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Reconstruction of plasma edge density profile from Lithium beam data using statistical analysis

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The determination of the space- and time-resolved electron density profile in the edge region of magnetically confined fusion plasma are of great interest for more detailed understanding of several phenomena, such as the L- to H-mode transition, edge localized mode and transport barrier physics. A routinely used method for measuring the edge electron density profile is Lithium beam emission spectroscopy.

The injected Lithium atoms are exited and ionized by collision with plasma particles (electrons, hydrogen and impurity ions). The spatial intensity profile of Li-I(2s-2p) line is obtained from a collisional-radiative model[1]. The occupation density of the exited state of Li atoms are described by the following system of differential equations:

$$\frac{dN_i(z)}{dz} v_{Li} = \sum_{j=1}^{M_{Li}} \left[n_e(z) a_{ij}(T_e(z), Z_{eff}(z), v_{Li}) + b_{ij} \right] N_j(z) \qquad N_{2p}(z=0) = \beta$$

where N_i is the occupation density of the (i-th) exited state, z is the coordinate along the lithium beam. $z = 0$ is the first measuring point. In the boundary condition it is assumed that until this point only the (2p) level is populated. The measured light profile $f(z)$ is determined by the 2p level: $f(z) = \alpha N_{2p}(z)$ where α is the unknown absolute calibration factor. The $i-th$ measurement d_i is than $d_i = f(z_i) + \varepsilon_i = D_i(n_e, \alpha, \beta) + \varepsilon_i$. It is supposed that the measurement errors ε_i are independent and normally distributed with zero mean and known σ_i^2 variance.

Estimation of the electron density profile from the measured data is done with the help of Bayesian Probability Theory [2], which can be formulated for the present application:

$$P(n_e, \alpha, \beta | \mathbf{d}, I) = \frac{P(\mathbf{d} | n_e, \alpha, \beta, I) \cdot P(n_e, \alpha, \beta | I)}{P(\mathbf{d}, I)}$$

The posterior probability density of the parameters (the left-hand side of the equation) is related to known quantities: the likelihood pdf and the prior pdf (the numerator is on the right hand side). The recent problem is parameter estimation, where the evidence of the data (denominator) which solves the normalization of the posterior pdf. I is the collection of additional knowledge such as the physical model, the electron temperature profile, constrains for the physically expected density profile, the error of the data, etc. The posterior pdf contains all the information available from the data and the prior knowledge. The posterior pdf is characterized by its minimum which gives the best estimate of the parameter and its width gives the reliability of the estimation.The likelihood describes the statistics of the measurement, putting all together gives:

$$P(d | n_e, , \alpha, \beta, I) = \frac{1}{\Pi_i \sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{1}{2}\chi^2 \right) \qquad \chi^2 = \sum_i^N \left(\frac{d_i - D_i(n_e, \alpha, \beta)}{\sigma_i} \right)^2$$

According to our knowledge, the expected density profile is smooth, a week prior is applied which penalize the curvature of the density profile:

$$P(n_e,\alpha,\beta|I) = exp\left[-\frac{1}{\lambda^2}\int\frac{d^2}{dz^2}n_e(z)dz\right]$$

The probabilistic method enables us to separate the significant information in the data from the statistical fluctuation. It provides robust method for the determination of density profile in any density regime. Due to the applied nonlinear optimization algorithm - in which each iteration the forward model has to be solved several times - the computation is time consuming.

The recently installed Li-beam diagnostics on TEXTOR tokamak can provide Li light profile up to $2\mu s$ sampling time during the whole discharge. To analyze these data it is a great advantage to decrease the time of the computation. A new method was developed which based on linearization of the original problem. Starting from an initial guess on the density profile (n_e^0), the calibration constant α and the population of 2p level at the first measurement point, one can write: $n_e = n_e^0 + \delta n_e$. Denoting the forward calculation by the operator $D_i(n_e)$ the measurement is $d_i = D_i(n_e^0 + \delta n_e)$ It was found that the operator $D_i^L(\delta n_e) = D_i(n_e^0 + \delta n_e) - D_i(n_e^0)$ is nearly linear if δn_e is not too large. Let $d_i^* = d_i - D_i(n_e^0)$ one can write: $d_i^* = D_i^L(\delta n_e)$. Let us expand the δn_e as a linear combination of m base functions $b_i(z)$:

$$\delta n_e = \sum_{i=1}^{m} p_i b_i(z)$$

Half overlapping triangular base function used at a positions z_i. Thus this linear combination is a linear interpolation between the z_i points with p_i values. Now the vector $\mathbf{p}$ parameterizes the density profile. Using flat prior for the parameter $\mathbf{p}$ the logarithm of the posterior is:

$$L = ln\left[P(|d_i^*,I)\right] = C - \frac{1}{2}\chi^2 - \frac{1}{\lambda^2}\mathbf{pRp} \qquad \chi^2 = \sum_{i=1}^{n}\left[\frac{d_i^* - \mathbf{Mp}}{\sigma_i^2}\right]^2$$

The matrix $\mathbf{M}$ is $M_{ij} = D_i(b_j(z))$, while the elements of $\mathbf{R}$ is:

$$R_{lk} = \frac{1}{\lambda^2}\int\frac{d^2}{dz^2}b_l(z)\frac{d^2}{dz^2}b_k(z)dz$$

The best estimate of the parameter $\mathbf{p}$ is where the posterior has a maximum $\mathbf{p}^s = argmin[L(\mathbf{p})]$. It can be determined from the condition $\partial L/\partial\mathbf{p} = 0$. This leads to the following matrix equation:

$$\mathbf{Np}^s = \mathbf{Ld}^*$$

where the matrix $\mathbf{N}$ and $\mathbf{L}$ are

$$N_{ik} = R_{ik} + \frac{1}{\lambda^2}\sum_k\frac{M_{jk}}{\sigma_j^2}M_{ij} \qquad L_{ik} = \frac{1}{\lambda^2}\frac{M_{ki}}{\sigma_j^2}.$$

As matrix $\mathbf{N}$ is quadratic and well conditioned the solution is $\mathbf{p}^s = (\mathbf{N}^{-1}\mathbf{L})\mathbf{d}^*$. **In this way the reconstruction of the density in a single time slice reduces to a few matrix manipulations which can be done numerically very quickly.** The reliability of the estimation is the width of marginal distribution of the posterior pdf. It is estimated as the diagonal elements of the inverse of the Hessian $\mathbf{H}$:

$$H_{ij} = \frac{\partial^2}{\partial p_i \partial p_j}L = -\sum_k\frac{M_{ik}M_{kj}}{\sigma_i^2} + \frac{1}{\lambda^2}R_{ij}$$

This method can be used as long as the density profile and the absolute calibration factor do not change significantly. For a new reconstruction series the initial guess on the parameter has to be determined by the nonlinear optimization.

The TEXTOR Li-beam diagnostics has been upgraded with the following main components: newly designed high throughput optics with in-vessel elements, two parallel observation systems containing a scientific CCD camera and a 16 channel Avalanche Photodiode (APD) fast camera unit and a fast deflection system [3].

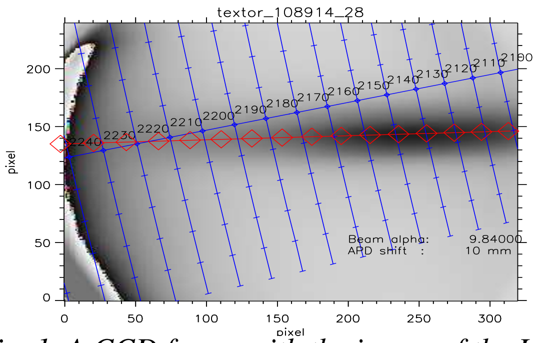

Fig. 1. A CCD frame with the image of the Li beam. The red diamonds mark the positions of the APD channels.

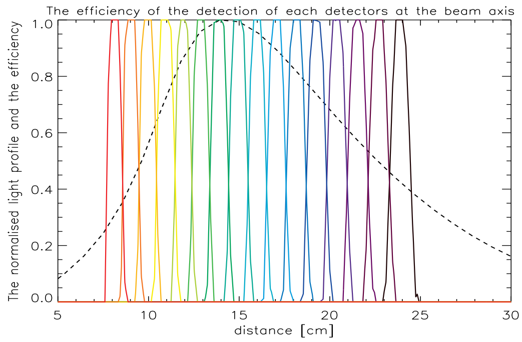

Fig. 2. Uncalibrated sensitivity of the APD channels (colored curves) along the beam axis.

The well calibrated CCD image (Petravich et al.P1.187 in this conference) was used to determine the density profile and the calibration factor. The simultaneously running APD diagnostics samples the Li light profile, too. As the light profiles measured by the CCD and the APD should be same, the calibration of the APD channels can be done in the CCD frame rate. This method enables us to reduce the systematic uncertainty of the light profile determined from the APD. *Fig. 1* shows the image of the CCD camera with 30ms exposure time. The position of the APD channels are also marked in the figure. For spatial cross-calibration of the CCD image and the APD channels the transfer matrix of the APD channels were constructed with the ZeMax optical design software. The uncalibrated sensitivity of the APD channels along the beam axis is depicted on *Fig. 2*

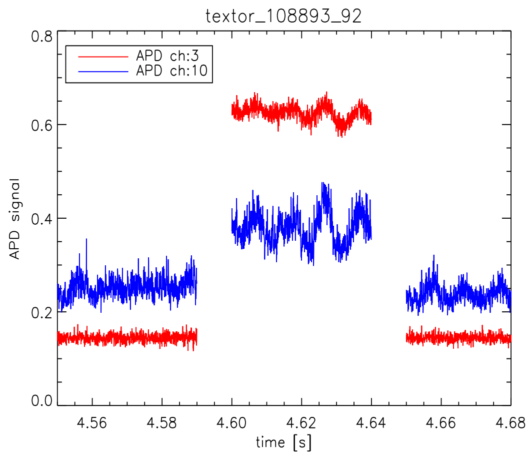

Fig. 3. Time traces of the measured Li2p line intensity for two APD channels.

Fig. 3 shows time traces of the measured Li2p line emission intensity of two APD channels with 4.8 μs time resolution. The time window comprises 1 on-beam and two off-beam periods. The off-beam signals on the sides are used to determine the background during the on-beam interval in the middle. Channel 3 measures the plasma core while channel 10 is close to the plasma edge. In the presented case, the background can be estimated from the off-beam intervals with acceptable accuracy. The statistical error of the measured line intensity is $\sigma_i = \sqrt{\sigma_i^b + \sigma_i^s}$, where σ_i^b is the error of the background subtraction while σ_i^s is the fluctuation of the signal during the on-beam interval caused by the photonic noise. Note that in the determination of σ_i^b the non-photonic fluctuation was taken into account as well.

To demonstrate the data analysis method an ohmic TEXTOR discharge was chosen. The line intensity profile determined from the CCD image and from the APD signals are compared on

Fig. 4 while *Fig. 5* depicts the corresponding reconstructed density profile from a single CCD image and from a single time slice of the APD signals, respectively. The uncertainty of the reconstructed density profile from the fast APD measurement (density profile every 10 μs) is also indicated.

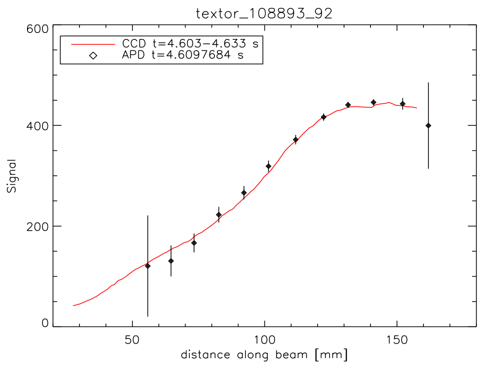

Fig. 4. The Li2p line intensity profile along the axis of the Li beam determined from the CCD image (red curve) and from the APD signals (black dots). The error bars indicate the statistical error of the measurements.

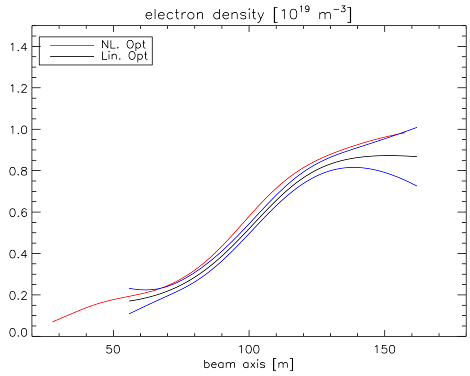

Fig. 5. Electron density profiles reconstructed from the measured Li2p intensity profiles shown in Fig. 4. Red curve - from the CCD image, black curve - from the APD measurements. The blue curves is the reliability range ($+/-\sigma$) of the density profile reconstructed from the APD measurement.

In most important non-stationary plasma discharges, averaging the off-beam signals for background subtraction causes unacceptable systematic error for the Li2p profile, hindering the reconstruction of the electron density profile. Developing a more sophisticated method for background subtraction for non-stationary plasma conditions taking advantage of the capability of the fast Li beam system, that the beam can be switched on and off up to a frequency of 200 kHz is an ongoing task. In the near future the simultaneous estimation of the absolute calibration factor and the initial Li2p occupation number will be solved and it enables the method taking into account prior informations for these quantities.

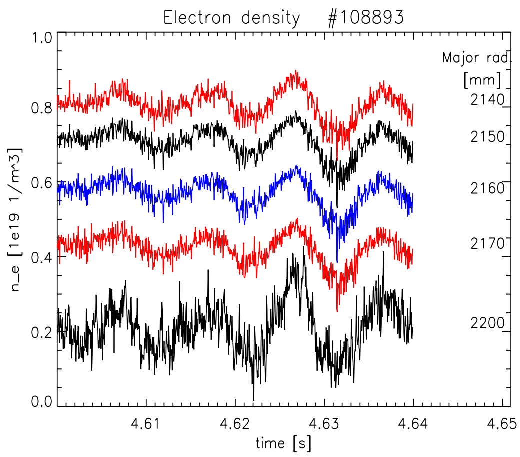

Fig. 6. Time traces of the electron density at different radial positions.

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[2] R Fischer, E Wolfrum, J Schweinzer and ASDEX Upgrade Team, *Plasma Phys. Control. Fusion* 50 (2008) 085009

[3] G. Anda et. al, 35th EPS Conference on Plasma Phys. Hersonissos, 9 - 13 June 2008 ECA Vol.32D, P-5.076 (2008)