HARMONICS GENERATION IN THE REFLECTION OF A LINEARLY POLARIZED LASER BEAM NORMALLY INCIDENT ON AN OVERDENSE PLASMA

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We study the harmonics generation in the reflection of a high intensity linearly polarized laser wave normally incident on an overdense plasma. We use an Eulerian Vlasov code for the numerical solution of the one-dimensional relativistic Vlasov-Maxwell equations for both electrons and ions [1]. The oscillation of the laser wave at the plasma edge creates an oscillating space-charge, giving rise to an oscillating electric field. If the intensity of the wave is sufficiently high to make the oscillation of the electrons relativistic, then the plasma edge oscillates nonlinearly in the field of the high intensity laser beam (similar to the relativistic oscillating mirror ROM), which results in an important distortion in the reflected wave, associated with the generation of harmonics. The combined effects of the edge electric field with the incident ponderomotive pressure have also important consequences on the ion dynamics, with the ion density profile forming a solitary-like structure close to the plasma edge. We consider the case when the laser beam wavelength \( \lambda \) is greater than the scale length of the jump in the plasma density at the edge \( L_{\text{edge}} \) (\( \lambda \gg L_{\text{edge}} \)), and \( n/n_e = 100 \).

The relevant equations

The one-dimensional Vlasov equations for the electron distribution function \( f_e(x, p_x, t) \) and the ion distribution function \( f_i(x, p_x, t) \) are given by [1]:

\[
\frac{\partial f_{e,i}}{\partial t} + \frac{p_{x,e,i}}{\gamma_{e,i}} \frac{\partial f_{e,i}}{\partial x} + (\mp E_x - \frac{m_{e,i}}{2\gamma_{e,i}} \frac{\partial a_x^2}{\partial x}) \cdot \frac{\partial f_{e,i}}{\partial p_{x,e,i}} = 0.
\]

(1)

Time \( t \) is normalized to \( \omega_{pe}^{-1} \), length is normalized to \( c\omega_{pe}^{-1} \), velocity and momentum are normalized respectively to the velocity of light \( c \), and to \( M_e c \). The indices \( e \) and \( i \) refers to electrons and ions. In our normalized units \( m_e = 1 \) for the electrons, and \( m_i = M_e / M_i \) for the ions. In the direction normal to \( x \), the canonical momentum written in our normalized units as \( \tilde{p}_{x,e,i} = \tilde{p}_{x,e,i} \mp \tilde{a}_x \) is conserved (the vector potential \( \tilde{a}_x \) is normalized to \( M_e c / e \)). \( \tilde{p}_{x,e,i} \) can be
chosen initially to be zero, so that \( \dot{p}_{\perp, e,i} = \pm \dot{a}_\perp \cdot \). \( E_x = -\frac{\partial \varphi}{\partial x} \) and \( \vec{E}_\perp = -\frac{\partial \vec{a}_\perp}{\partial t} \),

\[
\gamma_{e,i} = \left( 1 + (m_{e,i} p_{we,i})^2 + (m_{e,i} a_{\perp,i})^2 \right)^{1/2}.
\]

The transverse EM fields \( E_y, B_z \) for the linearly polarized wave obey Maxwell’s equations. With \( E^\pm = E_y \pm B_z \) we have:

\[
\left( \frac{\partial}{\partial t} \pm \frac{\partial}{\partial x} \right) E^\pm = -J_y ;
\]

which is integrated along the vacuum characteristic \( x=t \). In our normalized units:

\[
\vec{J}_\perp = \vec{J}_{\perp, e} + \vec{J}_{\perp, i} ; \quad \vec{J}_{\perp, e,i} = -\vec{a}_\perp m_{e,i} \int \frac{f_{e,i}}{\gamma_{e,i}} dp_{we,i} ; \quad J_{xe,i} = \pm m_{e,i} \int \frac{p_{we,i}}{\gamma_{e,i}} f_{e,i} dp_{we,i} \quad (3)
\]

Eq.(1) is solved using 2D interpolation along the characteristics [1,2]. We calculate \( E_{x}^{n+1/2} \) from Ampère’s equation: \( E_{x}^{n+1/2} = E_{x}^{n-1/2} - \Delta t J_{x}^{n} \), \( J_{x} = J_{xe} + J_{xi} \).

\textbf{Results}

The forward propagating linearly polarized laser wave is penetrating at \( x=0 \) at the left boundary, where the fields \( E^+ = 2E_{0} P(t) \cos(\omega t) \). The shape factor \( P(t) = \sin(n t/(2\pi)) \) for \( t < T = 50 \), \( P(t) = 1 \) otherwise. We choose for the amplitude of the potential vector \( a_{0} = 18\sqrt{2} \cdot \omega = 0.1\omega_{p} \), which corresponds to \( n/n_{e} = 100 \). The initial temperature for the electrons is \( T_{\perp}=1 \) keV and for the ions \( T_{\parallel}=0.1 \) keV. The total length of the system is \( L = 188.5c/\omega_{p} \). We use \( N = 10000 \) grid points in space and 2800 in momentum space for the electrons and ions (extrema of the electron momentum are \( \pm 5 \), and for the ion momentum \( \pm 171.06 \) ). We have a vacuum region on each side of the plasma of length \( L_{\text{vac}} = 73.986 \). The jump in density at the plasma edge on each side of the slab is \( L_{\text{edge}} = 2.8275 \), and the top slab density of 1 is of length 34.89. The incident wave wavelength is \( \lambda = 62.3 \), \( \text{i.e. } \lambda \gg L_{\text{edge}} \).

Figs.(1) show the plot of the density profiles (full curves for the electrons and dash curves for the ions) at \( t=150.8 \) (left frame), 160.2 (middle frame) and 169.6 (right frame). The incident laser wave is literally pushing the plasma edge, which is acquiring a steep profile, with electrons accumulating at the edge of the plasma. Then from \( t=150.8 \) to 169.6, \( \text{i.e. } \) over a period of time of only 18.8 (about a quarter of the laser wave period), the ions show a very rapid acceleration at the edge, as indicated in Figs.(1), forming a solitary-like structure. The electrons and ions accumulation at the edge increase the opacity of the plasma to the laser wave. Figs.(2) show the density profiles at \( t=282.7, 301.6 \) and 320.4, concentrating on the region of the plasma edge, together with the electric field \( E_{x} \) (dash-dot curves). Figs.(3) show
the incident wave $E^+$ (full curves) and reflected wave $E^-$ (dash curves) at the same times. In the first of Figs.(2) at $t=282.7$, we see the electron density profile expanding to the left, corresponding to a minimum of the incident wave $E^+$ in the first of Figs.(3). In the second of Figs.(2) at $t=301.6$ the electrons are compressed towards the right by the ponderomotive pressure of the wave, corresponding to a maximum of the incident wave $E^+$ at $t=301.6$ in Figs.(3), and an important build-up of the longitudinal electric field at the plasma edge (dash-dot curve in Fig.(2)). At $t=320.4$ in Figs(2) the electrons are again moving towards the left, corresponding to a minimum of the incident wave in Figs.(3). During the same time the ion profiles (following the initial rapid acceleration discussed in Fig.(1)) are evolving slowly, and moving slowly to the right. This can also be verified looking to the phase-space of the ions in Figs.(5) at $t=301.6$ and 320.4, while at the same times the electron phase-space plots in the second and third of Figs.(4) show the rapid motion of the edge electrons, oscillating rapidly from positive to negative directions, as well as a heating of the distribution function. The relativistic oscillation of the electron density surface is nonlinear, and can thus generate harmonics (similar to the relativistic oscillating mirror ROM). Even though we see an electron population in the first of Figs.(4) accelerating inside the plasma, the incident laser wave in Fig.(3) does not penetrate the plasma and is strongly damped at the edge within a skin depth. Fig.(6) shows the edge oscillating electric field, the peak at $t=301.6$ (dash curve) corresponds to a peak value of $E^+$ at the edge (see Fig.(3)), with the ponderomotive force compressing the electrons (see Fig.(2)). Fig.(7) shows the frequency spectrum of the incident signal (full curve) with a peak at $\omega = 0.1$, and the odd harmonics of the reflected signal (dash curve) [3].

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References
Fig. 2 Electron (full curve), ion (dash curve) density, electric field (dash dot curve) at $t=282.7, 301.6, 320.4$

Fig. 3 Incident wave $E^+$ (full curve), reflected wave $E^-$ (dash curve) at $t=282.7, 301.6, 320.4$

Fig. 4 Phase-space for the electron distribution function at $t=169.6, 301.6, 320.4$

Fig. 5 Phase-space for the ion distribution function at $t=169.6, 301.6, 320.4$

Fig. 6 Electric field at $t=282.7$ (full curve), $t=301.6$ (dash curve), $t=320.4$ (dash-dot curve)

Fig. 7 Frequency spectrum for $E^+$ (full curve) and $E$ (dash curve).