A Novel Approach for Dust Charging in RF Discharges

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Introduction

Dusty plasmas are attracting increasing interest due to their applications both in industry and in basic physics research [1]. One of the fundamental properties of a dust grain which is central to every model of its dynamic behaviour in a plasma environment is its charge. The most common approach to determining this property is through the OML (Orbital Motion Limited) theory [2]. In OML, the unperturbed ion and electron distribution functions far from the dust grain, are used in conjunction with the trajectories of particles in the vicinity of the solid particle to compute the currents onto it. This result together with the condition that in steady state the ion and electron currents must be equal, gives us the floating potential of the grain.

Many dusty plasma experiments and applications involve time varying fields and potentials, for instance experiments in RF discharges where dust particles are either in the sheath or the presheath of the discharge [3, 4] and the Earth’s upper atmosphere [5]. The basic formulation of the OML is one of a steady state model. In this work the OML approach will be applied for a drifting Maxwellian distribution of electrons with a time varying drift velocity. The results of this method will be analysed and a comparison with the corresponding results for the time average distribution will be performed.

The OML approach for a Maxwellian with a Time-Varying Drift Velocity

In this Section, the charging of a dust grain immersed in a plasma described by a Drifting-Maxwellian distribution with a time varying drift velocity will be considered, where the drift velocity, \(v_d\), is parallel with the \(z\)-axis

\[
f_j(v_j, \theta) = n_j \left( \frac{m_j}{2\pi k_B T_j} \right)^{3/2} \exp \left( -\frac{v_j^2 + v_d^2 - 2v_j v_d \cos(\theta)}{v_T^2} \right),
\]

where the time varying drift velocity has the form

\[
v_d = v_{d,0} \cos(\omega_{RF} t)
\]

where \(v_{d,0}\) is the amplitude of the drift velocity and \(\omega_{RF}\) is the angular frequency of the variation of the drift velocity.
Figure 1: The dust grain’s average floating potential normalised to the corresponding OML result for a stationary Maxwellian as a function of the drift velocity normalised to the electron thermal speed for a constant frequency (left) and constant $\beta$ (right).

The general integral using the OML approach to compute the particle current is

$$I_j = 2\pi q_j \int_{v_{j,\text{min}}}^{\infty} \int_0^\pi v_j^3 \sigma_{j,v_j} f_j(v_j) \sin(\theta) d\theta dv_j,$$

performing the integration, see [6], the electron current takes the form

$$I_e = \pi r_d^2 e n_0 \left( \frac{8k_B T_e}{\pi m_e} \right)^{1/2} \left[ G_1(\chi_e) + G_2(\chi_e) \phi_d' \right],$$

where $r_d$ is the radius of the dust grain, $\chi_e = \left( \frac{m_e v_d^2}{2k_B T_e} \right)^{1/2}$ is the drift velocity normalised with respect to the electron thermal velocity, $G_1(\chi_e)$ and $G_2(\chi_e)$ are functions defined in [6] and $\phi_d' = -\frac{e\phi_d}{k_B T_e}$ is the potential of the dust grain normalised with respect to the electron temperature in eV. For this current and using for the ions the standard OML current, the differential equation for the evolution of the dust grain’s charge can be written as

$$\frac{dq_d}{dt} = \Sigma I_j = I_{i,\text{OML}} + I_e,$$

normalizing the above equation with respect to the electron temperature $T_e$, and substituting the charging currents, the differential equation giving the evolution of the normalized dust potential can be derived

$$\frac{d\phi_d'}{dt} = -\frac{\omega_p r_d}{\sqrt{2\pi} \lambda_{De}} \left[ \frac{\beta}{\mu} \left( 1 + \frac{\phi_d'}{\beta} \right) - \left( G_1(\chi_e) + G_2(\chi_e) \phi_d' \right) \right],$$

where $\beta = T_i/T_e$ and $\mu = m_i/m_e$ are the ion to electron temperature and mass ratios respectively. Following [7], the floating condition for the average floating potential for time varying particle
currents can be written as

\[
\frac{1}{T_{RF}} \int_0^{T_{RF}} \left[ - \left( G_1(\chi_e) + G_2(\chi_e)\phi'_{d,\text{float}} \right) \right] dt = \sqrt{\frac{\beta}{\mu}} \left( 1 + \frac{\phi'_{d,\text{avg}}}{\beta} \right),
\]

where \( T_{RF} \) is the period of the drift velocity and \( \phi'_{d,\text{float}}(t) = \phi'_{d,\text{avg}} + \phi'_{d,RF}(t) \). Using Eq. 7, the dust grain’s normalised floating potential \( \phi'_{d,\text{avg}} \) can be calculated. The dust grain’s potential normalised to the corresponding OML value, calculated for a stationary Maxwellian of the same temperature, as a function of the magnitude of the drift velocity normalised with respect to the electron thermal velocity can be seen Fig. 1. Performing the calculation for both a constant frequency, with varying the value of \( \beta \), and a constant value of \( \beta \), with varying the value of the frequency, it was found that the dust grain’s potential increases compared with the stationary OML case as a function of the drift velocity but has only a weak dependence to \( \beta \) and the frequency.

**Comparing OML for Instant and Average Distributions**

In the previous section the average potential on a dust grain has been computed using the OML approach for a drifting Maxwellian distribution of electrons with a time varying drift velocity. However, in most experiments the average electron distribution is measured instead of the time resolved one. Motivated by this, the time averaged electron distribution will be used to calculate the dust grain’s normalised floating potential using the OML approach. By performing the appropriate numerical integration the time averaged distribution can be derived, see Fig. 2.
This distribution, for values of the drift velocity $v_{d,0} < v_{T,1}$, can be approximated by a stationary Maxwellian distribution with a temperature $T_{e,2} > T_{e,1}$, see Fig. 2. It can be seen that there is very good agreement between the computed average distribution and the fitted Maxwellian.

Using this fitted distribution, the OML floating potential for the dust grain can be calculated. For a time varying electron distribution having a temperature $T_{e,1} = 1\text{eV}$ and a normalized drift velocity with respect to the electron thermal speed of $v'_d = v_d/v_{T,e} = 0.5$ and an RF frequency $f_{RF} = 6.6\text{MHz}$, the average temperature of the time averaged distribution is $T_{e,avg} = 1.082\text{eV}$. For an Argon plasma having an ion temperature $T_i = 0.01\text{eV}$ and a number density $n_i = n_e = n_0 = 10^{16}\text{m}^{-3}$ and a dust grain with a radius of $r_d = 15\mu\text{m}$ the floating potential for the two distributions, the time varying and the time average, can be calculated, the results can be seen in Fig. 2. It can be seen that the resulting normalised potential for the time average distribution is less negative than the time varying one.

**Conclusions**

It was found that the floating potential of a dust grain calculated for a drifting Maxwellian electron distribution with a time varying drift velocity is larger compared with the corresponding OML result for a stationary distribution and it increases for larger values of the magnitude of the drift velocity. Furthermore, this result is still more negative compared with the one calculated by the corresponding time average distribution. This result was to be expected as the directed motion of electrons would lead to a more negative potential whereas an increase in the electron temperature would decrease the $\beta = T_i/T_e$ ratio which in turn would lead to a decrease of the normalised floating potential.

**References**


