Modelling of (2,1) NTM threshold in JET

P. Maget\textsuperscript{1}, H. Lütjens\textsuperscript{2}, R. Coelho\textsuperscript{3}, B. Alper\textsuperscript{4}, M. Brix\textsuperscript{4}, P. Buratti\textsuperscript{5}, R.J. Buttery\textsuperscript{4}, H. De la Luna\textsuperscript{6}, N. Hawkes\textsuperscript{4}, I. Jenkins\textsuperscript{4}, C.D. Challis\textsuperscript{4}, C. Giroud\textsuperscript{4}, X. Litaudon\textsuperscript{1}, J. Mailloux\textsuperscript{4} and EFDA Contributors\textsuperscript{*}

\textit{JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK}

\textsuperscript{1}CEA, IRFM, F-13108 Saint Paul-lez-Durance, France. \textsuperscript{2}Centre de Physique Théorique, Ecole Polytechnique, CNRS, France. \textsuperscript{3}Inst Plasmas & Fusao Nucl, EURATOM Assoc, IST, P-1049001 Lisbon, Portugal. \textsuperscript{4}Euratom/UKAEA Fusion Association, Culham Science Centre, Abingdon OX14 3DB, UK. \textsuperscript{5}ENEA Fus, EURATOM Assoc, I-00040 Frascati, Italy. \textsuperscript{6}Laboratorio Nacional de Fusión, Asociación EURATOM-CIEMAT, Madrid, Spain.

Introduction

High performance, advanced scenario discharges with $q_{\text{min}}$ above unity are often limited by the $n=1$ MHD mode in JET \cite{1, 2}. Although the resistive nature of the mode is not clear in its initial phase, it evolves to a large island on $q=2$, with significant confinement degradation ($\sim 15\%$ in $H_{89}$ in our example). We investigate here the possibility that this mode is a (2,1) Neoclassical Tearing Mode (NTM) by computing the critical island width at which such mode would be unstable, using the non linear MHD code XTOR \cite{3, 4} where the relevant bootstrap current physics is accounted for, as well as perpendicular and parallel heat transport. Two different situations observed in the JET Tokamak are studied. In the first case, where the normalized kinetic pressure is at $\beta_N \sim 2.5$, slightly above the computed no-wall limit (with MISHKA), $n=1$ MHD activity consists of short bursts that do not impact the performances, while in the second experimental case, with $\beta_N \sim 2.8$, still above the computed no-wall limit, bursts of $n=1$ mode are followed by a transition to a long-lived mode with significant confinement degradation (figure 1). Quasi-linear calculations with XTOR show that the (2,1) NTM threshold is decreasing when approaching the observed mode triggering time, the main reason being the increase of the local magnetic shear at $q=2$ as the plasma current diffuses. Although sensitive to the input equilibrium, these results are consistent with a limitation of these discharges by (2,1) NTM, assuming a seeding process of constant amplitude.

Non-linear MHD model and Rutherford equation

We solve the standard fluid MHD model equations with fixed density implemented in XTOR:

\[
\begin{align*}
\rho (\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v}) &= \mathbf{J} \times \mathbf{B} - \nabla p + \mathbf{v} \nabla^2 \mathbf{v} \\
\partial_t p + \mathbf{v} \cdot \nabla p + \Gamma p \nabla \cdot \mathbf{v} &= \nabla \cdot \mathbf{\chi}_\perp \nabla p + \mathbf{B} \cdot \nabla \left[ \frac{\mathbf{\chi}_\parallel}{\mathbf{B} \cdot \nabla p} \right] + H \\
\partial_t \mathbf{B} &= \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \eta (\mathbf{J} - \mathbf{J}_{NI})
\end{align*}
\]

where \( \mathbf{J}_{NI} = \mathbf{J}_{cd} + \mathbf{J}_{bs} \). \( \mathbf{J}_{bs} \) is the bootstrap current, \( \mathbf{J}_{cd} \) is the imposed current source \( (\mathbf{J}_{cd} = (\mathbf{J} - \mathbf{J}_{bs})_{t=0}) \), and \( H = -\nabla \cdot \mathbf{\chi}_\perp \nabla p(t = 0) \) is the heat source term. The bootstrap current is modelled as \( \mathbf{J}_{bs} = f_s J_{bs}^q (\nabla p(t)/\nabla p^q) \mathbf{B}/B \) with \( f_s \) a free parameter for rescaling the total bootstrap current \( J_{bs}^q \). Note that the equilibrium is not modified when varying \( f_s \). The threshold for the (2,1) NTM is obtained by varying the seed island size until the NTM branch is found.

The result is compared to the following form of the Rutherford equation, that covers the same physics as the code, i.e. includes curvature [5] and bootstrap [6] contributions:

\[
0.825^{-1} \frac{dW}{dt} = a \Delta' - 6.35 \frac{D_R}{W^2 + 0.65 W^2} + 6.35 f_s J_{bs}^q \frac{W}{s W^2 + (1.8 W)^2}
\]

where the various terms are evaluated at \( q = 2 \), \( S \) is the Lundquist number \( (S = \tau_R/\tau_A) \), with \( \tau_R = \mu_0 a^2 / \eta \) and \( \tau_A = R_0 \sqrt{\mu_0 p / B_0} \), \( J_{bs}^q = (\mu_0 R_0 / B_0) J_{bs}^q \) with \( R_0 \) and \( B_0 \) the major radius and magnetic field at geometric axis, and \( W \equiv w/a \). The classical tearing parameter is approximated by \( a \Delta' = -2m/x \) where \( x = \sqrt{\Phi} \) and \( \Phi \) is the normalized toroidal flux. We have also defined \( W_\chi = 2 \sqrt{2} (\chi_\perp / \chi_\parallel)^{1/4} / \sqrt{\text{ens}} \) with \( s \) the magnetic shear and \( \text{ens} = a^2 / R_0 \). The critical island width is mainly determined by the balance between the curvature term and the bootstrap term.

Critical island width: comparison to Rutherford evaluation

Our study is based on equilibrium reconstructions using the EFIT code constrained by MSE, Polarimetry and pressure measurements. We consider a first discharge where \( n = 1 \) activity is limited to short bursts (74226 at \( t = 7s \), figure 2), and another discharge where after similar bursts a long lived \( n = 1 \) mode develops (72668 at \( t = 6s \) and \( t = 7s \), figure 1). The corresponding safety factor profiles are shown in figure 3. For discharge 72668, the main evolution of the equilibrium when approaching the mode triggering time is the diffusion of the plasma current, resulting in an outward displacement of the \( q = 2 \) surface. The heat diffusivity values are strongly constrained for NTM studies. Indeed, the relative dynamics of pressure and plasma current must be similar to that in the experiment, since the bootstrap current is driven by pressure gradient. This imposes that \( \delta \chi_\perp = (\delta \chi_\perp)_{\text{exp}} \), where \( \chi_\perp \) is the perpendicular diffusivity (normalized to \( a^2 / \tau_A \)). The experimental diffusivity is evaluated in TRANSP for pulse 7268,
and we deduce $S\chi_\perp \approx 400$. For pulse 74226, which is at higher magnetic field and plasma current, and therefore better confinement, we take $S\chi_\perp = 100$. In both cases, we have checked that the critical island width is weakly dependent on $\chi_\perp$ around the value taken in the simulation.

The NTM threshold is calculated from quasi-linear runs ($n=0$ and $n=1$ only) at $S(0) = 10^7$ as a function of the bootstrap factor $f_x$, and compared to the evaluation coming from the simple Rutherford model described earlier. For the equilibrium of pulse 72668 at $t=7s$, the correspondence between $f_x$ and the bootstrap fraction is $f_{bs} = I_{bs}/I_p \approx 0.76 f_x$, and the TRANSP bootstrap fraction is $f_{bs} \approx 0.34$, resulting in an appropriate coefficient $f_x \approx 0.4$. We find that the (2,1) NTM has a higher threshold for the 2 equilibria where the $n=1$ mode is limited to short bursts compared to the equilibrium where the resistive mode develops. This difference is found from the Rutherford evaluation and is even more pronounced in the XTOR calculations (figure 4). The dominant effect that can be identified is the increase of the magnetic shear between $t = 6s$ and $t = 7s$. Taking only the dominant radial position and magnetic shear dependencies of the parameters involved in the Rutherford equation, i.e. $a\Delta' \propto 1/x$, $D_R \propto -xp'/x^2$, $W_\chi \propto 1/\sqrt{x}$, $J_{bs} \propto -p'/\sqrt{x}$ and assuming a typical pressure profile $p = p_0(1-x^2)^2$, we can calculate the critical island width variation around the point given by the reconstruction of pulse 72668 at $t = 7s$. We then find that moving the resonance outward increases the critical width, while increasing the magnetic shear lowers it. Between $t = 6s$ and $t = 7s$, the increase of shear has a more important effect that the outward displacement, resulting in a lower threshold consistent with the mode triggering (figure 5), if we assume a seeding process of constant amplitude.
Figure 4: Critical island widths from XTOR ([σ] ) and Rutherford equation (− ▲ ).

Figure 5: Effect of magnetic shear and position of q = 2 surface on critical island width.

Figure 6: Non-Linear XTOR runs: q = 2 island size (top) and maximum pressure (bottom).

Non-linear simulations and comparison to experiment

Non-linear simulations have been performed in order to make more complete comparison with experimental observations, in particular with the impact of the mode on the confinement. In the non-linear regime, the (2,1) NTM is found to generate a large stochastic region, resulting in a significant confinement degradation. For \( f_{bs} = 0.38 \), the core pressure is reduced by about 12% at the latest simulation step (figure 6). Note however that the saturation is not yet reached in the simulation (\( 10^4 \tau_A \) corresponds to \( \sim 300ms \)). In the experiment, the confinement degradation takes place in about 1 second, and is in the range 20-30% for temperatures and 30-40% for density (figure 1), but we remind that the latter is not evolved in the simulations presented here. Comparison of mode structure with experimental measurements in the non-linear regime shows than the \( n = 1 \) mode is well reproduced, apart from a radial shift that could originate from inaccuracy of the magnetic equilibrium. We also find that the (3,2) NTM is destabilized in the simulation, in contrast with the experiment where the \( n=2 \) mode develops at \( q = 5/2 \) only.

References