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Fast Ion Driven Alfvén Eigenmodes within the $q = 1$ Radius

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I. Introduction

In JET discharges with high-power ICRH and a central safety factor lower than unity, monster sawtooth crashes are preceeded by so called *tornado modes* [1], see figure 1 (left), observed experimentally using both external Mirnov coils and interferometry diagnostics. These modes, with frequencies in the toroidal Alfvén eigenmode (TAE) range, appear with decreasing toroidal mode numbers one by one and they exhibit a peculiar downward sweep in frequency, initially deviating from the usual TAE scaling $f_{TAE} \propto B/\sqrt{n}$. Moreover, the tornado modes appear simultaneaously with a decrease in the intensity of γ-rays emitted from hot ions in the plasma center, implying that the tornado modes are responsible for a reduction of the hot ion population inside the $q = 1$ magnetic surface, and that this redistribution may in fact be the cause of the monster sawtooth crashes.

O-mode interferometry reveals another interesting feature of the magnetic spectrum on JET: The tornado modes are often preceeded by a set of core-localized modes in the range $50 - 200$ kHz. These modes behave similar to Alfvén cascades (ACs) in reversed-shear plasmas [2], in that their frequencies sweep up more or less linearly in time, seemingly from the same initial frequency but with different sweeping rates. Similar modes have been observed on the Alcator C-Mod tokamak using phase constrast imaging diagnostics [3].

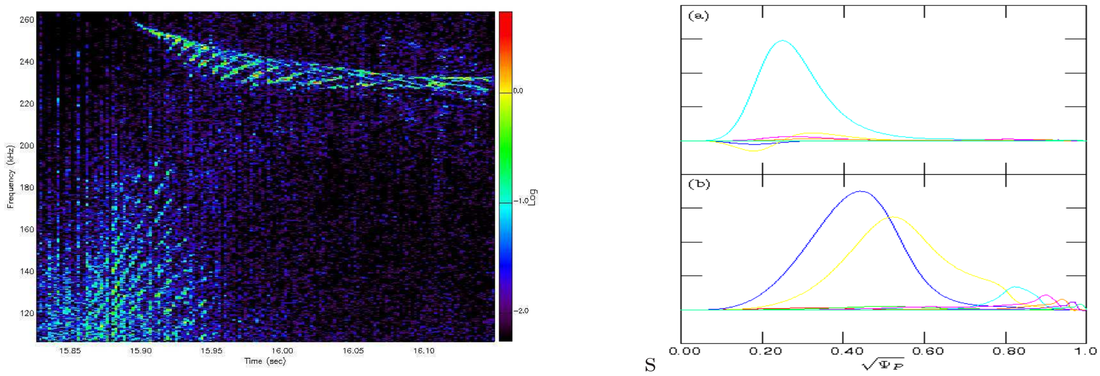

Figure 1: Left: Magnetic spectrum of the plasma center during shot 66203 on JET, measured by O-mode interferometry diagnostics. Both tornado modes and cascading modes are clearly visible. Right: Mode structure of (a) cascading mode with $q_0 = 0.95$ and (b) tornado mode with $q_0 = 0.91$ obtained numerically using MISHKA-1.

Two important features of tokamak plasmas undergoing monster sawtooth oscillations is the smallness of the magnetic shear inside the $q = 1$ radius and the proximity to the magnetic axis. When the shear $s = r/q\ dq/dr$ satisfies the condition $s < \epsilon$, with the inserve aspect ratio

$\epsilon = r/R_0$, so called multiple low shear TAEs (MLSTAEs) may be supported by the tokamak geometry itself. Moreover, a flat q-profile implies broader radial mode structures, with the result that resonating particles interact with the waves during longer times.

It is of high importance to clarify what kind of Alfvén eigenmodes may exist within the $q = 1$ radius, how these eigenmodes are connected to the behavior of the central safety factor profile and how they affect hot ions inside the $q = 1$ surface. The aim of the present EPS contribution is to identify the observed modes and their connection to the safety factor. Calculations of the transport of fast ions through the $q = 1$ surface during monster sawtooth crashes in the presence of the modes identified in this work will be the aim of future investigations.

II. Mode Identification

In order to explain the two types of modes observed, two different paths were explored. First, a kinetic Alfvén wave, residing in a potential well at the magnetic axis, was considered analytically. Second, a slightly reversed low-shear equilibrium was created and studied numerically to find the possible modes within ideal MHD theory.

A. Kinetic Modes

Kinetic modes close to the magnetic were first predicted theoretically by Rosenbluth and Rutherford in 1975 [4]. By examination of a narrow region well off the magnetic axis, where $d/dr \gg 1/r$, they used WKB theory to solve a generalization of the MHD equation for the radial dependence of the mode amplitude (including finite ion Larmor radius effects and finite parallel electron conductivity), and finally obtain a fast varying "kinetic" Alfvén wave. A similar approach was used by Konovalov et al. [5] to explain certain modes with sweeping down frequencies which are sometimes observed following ACs in reversed-shear plasmas.

In the close proximity of the magnetic axis, the ordering $d/dr \sim 1/r$ and the high values of T_e and T_i suggests that non-ideal effects may play an important role. For large mode numbers, $m^2 \gg 1$, the non-ideal differential equation for the mode amplitude ξ is given by (cf. [4])

$$\omega^2 \frac{7}{4} \rho_i^2 \frac{d^4\xi}{dr^4} - \frac{m^2}{r^2} \left(\omega^2 - \omega_A^2\right) \xi = 0 \ , \tag{1}$$

with the additional restriction that the solutions are regular at $r = 0$ and everywhere finite. Here, ω is the mode frequency, ρ_i is the ion Larmor radius, m is the poloidal mode number and $\omega_A = k_\parallel v_A$ is the Alfvén frequency, $k_\parallel$ and the Alfvén velocity v_A being given respectively by

$$k_\parallel = \frac{1}{R_0} \left(\frac{m}{q} - n\right) \ , \quad v_A = \frac{B}{\mu_0 \rho} \ .$$

Expanding $k_\parallel^2$ to second order in r around the magnetic axis and looking for modes with frequencies close to the on-axis Alfvén frequency $\omega_{A0} = \omega_A \left(r = 0\right)$, equation (1) becomes

$$\Lambda^2 \frac{d^4\xi}{dr^4} - \frac{1}{r^2} \left(\Delta + \alpha r^2\right) \xi = 0 \ , \tag{2}$$

where the mode frequency has been written as $\omega = \omega_{A0} + \delta\omega$, $\delta\omega \ll \omega_A$, and we have defined

$$\Lambda^2 \equiv \frac{7\rho_i^2}{4m^2} \ , \quad \Delta \equiv \frac{2\delta\omega}{\omega_{A0}} \ , \quad \alpha \equiv \frac{mq_0''}{R_0 q_0^2 k_{\|0}} \ .$$

Note that all quantities denoted by 0 are to be calculated at the magnetic axis, $r = 0$. If we transform to the dimensionless quantities

$$z = \left(\frac{|\alpha|}{\Lambda^2}\right)^{\frac{1}{4}} r \ , \quad E = \frac{\Delta}{\Lambda\sqrt{|\alpha|}} \sim 1 \ ,$$

equation (2) takes on the form

$$\frac{d^4\xi}{dz^4} - \frac{1}{z^2}\left(E \pm z^2\right)\xi = 0 \ , \tag{3}$$

where the $\pm$ in front of z^2 represents the sign of α. The existence of kinetic eigenmodes satisfying equation (3) may very well depend on the sign of α, which is in turn determined by the signs of q_0'' and the mode numbers m and n. Assuming that q_0 decreases in time from $q_0 = 1$, $\alpha > 0$ implies that the mode frequency experiences upward sweeping for $q_0'' > 0$ and downward sweeping for $q_0'' < 0$, while $\alpha < 0$ implies that the mode frequency sweeps up for $q_0'' < 0$ and down for $q_0'' > 0$. Finally, the frequency shift $\delta\omega$ of the kinetic modes may be estimated via the rescaling expression for Δ, resulting in

$$\delta\omega = E\frac{v_A\rho_i}{4q_0}\sqrt{\frac{7}{R_0}\left|\frac{mk_{\|0}q_0''}{m}\right|} \sim 1 \text{ kHz} \ .$$

Modes with such small $\delta\omega$ are not distinguishable from shear Alfvén continuum modes, satisfying $\omega^2 = \omega_A^2$, by present day diagnostics. Unambiguous observations of these modes would require knowledge of the signs of q_0'' and the mode numbers. However, the dependence of $\delta\omega$ on q_0'' suggests the possibility that observations of up-sweeping kinetic modes could offer new diagnostic oppurtunities for the magnetic shear.

B. Ideal MHD Modes

Further away from the magnetic axis, ϵ^2-terms become more important than the non-ideal effects already considered, and AC-like Alfvén eigenmodes can be found within ideal MHD theory [6]. In order to study this regime, the equilibrium code HELENA was used to construct a low-shear equilibrium with a slighlty reversed safety factor profile, see figure 2 (left). Solutions for the ideal MHD eigenmode equations were then obtained using MISHKA-1, a full geometry ideal MHD stability code, to scan through the relevant values of q_0, ranging from $q_0 = 1$ and down.

Both sweeping up, core-localized ACs and tornado-like TAEs, located somewhat more off axis and moving radially outwards, were found for all toroidal mode numbers considered. As can be seen in figure 2 (right), the frequencies of the ACs sweep from 0 at $q_0 = 1$ all the way up to the TAE frequency range, where the TAEs appear with frequencies that evolve in a manner quite similar to the experimentally observed spectrum in figure 1 (left). It should be mentioned that the TAEs develop strong interaction with global AEs as they move outwards, effectively prohibiting identification of the radial asymmetry observed in [1], see figure 1 (right).

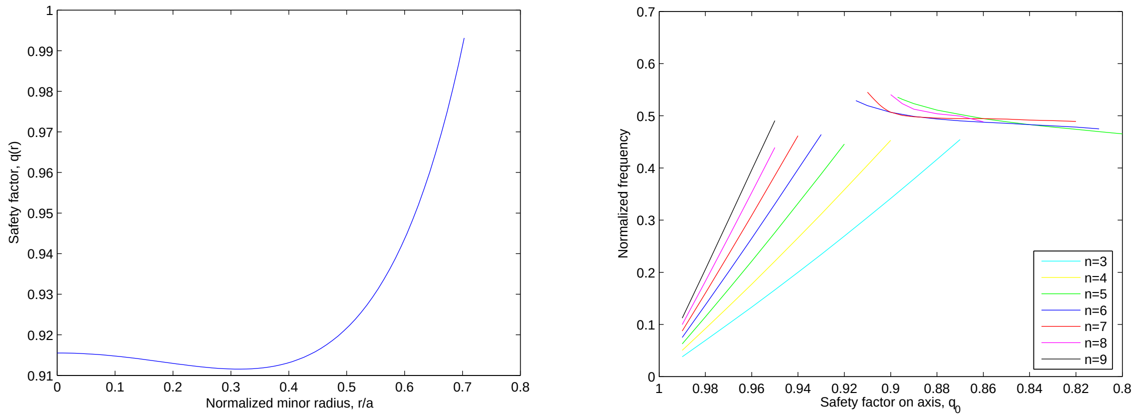

Figure 2: Left: Zoom in $(q(r) < 1)$ of the slightly reversed safety factor profile of the low-shear equilibrium created by HELENA. Right: Mode frequencies, with toroidal mode number n, for the modes identified with MISHKA-1.

III. Concluding Remarks

Two paths were explored in order to explain core-localized, cascading modes and tornado modes, both observed just before monster sawtooth crashes on JET and Alcator C-Mod. First, kinetic Alfvén modes with large mode numbers were treated analytically in the proximity of the magnetic axis. Second, core-localized ACs as well as TAEs exhibiting tornado behavior were found numerically within ideal MHD theory using a slightly reversed, low-shear safety factor profile. Both types of modes are plausible candidates for explaining the observations, and in order to draw unambiguous conclusions, measurements of mode localization, mode number sign and magnetic shear,s as well as better frequency resolution in the experiments are needed. The modes identified here will be used in future modelling of fast ion redistribution through the $q = 1$ surface during monster sawtooth crashes.

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