Electron Langmuir Probe Current in Tokamak Edge Plasma

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Langmuir probes are one of the oldest and simplest diagnostic tools used for measuring the edge plasma parameters with sufficiently high temporal and spatial resolution. In this work we will use the kinetic theory for the electron probe current in non-local approach when the electrons reach the probe in diffusion regime [1]. The tokamak’s edge plasma is usually considered as non collisional but strongly turbulent. In the turbulence scenario the vector of the electric field changes its orientation in arbitrary manner. This causes arbitrary changes in the direction of the electron’s motion but does not change their kinetic energy. The result is similar to the one when elastic collisions are taken into account [2].

In this paper we discuss the application of an extended formula for the electron probe current of turbulent tokamak edge plasma in the presence of a strong magnetic field.

The electron probe current density may be found solving the Boltzmann kinetic equation for the isotropic part of the Electron Energy Distribution Function (EEDF), $f_0$, which takes the form of a diffusion equation [3]:

$$\nabla_r D(W - e\varphi(r))\nabla_r f_0(W, r) = 0$$

with the corresponding boundary conditions [3]:

$$f_0(W, r = a) = f_0(W, a)$$

where $a$ is the radius of the probe, $e$ is the electron charge, $W = \varepsilon + e\varphi(r)$ is the total electron energy, $\varepsilon$ is the kinetic energy of the electrons and $\varphi(r)$ is the potential distribution introduced by the probe. The diffusion coefficient in the non local approach at the probe’s vicinity is $D(\varepsilon) = D(W - e\varphi(r)) = vD = \frac{v^2\lambda_e}{3}$. Far from the probe, $f_0$, should coincide with the undisturbed EEDF $f_\infty(W)$. Far.

Now let us consider the solution of equation (1) with the boundary conditions (2). Starting with:

$$D(W - e\varphi(r))\nabla_r f_0(W, r) = \text{const} = C_1$$
we note that the gradient $\nabla_r$ depends on the shape of the probe, that is $\nabla_r = r^n \frac{d}{dr}$, where $n = 0$ for plane probe, $n = 1$ for cylindrical probe and $n = 2$ for spherical probe. The integration from the probe surface $a$ to $r$, gives:

$$f_0(W, r) = C_2(W) + C_1(W) \int_a^r \frac{dr}{r^n D(W - e\varphi(r))},$$

(3)

where the constants of integration $C_1(W)$ and $C_2(W)$ depend on the boundary conditions (2). As a consequence, the solution of this equation is finally given by:

$$f_0(W, r) = f_0(W, a) + \left[ f_0(W) - f_0(W, a) \right] \frac{\int_a^r \frac{dr}{r^n D(W - e\varphi(r))}}{\int_a^r \frac{dr}{r^n D(W - e\varphi(r))}}.$$  

(4)

The isotropic EEDF, $f_0(W, a)$ at the probe surface ($r = a$, when $W \geq eU$) can be found, using equation (3) and the effective boundary conditions [3]:

$$f_0(a, W \geq eU) = \gamma f_i(a, W)$$

$$f_i(r, W) = -\lambda_c \nabla_r f_0(r, W) = \frac{\nu df_0}{d r},$$

(5)

where $f_i(r, W)$ is the anisotropic part of the distribution function, $U = \varphi(a)$ is the potential of the probe and $\lambda_c = \frac{\nu}{\gamma}$ is the free path of the electrons. The geometric factor $\gamma = \gamma(R/\lambda)$ takes values in the range $0.71 \leq \gamma \leq 4/3$ [1].

Differentiating (3) with respect to the radial variable and using (5), the isotropic EEDF $f_0(W, a)$ on the probe’s surface becomes:

$$f_0(W, a) = \frac{f_i(W)}{1 + \frac{2}{3} \left( \frac{W - eU}{m\gamma_0} \right)^2 \int_a^r \frac{dr}{r^n D(W - e\varphi(r))}}.$$  

(6)

Now we can evaluate the probe current density from the anisotropic part of the distribution function $f_i$:

$$f_i = \frac{8\pi e}{3m^2} \int_{eU}^W (W - eU) f_i(a, W) dW.$$  

(7)

To obtain the expression, when $W \geq eU$ we use the boundary condition (5):
\[ j_e = \frac{8\pi e}{3m^2 \gamma_0 eU} \int_{-\infty}^{\infty} \frac{(W-eU)f_e(W)dw}{1+\frac{2}{3} \frac{m \gamma_0}{(r/a)^n} D(W-e\varphi(r))}. \]

Having in mind that away from the probe’s sheath \( W = \frac{mv^2}{2} \Leftrightarrow \frac{2}{m} = \frac{v^2}{W} \) and the coefficient of the global diffusion in the non local approach is \( D(W) = \frac{v^2 \lambda_e(W)}{3} \), we write the probe current density:

\[ j_e(U) = \frac{8\pi e}{3m^2 \gamma_0 eU} \int_{-\infty}^{\infty} \frac{(W-eU)f_e(W)dw}{1+\frac{1}{W} \frac{\lambda_e(W) \gamma_0}{(r/a)^n} D(W-e\varphi(r))}. \] (8)

The above expression can be further simplified, denoting with \( \psi(W) \) the diffusion parameter:

\[ \psi(W) = \frac{1}{\lambda_e(W) \gamma_0} \int_{-\infty}^{\infty} \frac{D(W)dr}{(r/a)^n D(W-e\varphi(r))}. \] (9)

Now let us consider a cylindrical Langmuir probe \(( n = 1)\) with radius \( a \), length \( L \) and probe area \( S \), oriented perpendicularly with respect to the magnetic field in tokamak edge turbulent plasma. We can write the diffusion parameter \( \psi_\perp \) as follows:

\[ \psi_\perp(W) = \frac{1}{\gamma_0 \lambda_e(W)} \int_{a}^{A} \frac{D(W)dr}{(r/a)^n D(W-e\varphi(r))}. \] (10)

In turbulent magnetized plasma the upper limit of the integral has the value \( A = \frac{\pi L}{4} \) which justification is discussed in [4]. The diffusion coefficient in (10) is usually assumed constant [3]. For the turbulent tokamak edge plasma it is not a constant. We have the ratio of the coefficient of the global diffusion \( D(W) \) over the diffusion in the vicinity of the probe \( D(W-e\varphi(r)) \):

\[ \frac{D(W)}{D(W-e\varphi(r))} = \frac{D_{\text{Bohm}}}{D} = \frac{1}{16} \frac{\lambda_e(W)}{R_a(W,B)}. \] (11)

We obtain the following value for the diffusion parameter for probe perpendicular to the magnetic field:

\[ \psi_\perp(W) = \frac{a \ln \left( \frac{\pi L}{4a} \right)}{16 \gamma_0 R_a(W,B)}. \] (12)
In this case the expression for electron probe current is:

\[ I_e(U) = \frac{8\pi e S}{5m^2\gamma_0} \int \left( W-eU \right) f_e(W) dW + \left( W-eU \right) \psi_\perp(W) \]  \hspace{1cm} (13)

We refer to Figure 1 for a comparison of the results from the model calculations for the electron probe current (13) and the experimental total probe current obtained by perpendicular to the magnetic field probe in CASTOR tokamak edge plasma.

These model calculations were made with the electron temperatures and densities evaluated from the same experimental volt – ampere (IV) curve, using first derivative probe method [4]. Since the ion probe current is less affected than the electron probe current, a better comparison is made using its first derivative (see Figure 2). One finds a satisfactory agreement. Moreover, the comparison of the first derivatives allows us to calculate the value of the plasma potential [4].

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References