

## TOKAMAK PLASMA INDUCTANCE CONTROL AT JET

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### I. INTRODUCTION

Plasma inductance control is an essential profile control tool for tokamaks that can be used to extend pulse duration, access to advanced regimes, reduce vertical instability growth rate, and improve experiment reproducibility. To better understand the inductance control problem, we derive a lumped parameter model that approximates a process that is inherently a distributed parameter system. This model is then used to design an inductance control system, and its performance studied with simulations. Experiments done at JET in support of the controller have help to identify some of the issues regarding the use of boundary flux as actuator. Inductance control with neutral beams has been achieved with different degree of efficiency.

### II. STATE SPACE MODEL

The dynamics of the flux diffusion, plasma inductance and plasma current can be described by state space model in the form

$$\begin{aligned} dx/dt &= f(x) + g(x)u(V_b, R, \hat{I}, x) \\ y &= h(x) \\ h(x) &= \left( x_1 x_3^2, \frac{x_2}{x_3}, x_3 \right) f(x) = \left( 0, 0, \frac{(x_3^0 - x_3)}{\tau} \right)^T g(x) = \left( \frac{2(x_3 - 1)}{x_2 x_3}, \frac{(2 - x_3)}{x_1 x_3}, -\frac{x_3}{x_1 x_2} \right)^T \end{aligned} \quad (1)$$

The dynamics is captured by a first order differential equation for a state space vector  $x = (x_1, x_2, x_3)^T$ . The driving term is an effective voltage input  $u(V_b, R, \hat{I}, x) = V_b - Rx_2/x_3 + R\hat{I}$  encompassing applied boundary voltage  $V_b$ , plasma resistance  $R$ , and current drive  $\hat{I}$  and state vector components. The output equation of this model gives inductance, plasma current and an equilibrium related state.  $y = (L_i, I, (\psi_C - \psi_B)/(\psi_R - \psi_B))^T = (x_1 x_3^2, x_2/x_3, x_3)^T$ . The resistive  $\psi_R$  and equilibrium  $\psi_C$  fluxes are defined from poloidal flux  $\psi$  and current density  $j$  as

$$I = \int_{\Omega} j dS \quad \psi_C = \frac{\int_{\Omega} \psi j dS}{I} \quad R = \frac{\int_{\Omega} \eta j^2 dS}{I^2} \quad \frac{\hat{I}}{I} = \frac{\int_{\Omega} j \hat{j} dS}{\int_{\Omega} \eta j^2 dS} \quad \psi_R = -\int_0^t V_R dt = -\int_0^t R(I - \hat{I}) dt \quad (2)$$

Where  $\eta$  is plasma resistivity and  $\hat{j}$  is the non inductive current density. At the heart of this model is the equilibrium state  $x_3 = (\psi_C - \psi_B)/(\psi_R - \psi_B)$ . The dynamics of the equilibrium state is directly linked to flux diffusion, and is approximated by a first order system around a steady state equilibrium solution  $x_3^0$ . If a single operation point  $x_3^0$  for the equilibrium exists for a single plasma discharge, a time constant  $\tau$  measures how the

equilibrium approaches or departures from this operation point. This time constant is related to ( but is not) the skin time. The model presented fulfils Poynting's energy theorem in tokamak geometry [1]. The advantage of the state space formulation is that the output equation can be augmented with any combination of the system states and inputs to give any desired output, without changing the dynamic equations associated to the states. In our model, for instance, the magnetic energy is  $W = \frac{x_1 x_2^2}{2}$ , the inductive flux [1] is  $\psi_{ind} = x_1 x_2$ , the voltage at the equilibrium flux surface  $\psi_C$  is  $V_C = (x_3 - x_3^0) x_1 x_2 / \tau + V_B$ , etc. The validity of the state space model in the ohmic regime for two discharges with step up/down on plasma current and negligible current drive is illustrated in Fig. 1. Due the slow response of the system, there is a large sensitivity to initial conditions. Hence, model parameters and initial conditions for the states are found by running an optimization algorithm to find the best match to the experimental data. This leads to  $x_3^0 \cong 0.98$  and  $\tau \cong 1.25$ .

### III. CORRELATION WITH RAMP-RATES

Taking the ratio between inductance and current time derivatives at the state space model, the correlation between plasma current ramp rates and inductance changes is unveiled.

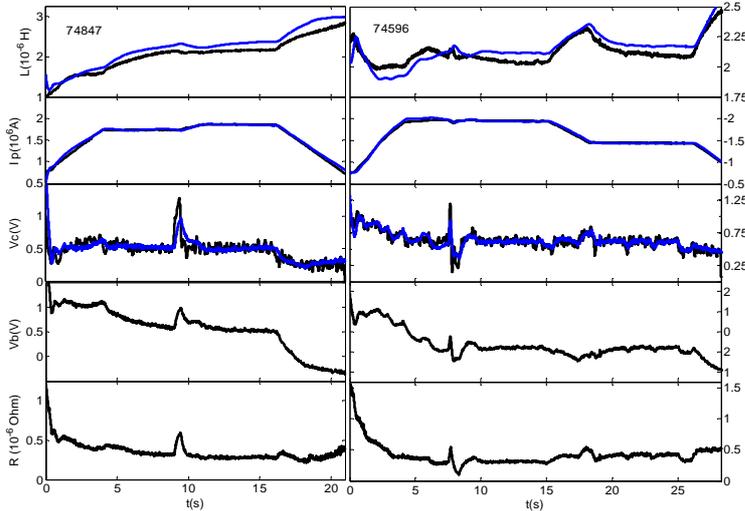


Fig. 1. Comparison between experimental readings (black) and state space model outputs (blue). In top-down order are shown the plasma internal inductance, plasma current, voltage  $V_C$ , boundary voltage  $V_B$  and plasma resistance  $R$ .

$$\frac{dL_i}{dt} = \frac{2(x_3 - 1)}{(2 - x_3)} \frac{L_i}{I} \frac{dI}{dt} + \frac{2L_i}{x_3(2 - x_3)} \frac{dx_3}{dt} \quad (3)$$

Contrary to what is commonly believed, this correlation does not imply a cause and effect. But it can be used for the design of an inductance control system based on plasma current ramp rate regulation [2], provided a reliable observer for the equilibrium state is available.

### IV. CONTROLLABILITY

If all the system states can be driven to a desired value using a finite sequence of actuator values, the system is said to be controllable. In our case this

translates in  $\det |g(x), ad_{f(x)}g(x)| = (x_3^0 - x_3) / (x_1^2 x_3^2 x_3^2) \neq 0$  [3].

In other words, controllability is lost when the profiles stop evolving. The best opportunity window for full control of the system states is when the plasma is not very hot and the equilibrium state is in full evolution, e.g. plasma current ramp-up/down phases.

### V. NON LINEAR CONTROL WITH BOUNDARY FLUX ACTUATOR

To save time, efforts and development costs, a previously developed boundary flux control tool [4] was made available as actuator to a real time network [5] for the inductance control application. The main requirement for the control is to work during the ramp-up phase of the discharge, where the plasma current changes from small to large values. The open loop gain of the system changes approximately with the inverse of the plasma current as the ramp up phase progresses. This dynamic range can be quite important for ITER. This work deals with

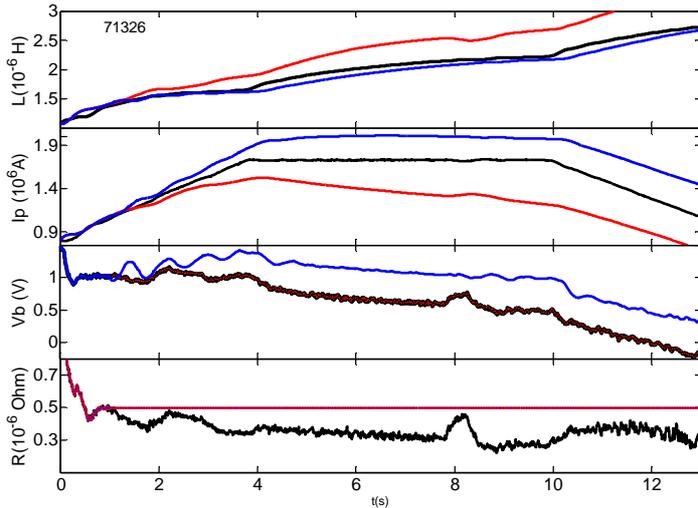


Fig.2 Simulation of inductance tracking. Reference discharge is shown in black. Red traces correspond with an open loop simulation with identical boundary voltage to the reference discharge. Blue traces are feedback simulations. In both red and blue cases Ip and L are obtained from the state space model subject to the same constant plasma resistance shown. In the red case the boundary voltage matches the reference discharge, while in the blue case the boundary voltage is issued by the control.

this non linearity using feedback linearization techniques [3]. The resulting control law for boundary voltage is  $V_b^{req} \cong RI - v/I$ , and from this  $\psi_b^{req} = -\int V_b^{req} dt$ , which can then be handled by boundary flux control [4]. A control based on boundary flux does not depend critically on an observer for the equilibrium state, as it would a control based on plasma current ramp rate regulation. Alternatively, a reference for transformer current can be issued as

$$I_1^{req} = (\psi_b^{req} - L_e I - \sum M_j I_j) / M_1$$

where  $L_e$  is the external inductance [6] and  $M_j$  are the mutual inductances between  $I_j$  poloidal field currents and the plasma. Fig. 3. shows the feasibility of

the controller proposed. A reference discharge data is shown in black. The red traces correspond with an open loop simulation with identical boundary voltage to the reference discharge, but a larger plasma resistance. Consequently, inductance increases and plasma current decreases respect to the reference case. The blue traces are feedback simulations, in which the boundary voltage is adjusted to give the inductance from the reference discharge. The change on plasma resistance is compensated by a larger voltage, resulting on larger plasma current. The full plant with non linear feedback and a PID control with  $K_p=2$ ,  $K_i=0.001$  and  $K_d=0.45$   $PID(s) = K_p (1 + K_i/s + K_d s / (0.01s + 1))$  and flux control transfer function with 160ms delay is used for the simulations.

VI. EXPERIMENTS OF INDUCTANCE CONTROL WITH BOUNDARY VOLTAGE

A version of the non linear control outlined before, including additional limits and

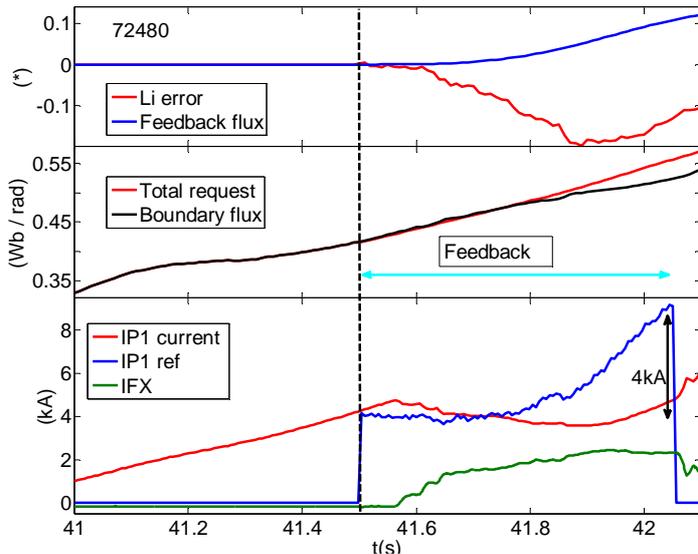


Fig. 3. Inductance control experiments using the boundary flux control tool as actuator. From top down are shown the Internal inductance error signal, flux request and relevant PF currents.

protections, timers, etc has been tested using JET's XSC [4] and RTPC [5] systems. The normalised internal inductance  $l_i = 2x_1 x_3^2 / \mu_0 r_0$  was used as reference. The experiments were performed during the ramp-up phase with strong plasma shaping. IFX current being ramped up drove boundary flux in addition to the flux being driven by the transformer coil IP1. Since its flux reference was being attained, flux control loop relaxed its current request to IP1. At some point the flux request issued by inductance control exceeds the actual boundary flux value, so flux control demands

more current from IP1, but is too late. IP1 coil is on a free fall from which will not recover quickly enough. Flux control continues to demand more current to the coil to achieve the flux request until exceeds exceeded the actual coil current by more than 4kA. At this point, the coil protection system issues a soft stop. Future experiments using the present flux control will have to be performed without simultaneous plasma shaping.

## VII. EXPERIMENTS OF INDUCTANCE CONTROL WITH BEAMS

The use of NBI during the ramp-up phase is useful to shape the profiles at an early stage. First

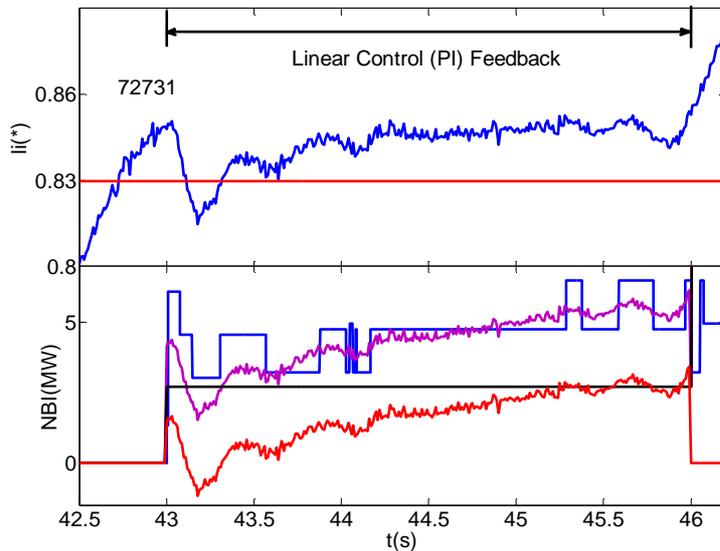


Fig. 4. Control of inductance with NBI. Top: Internal inductance reference is shown in blue, and actual real time values in red. Middle: Actual NBI power being injected. Bottom: Plasma current.

## VIII. CONCLUSIONS

A model for plasma current and inductance time evolution as function of plasma resistance, non-inductive current drive sources and boundary voltage has been obtained. It has been used to design a plasma inductance control with boundary flux as the actuator. Numerical results have show the feasibility of the proposed scheme. Experimental tests suggest to upgrade the flux control to compensate the IP1 current request with known perturbations introduced by the shaping coils currents. The ideal realisation, however, would be to act directly on the transformer primary coil voltage. Preliminary test of inductance control with beams have been successful, and will have to be integrated along with RF and LHCD in the control scheme. Further work will be devoted to integrate non inductive current drive actuators in the control, and to extend the non linear controller design to full control of the state space vector.

## IX. REFERENCES

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experiments with NBI have shown the feasibility of feedback to control the inductance. Fig 4 shows the results of a feedback experiment during the ramp-up phase. The natural tendency of  $l_i$  to increase with time in ohmic shots is compensated by NBI heating reducing the plasma resistance, and then lowering the inductance. This control test was performed with a linear PI control without with  $K_p=80$ ,  $K_i = 0.5$ , during the ramp-up phase, without compensation of non linearities, and tuned experimentally. There is room for improvement.