Comparison of rotation damping by MHD with NTV theory in MAST

M.-D. Hua\(^1,2\), I.T. Chapman\(^3\), A.R. Field\(^3\), L. Garzotti\(^3\), S.D. Pinches\(^3\)

\(^1\)Imperial College, SW7 2AZ, London, UK.
\(^2\)Ecole Polytechnique, Route de Saclay, 91128, Palaiseau, France.
\(^3\)EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, OX143DB, UK.

1. Introduction

Plasma rotation is of particular interest in tokamak research as it is believed to help suppress macro- and micro-instabilities. In MAST, torque from Neutral Beam Injection heating spins the plasma up to velocities \(\sim300\ \text{km.s}^{-1}\). Some discharges with a reversed core safety factor profile \((q)\) are affected by a sustained internal MHD mode which can be observed on the data from Soft X-Ray cameras (SXR). This mode occurs in a number of L- and H-modes with the usual MAST toroidal field \(\sim0.5\ \text{T}\), plasma current \(\sim700\ \text{kA}\), and NBI heating power above 1MW. Analysis of the external magnetic Mirnov coil signals shows that this mode has a toroidal number of \(n=1\). Simultaneously, strong damping of the core rotation is measured by Charge Exchange Recombination Spectroscopy (CXRS). The short time scale of this process (\(\sim10\text{ms}\)) compared to MAST confinement times (\(\sim50\text{ms}\)) excludes MHD-induced transport as a cause for the plasma braking. Fast Thomson Scattering (TS) temperature profiles and the relative phases of different SXR channels show no evidence of the presence of magnetic islands, which rules out electromagnetic torques associated with resistive effects at resonant layers. Moreover, these torques are strongly localised, inconsistent with a simultaneous slowing down of the entire plasma core. In contrast, the non-resonant instantaneous distributed torque predicted by Neoclassical Toroidal Viscosity (NTV, [1]) seems well suited to describe the measured braking of the plasma by this MHD mode.

The objective of this study is to investigate whether the torque predicted by NTV is consistent with experimental observations. The structure of this mode is first determined using the CASTOR linear code [2], its saturated amplitude is estimated by comparing results of forward simulations of the SXR emission to the measurements. Based on this information, the NTV torque is calculated and compared to experimental data.

2. Structure of the MHD mode from CASTOR and Soft X-ray measurements

The structure of the MHD mode is determined using the linear CASTOR code with zero resistivity, a setting justified by the absence of any island in the plasma. The calculation is performed for a toroidal number of \(n=1\) and in complete toroidal geometry, thus including a full spectrum of poloidal numbers. It is based on an equilibrium reconstruction constrained to the pressure from TS and CXRS, and the field line pitch angle from the Motional Stark Effect diagnostic (MSE). This structure is a linearly growing structure, hence the need to determine its amplitude.

The simulated SXR emissivity is based on the theoretical expression of bremsstrahlung emission, \(\varepsilon_{\text{SXR}} \propto \left(n_i n_e Z_{\text{eff}}^2 T_e^2 \sqrt{T_e} \right) \int \exp(-h\nu/T_e) \nu \text{d}\nu\). It is a flux function, with \(n_i, n_e\) and \(T_e\) the ion density, electron density and temperature measured by TS, and assuming quasi-neutrality. \(\nu\) is the emission frequency and \(Z_{\text{eff}}\) is obtained by analysis of the bremsstrahlung emission on an equilibrium timescale. The shape of the flux surfaces can be inferred from the equilibrium field, the magnetic perturbation and its assumed amplitude. Since the resulting structure rotates toroidally, it produces fluctuations measured
by the SXR cameras. For the simulation, this rotation is deduced from the experimental SXR data by cross-correlation techniques. This allows the mapping of the distorted flux surfaces seen by the cameras, hence the simulation of the line integrated SXR measurements. This forward analysis is carried out for different mode amplitudes and compared to the experimental data. Since the MHD perturbation does not affect the average SXR signal but only results in its variation, the comparison with the experimental data is based on this fluctuation only. This method reduces the influence of parasitic SXR sources, as for example impurity line emission. The simulation with the lowest residuals with respect to the measurement is then considered a good estimate of the mode’s amplitude.

3. Torque according to the Neoclassical Toroidal Viscosity theory

NTV theory has been applied extensively to externally-driven, static, magnetic perturbation cases [3]. This theory can also be used if the field’s axisymmetry is broken by the presence of an MHD instability [4], in which case the torque arises from the differential flow of the plasma through the non-axisymmetric perturbation. The flow considered here is the motion of the ion fluid, the velocity of which is assumed to be equal to that of the carbon fluid measured by CXRS.

An intuitive understanding of the slowing down process is as follows: the mode’s structure can only remain coherent if it rotates as a rigid body, while single-fluid MHD theory prescribes the plasma, despite its sheared rotation, to be frozen in the magnetic field. In effect, multi-fluid MHD equations allow the motion of the magnetic field lines to depart from that of the fluid [5], [6]. This shift is of the order of the diamagnetic frequency, and results in a less strict, shifted, frozen-in condition. These two competing velocity profiles, the one of the mode and that of the fluid, are brought in agreement during a transition phase: the slowing down of the plasma. In a collisional plasma, this is because the fluid contracts and expands as it goes through the distorted flux surfaces, thus leading to viscous dissipation in a mechanism similar to magnetic pumping [7]. In collisionless cases, the orbit of banana-trapped particles drifts radially, creating a radial current which acts on the flow by generating a \( \vec{j} \times \vec{B} \) torque.
The NTV theory is expressed in straight field line, constant Jacobian, Hamada coordinates \((v, \theta, \zeta)\), and requires the Lagrangian expression of the total magnetic field’s modulus to be written as:

\[
\mathcal{B}(\tilde{X} + \tilde{\zeta}) = \left| \mathcal{B}_0 + \sum_{n,m} (b_{n,m}/B_0) e^{i(n\theta - n\zeta)} \right| = B_0 \left( 1 + \sum_{n,m} (b_{n,m}/B_0) e^{i(n\theta - n\zeta)} \right).
\]

Here, \(\xi\) is the displacement and \(B_0\) the equilibrium field. This is done firstly by building the Hamada coordinates from the usual cylindrical ones, and secondly by exploiting the equilibrium axisymmetry while rewriting the third term as \(\left| \left( \tilde{X} + \tilde{\zeta} \right) \right| = \left( \tilde{B}_0 \left( \tilde{X} + \tilde{\zeta} \right) - \tilde{B}_0 \left( \tilde{X} \right) + O(\xi^2) \right).\) In the low collisionality regime relevant for MAST plasmas, a convenient formula for the NTV torque can be found in [3] and adapted to a perturbation rotating at a frequency \(\omega_{MHD}\):

\[
\tau_{NTV} = \frac{1}{2} n_i (eT_i/2v_i) \frac{R^2B_b \xi}{\epsilon^{3/2}} \left[ \langle R R^{-2} \rangle \frac{1}{2} \sum |n b_{n,m}|^2 W_{n,m} \left[ (\omega_\phi - \omega_{NC}^*) - \omega_{MHD} \right] \right]
\]

\(T_i, v_i, R, \epsilon, \) and \(\omega_\phi\) are the ion temperature, ion-ion collision frequency, local major radius, local inverse aspect ratio and ion toroidal angular frequency. Angle brackets denote average over the flux surface, \(\omega_{NC}^* = (3.5/ZeB_0R)(dT_i/dr)\) is an offset velocity with \(r\) the minor radius, and \(W_{n,m}\) is defined in [3]. The ion temperature is measured by CXRS, while geometrical factors are taken from the equilibrium reconstruction. The frequency term of this expression makes the NTV torque proportional to the difference of the plasma and mode’s frequencies \((\omega_\phi - \omega_{NC}^*)\) with a shift proportional to the ion temperature gradient \(\omega_{NC}^*\), consistent with the relaxed, shifted, frozen-in condition introduced earlier.

4. Results and discussion

The most unstable linear MHD structure calculated by CASTOR for this equilibrium is an internal \(n=1\) kink mode, shown in figure 1a). It is assumed that, as the mode saturates, its structure remains identical to this earlier one. This assumption, although quite strong, is likely to be verified outside the mode’s resonant surfaces and inertial layer. The best match between simulation and data, shown on figure 1b), is obtained for a displacement with a radial amplitude of 1.2 cm. The amplitudes of the fluctuations are well matched for each channel. The relative phases between the observed SXR channels are not only reproduced by the simulation, but are also consistent with those expected from a kink mode. Interestingly, the phases of SXR traces produced by an island would not be consistent with the observed phases. This further justifies the ruling out of resonant torque in this study.
The rotation frequency profiles of the plasma during the braking are shown in figure 2a). The rotation is unchanged for the radial location $R=1.15\,\text{m}$. It is tempting to interpret this point as the position where the mode’s frequency and that of plasma are equal, hence where one would expect the torque applied by the mode to vanish. This is actually not the case, and the dashed line in figure 2a) indicates the frequency of the mode at its onset. Although it decreases on equilibrium timescales, this latter frequency does not reach that of the plasma at $R=1.15\,\text{m}$ at any time. This frequency gap can be explained, in an intuitive manner, by the shifted frozen-in condition mentioned earlier, or, in a mathematical way, by the presence of the offset velocity $\omega_{\text{NC}}$ in the NTV formulation. Incidentally, these two shifts are of the order of the diamagnetic frequency, and so is the difference in frequency between the mode and the point of unchanged rotation.

The torque predicted by NTV at $t=255\,\text{ms}$ is plotted in figure 2b), together with the measured rate of change of angular momentum for each flux tube. The prediction and observation have the same order of magnitude. The profile shapes are similar, except in the vicinity of the rational surfaces and the inertial layer. The latter is situated at the position of minimal $q$, which is slightly above one. In these regions, large parallel magnetic field perturbations result in a high torque which is not observed on experimental data. Since the linear structure used here is likely to differ from the saturated one at these locations, this disagreement is not regarded as invalidating the applicability of the theory to the observations. The large number of plasma profiles involved in this calculation means that uncertainties are difficult to assess and may be large. These do not compromise the results presented here, but they do not allow more detailed comparisons. It is however worth mentioning that the inclusion of the offset velocity $\omega_{\text{NC}}$ is crucial in order to reproduce the measured torque profile with NTV theory. Furthermore, the Lagrangian term $\mathbf{\xi} \cdot \nabla \mathbf{B}$ must be taken into account to predict a torque of magnitude comparable to the one observed experimentally, its absence decreasing the calculated result by up to 70%, especially in the vicinity of the magnetic axis.

MHD-induced transport and resonant MHD torques appear inadequate to explain the observed damping of core rotation by MHD in MAST. In contrast, this study shows that plasma braking predicted by NTV theory presents strong similarities with experimental data, making it a possible mechanism for the flattening of the rotation profile. Further work will include the comparison of SXR-inferred saturation amplitudes to theoretical values, and additional stability analysis of reversed magnetic shear equilibria with $q_{\text{min}} \approx 1$, where the sustained kink mode described in this paper occurs.

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References


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