameter range in \( \rho_{p_i}^* \) and \( \nu_{ii} \) for the NTM onset**

A reference set for onset conditions for (3/2)-NTMs has been collected. A scan over a range of \( q_{95} \) values \( (q_{95} = 3.3 - 4.5) \), as well as discharges in pure hydrogen have been used to guarantee a larger variation in \( \rho_{p_i}^* \) and \( \nu_{ii} \). The \( \times \) symbols denote the previous data sets [12,2], the \( + \) symbols with shotnumbers the extended data set and the box the hydrogen data. Figure (c) shows the combined scaling with ASDEX Upgrade and JET data [2].

\[
\beta_{p, \text{onset}}/L_p = 355 \cdot \rho_{p_i}^* \pm 0.25 \\
\beta_{p, \text{onset}}/L_p = 258 \cdot \rho_{p_i}^* \pm 0.27 \cdot \nu_{ii} \pm 0.0493 \pm 0.0064
\]

or by taking \( \beta_p \) one gets (Fig. 1 (c) together with corrected (\( \rho_{p_i}^* = \rho_{p_i}/a \) consistently used) JET data [2])

\[
\beta_{p, \text{onset}} = 40.5 \cdot \rho_{p_i}^* \pm 0.20 \\
\beta_{p, \text{onset}} = 17.8 \cdot \rho_{p_i}^* \pm 0.41 \cdot \nu_{ii} \pm 0.127 \pm 0.041
\]

For consistency and the possibility to compare the onset and \( b_p, \text{marg} \) scaling law, we refer to the scaling with \( b_p/L_p \) as the reference for the onset [13]. For the onset scalings of the (3/2)-NTM the influence of \( L_p \) does not change the general dependence.

**Scaling of the marginal \( b_p \) for the NTM**

The \( b_p \) value is calculated from the temperature and density profiles from a Thomson scattering measurement. The radial location of the modes has been derived from soft-X-ray measurements and for the (2/1)-NTM mainly with the ECE diagnostic. The temporal
evolution of the mode amplitude has been derived from tracing the Fourier amplitude from magnetic pickup coils measuring $dB_0/dt$ ($W \sim \sqrt{B_0} = \sqrt{dB_0/dt} f$).

For $q_{95} \approx 4$ ($3.9 < q_{95} < 4.2$) NTMs decay away at the marginal $\beta_p$ while the discharge remains well in H-mode. For lower values of $q_{95} \leq 3.6$ it happens that the NTMs remains unstable until the discharge makes a transition back to L-mode. For JET a similar behaviour of the (3/2)-NTM has been observed for low $q_{95} \approx 3.4$ [3]. This means that the discharge is metastable towards the excitation of an (3/2)-NTM during the whole H-mode. The excitation would be mainly governed by the presence of a large enough seed-island [14].

The fit of the NTM amplitude to the local $\beta_p$-values is improved, if one includes the local gradient lengths considering $\beta_p \cdot L_q/L_p$ or $\beta_p/L_p$ for constant $q_{95}$ instead of $\beta_p$. Within a discharge $L_q$ is assumed to be constant. By taking $\beta_p/L_p$ a reasonable correlation between the island size and the $\beta_p/L_p$ can be achieved [13].

It is additionally important for the calculation of the bootstrap current to include the stronger influence of the density gradient $\nabla n$ compared to the temperature gradient $\nabla T$ in the calculation of $L_p$ [15]. This gives the best results for the fitting of the temporal evolution of the mode amplitude [13] and the clearest reduction of the data scatter for the scalings. The pressure gradient length according to [4] results in $1/L_p^{corr} \approx 1/3 \cdot 1/L_p + 2/3 \cdot 1/L_n$. For the onset it was possible to use the uncorrected $L_p$, as only discharges with negligible $n_e$-peaking at the modes onset have been included. During the rampdown the ratio between the different terms significantly differs.

Figure 2: Scaling of the marginal $\beta_{p,\text{marg}}$ for the (3/2) NTM (○) together with the onset scaling according to Eq. (4) (+). The left figure (a) shows again the parameter range, the middle (b) and the right figure (c) the resulting scalings with respect to $\rho_{pi}^*$ alone and $\rho_{pi}$ and $\nabla n$ respectively.

Looking at the scalings for the marginal $\beta_p$, it is found that $\beta_p/L_p$ is the combination of $\beta_p$, $L_p$ and $L_q$ giving exponents for $\rho_{pi}$ and $\nabla n$ with the smallest absolute errors both for $\rho_{pi}$ alone and the combined fit for $\rho_{pi}$ and $\nabla n$ together. For the exponent of $\rho_{pi}$ for $\beta_p/L_p$ a value close to 1 results, which is consistent with all the considered theories. The other combinations give larger values up to 1.48, which is not consistent with the models.

Scaling for the (3/2)-NTM In Fig. 2 the scalings for the marginal $\beta_{p,\text{marg}}$ for the (3/2)-NTM in comparison with the onset data according to Eq. (4) are shown. Clearly the hysteresis between the onset and the decoupling point can be seen. The results can be summarised as

$$\beta_{p,\text{marg}}/L_p = 137 \cdot \rho_{pi}^{1.08\pm0.31} \quad \text{or} \quad \beta_{p,\text{marg}}/L_p = 247 \cdot \rho_{pi}^{1.19\pm0.32} \cdot \nabla n^{0.115\pm0.12}. \tag{6}$$

Whereas for the onset scaling the influence of the $1/L_p$ term is almost negligible, it gets very important for the decoupling point and for JET always $L_p^{corr}$ has been used.

Scaling for the (2/1)-NTM The $\rho_{pi}$ values are smaller compared with the (3/2)-NTMs by about 25%, the $\nabla n$ values are higher by a factor of about 3. For (2/1)-NTMs the scalings taking $\rho_{pi}^*$ alone and for the combination of $\rho_{pi}^*$ and $\nabla n$ differ significantly (Fig. 3 (a) and (b)) and result in

$$\beta_{p,\text{marg}}/L_p = 100 \cdot \rho_{pi}^{1.10\pm0.52} \quad \text{or} \quad \beta_{p,\text{marg}}/L_p = 2453 \cdot \rho_{pi}^{2.037\pm0.32} \cdot \nabla n^{0.239\pm0.061}. \tag{7}$$

The influence of the collisionality $\nabla n$ is significantly stronger for the (2/1)-NTM. The higher $\nabla n$-values result in a different exponent for $\rho_{pi}^*$ for the combined scaling, whereas the fit for $\rho_{pi}$ alone results in an
almost linear dependence of $\beta_{p,\text{marg}}/L_p$ on $\rho_{pi}$. Looking at the dependence of $\beta_{p,\text{marg}}/L_p$ on $\bar{v}_i$, alone, no clear dependence on the collisionality could be concluded. Removing the dependence from $\rho_{pi}$ from the data and considering $\beta_{p,\text{marg}}/L_p/\rho_{pi}^{1.10\pm0.52}$, a clear increase with $\bar{v}_i$ can be observed (Fig. 3 (c))

$$\beta_{p,\text{marg}}/L_p/\rho_{pi}^{1.10\pm0.52} = 121 \cdot \bar{v}_i^{0.106\pm0.076}.$$  \hspace{1cm} (8)

![Figure 3](image)

Figure 3: Fig. (a) and (b) show the dependence of $\beta_{p,\text{marg}}/L_p$ for the (2/1)-NTM on $\rho_{pi}^*$ alone and $\rho_{pi}^*$ and $\bar{v}_i$ respectively. Fig. (c) shows the $\beta_{p,\text{marg}}/L_p/\rho_{pi}^{1.10\pm0.52}$ (taken from Fig. (a)) dependence on $\bar{v}_i$.

**Summary and conclusions** Scaling laws for the (3/2) and the (2/1)-NTMs on ASDEX Upgrade have been derived for the onset and the disappearance of the mode at sufficiently small local $\beta_p$. For the (3/2)-NTM, the onset $\beta_{p,\text{onset}}$ as well as the marginal $\beta_{p,\text{marg}}$ show a nearly linear dependence on $\rho_{pi}^*$ and a weak $\bar{v}_i$ dependence. In order to follow the island width evolution and to obtain a good fit for $\beta_{p,\text{marg}}$, the local pressure gradient length had to be included together with a refined theory for the bootstrap current [4]. A $q_{95}$-dependence has been observed, such that for low $q_{95}$-values the (3/2)-NTM remains present until the transition back to L-mode. For the (2/1)-NTM a stronger dependence on the local collisionality $\bar{v}_i$ has been found for $\beta_{p,\text{marg}}$, resulting in $\beta_{p,\text{marg}}/L_p = 100 \cdot \rho_{pi}^{1.10\pm0.52}$ and $\beta_{p,\text{marg}}/L_p = 2453 \cdot \rho_{pi}^{2.037\pm0.32} \cdot \bar{v}_i^{0.239\pm0.061}$. Yet a further increase of the (2/1)-NTM data base, also in combination with other experiments, is an important ongoing work for more reliable predictions. The linear $\rho_{pi}^*$ dependence is consistent with all the three considered models. The fact that the resulting fit favours the driving term $\beta_p/L_p$ instead of $\beta_p \cdot \sqrt{L_q/L_p}$ as in [16,2] suggests that the finite $\chi_{\perp}/\chi_{||}$ ratio or the finite orbit width effect on the ion bootstrap current are the main stabilising effects.

**Acknowledgement** O. Sauter has been supported by the Swiss National Science Foundation.

**References**