Microwave Corona Breakdown in Strongly Inhomogeneous Fields

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Abstract

We investigate the main physical properties of microwave corona breakdown in strongly inhomogeneous electric fields. The focus is put on numerical and approximate analytical calculations of the breakdown threshold in an exponential ionization profile, which is the basic form of the ionization profile caused by the field enhancement around irregular and curved conducting objects. The breakdown threshold is a value associated with a certain plasma density profile. We explain the major characteristics of the density profile, its dependence on pressure and the interplay of diffusion and attachment. For lower pressures the diffusion distributes the electrons from regions of high ionization frequency to regions of weak ionization, where the attachment acts as a sink. For high pressures the diffusion becomes negligible and the breakdown threshold is set by the equality of the ionization and attachment frequencies in the high field region.

1. Introduction

When working with radiating systems surrounded and/or filled with gases there are natural limitations to the amount of energy that can be put in before the device starts operating in unwanted modes. An important failure mechanism is the electric breakdown of the gas. The breakdown is caused by the avalanching effect of electrons being sufficiently accelerated by the electric field to make ionizing collisions with neutrals. The natural limiter on this sort of avalanche for air under finite pressures is set by the loss of electrons through attachment on neutrals and diffusion to absorbing surfaces. The electron avalanche creates a plasma, a conducting patch of gas which may be harmful to the device in a number of ways, ranging from noise to short circuits. Therefore the breakdown threshold electric field needs to be calculated so the device can be operated at a safe power level.

The calculation of the breakdown threshold in a homogeneous field implies a plasma profile which does not depend upon pressure, but solely on the geometry in question. When the field becomes inhomogeneous, other effects play in to make the actual plasma size and profile depend strongly on pressure. The reason for this is simply that free electrons are created in unequal amounts at different locations in the volume in question, and the way to redistribute the electrons
so they can be absorbed by walls or attachment is through diffusion. Diffusion becomes less and less effective for increasing pressure. When the diffusion length becomes very small in comparison to the field inhomogeneity and the system size we should expect that the breakdown threshold is set by the equality of ionization and attachment in the high field regions.

**General procedure**

The simplified way of calculating the breakdown threshold in air starts with the continuity equation, [1];

\[ \frac{\partial n}{\partial t} = \nabla^2 (Dn) + n(\nu_i - \nu_a) \]  

(1)

A more exact approach would start with kinetic equations, and taking into account the dependence of all the involved parameters on the electric field strength and such. Since we want to reveal the important physical aspects and establish rule of thumbs we choose to work with averaged quantities and approximate the behavior of \( D \) and \( \nu_a \) as independent of the electric field, but changing with pressure. The ionization frequency depend heavily on the electric field strength, but a practical and sufficiently accurate approximation which is often used in these circumstances is \( \nu_i \propto \nu_a (E/E_a)^{5.33} \), see e.g. [2]. Setting the time-derivative in the continuity equation to zero we can calculate the ionization frequency corresponding to a quasi-steady plasma profile, in a given system. From this value of the ionization frequency the electric field strength can be calculated, and this is what is generally considered as the actual breakdown threshold. With these considerations we rewrite the continuity equation as;

\[ \nabla^2 n + n(\lambda_s(\bar{r}) - q) = 0 \]  

(2)

Where \( \lambda = \nu_i(E_{\text{max}})/D \), \( s(\bar{r}) = E(\bar{r})/E_{\text{max}} \), and \( q = \nu_a/D \). The results from calculations of the breakdown threshold are often presented as plots of \( \lambda \) versus \( q \). One can get a notion of the size of the breakdown region by looking at the diffusion length \( L_D = \sqrt{D/(\nu_i - \nu_a)} \), or by plotting the actual plasma profile.

**Exponential ionization profile**

A number of situations bear a significant resemblance to the rather simple model of an exponential ionization profile, [3]. This could for example represent the presence of a conducting irregularity in an otherwise homogeneous field, or perhaps the field profile around wires, spheres etc., although the diffusion would be higher due to the non 1-D geometry.

Our ionization profile looks like;

\[ \nu_i(x) = \nu_{i0} \left[ (1 - \gamma) \exp(-2\mu x) + \gamma \right] \]  

(3)
Where the main characteristics are $v_i(0) = v_{i0}$, and $v_i(x \to \infty) \to \gamma v_{i0}$.

In previous calculations of breakdown thresholds the Rayleigh-Ritz direct variational method has shown excellent agreement with simulations and experiments, [4, 5]. For this problem we assume a trial function on the form $n_T(x) = x \exp(-\alpha \mu x)$. The expression for our ionization frequency becomes;

$$\lambda[\alpha] = \mu^2 \frac{q/\mu^2 + \alpha^2}{(1 - \gamma)\alpha^3 + \gamma(\alpha + 1)^3(\alpha + 1)^3} \tag{4}$$

Where the optimal value of $\alpha$ is determined by solving $\frac{\partial \lambda}{\partial \alpha} = 0$. This is readily done on computer, and using the result for $\lambda$ one can plot the diffusion length, as seen in Fig. 1.

![Figure 1: The diffusion length as a function of $q/\mu^2$ for three values of $\gamma = 1/81, 1/27, 1/3$ from top to bottom.](image)

The main features of the diffusion length are easily understood. When the pressure is low (low $q/\mu^2$) the diffusion length is long, and the influence of a small region with enhanced field becomes negligible. The breakdown threshold is set by equality of ionization and attachment in the far field, i.e. $\gamma v_{i0} = v_a$. This gives an expression for the diffusion length; $L_D = \sqrt{\mu^2 \gamma/q(1 - \gamma)}$. For a small value of $\gamma$ the logarithmic expression is $\ln \mu L_D = \frac{1}{2} \ln \gamma - \frac{1}{2} \ln q/\mu^2$, which is easily recognized in the left part of Fig. 1, where the lines are straight. In the right part of the figure the three curves merge into one which steadily decreases. Which is what should be expected, since when the plasma region is very small ($L_D \ll 1/\mu$) and close to $x = 0$, the exponential profiles look the same, and have no dependance on $\gamma$; $v_i(x \ll 1, \gamma \ll 1) \approx v_{i0}(1 - 2\mu x)$.

The diffusion length gives a good understanding of the physical size of the breakdown plasma region. In addition to this one can look at the actual profile of the plasma. In Fig. 2 one can see the output of computer simulations of the breakdown plasma. The smallest profile corresponds to a high pressure, whereas the two other profiles illustrate how this plasma grows in extension for lower pressures.
Figure 2: The breakdown plasma profile in the exponential field for different values of pressure.

Conclusion

This investigation of the breakdown plasma for an exponential ionization profile has two immediate implications. First, it is a fact that an inhomogeneous field, and concomitant inhomogeneous ionization profile, gives rise to an inhomogeneous breakdown plasma. This implies the possibility of small localized plasmas appearing in systems working well below the breakdown threshold corresponding to the ambient field strength. Second, there is an uncertainty in the very concept of breakdown. Since this concept is based on the notion that any plasma forming in an rf system is harmful. We show that a small breakdown region is possible, and this in itself need not be harmful. However, to evaluate the risk of a small plasma region one needs to consider several things, such as heating, reduction of local field, plasma expansion etc. This is as of yet an unexplored avenue of theoretical investigations.

References


