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LINE RADIATION TRANSPORT IN TOKAMAK EDGE PLASMAS: OPACITY AND FLUCTUATIONS

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1. Introduction

The advent of large scale divertor-equipped tokamaks (in particular with ITER being under construction) provides new challenges in numerical modelling. A problem currently under investigation is the role of atomic line radiation transport on the ionization–recombination balance of the edge plasma. Estimates show that the photon mean free path of the first hydrogen resonance lines is shorter than 10 cm at $N_{at} = 10^{13}$ cm^{-3} and $T_{at} = 10$ eV, i.e. in typical edge plasma conditions. First investigations of line radiation trapping have been made since a few years ago by using independent approaches [1,2]. All of them were made assuming a plasma background whose typical variation scales are much larger than the neutral and radiation transport scales. This approximation is questionable for tokamaks, where the turbulence radial correlation length l_{turb} (estimated as $10\rho_s$ [3]) can be of the same order or even smaller than the photon mean free path ($l_{turb} \sim 1$ mm at $T_e = 10$ eV and $B = 5$ T). In this work, we relax this assumption and propose a statistical parameterization of the fluctuations in the rates appearing in the radiative transfer equation. The radiative transfer formalism is first presented in Sec. 2, and then the statistical model is developed in Sec. 3. We next apply it to the calculation of Lyman α (which is the line the most affected by opacity) in the limiting case where l_{turb} is much smaller than the photon mean free path.

2. Radiative transfer modelling

The fundamental quantity of interest in radiative transfer is the "specific intensity" $I(\omega,\vec{n},\vec{r},t)$ [4], which denotes the energy flux in direction $\vec{n}$ per unit of frequency ω and solid angle Ω, at the location $\vec{r}$ and time t. This quantity obeys the radiative transfer equation

$$\left((1/c)\partial_t + \vec{n}.\vec{\nabla} + \chi(\omega,\vec{n},\vec{r},t)\right)I(\omega,\vec{n},\vec{r},t) = \eta(\omega,\vec{n},\vec{r},t)\,. \tag{1}$$

Here, $\chi(\omega,\vec{n},\vec{r},t)$ stands for the loss due to the photon absorption ("extinction coefficient") and is interpreted as the inverse of the photon mean free path; $\eta(\omega,\vec{n},\vec{r},t)$ is the source related to spontaneous emission. For an atomic line $u \rightarrow l$, these terms are proportional to the product between the atomic populations $N_u(\vec{r},t)$, $N_l(\vec{r},t)$ (which obey a set of collisional-radiative equations) and the line shape function $\phi(\omega,\vec{n},\vec{r},t)$: $\chi \propto N_l\phi$; $\eta \propto N_u\phi$. At LTE, these quantities depend on space and time through the plasma parameters ($N_e(\vec{r},t)$, $T_e(\vec{r},t)\ldots$) and, hence, are affected by fluctuations if the plasma is turbulent. In the most rigorous way, an investigation of the role of turbulence on radiation transport should involve the coupling of a transport code solving the radiative transfer equation to a turbulence code. Such a work is currently under progress with EIRENE, a Monte-Carlo code that takes account of both neutral and photon transport [5]. In the next section we propose a preliminary and complementary approach founded on a statistical formalism.

3. Statistical parameterization of the fluctuations

The statistical approach relies on a coarse grained description in time: we consider the photon transport at a time scale T much larger than the turbulence time t_{turb}, and we focus on the averaged specific intensity

$$\langle I\rangle(\omega,\vec{n},\vec{r},t)=\frac{1}{T}\int_t^{t+T}dt' I(\omega,\vec{n},\vec{r},t')\ . \tag{2}$$

According to the formalism developed by Pomraning [4], a closed transport equation can be obtained for this quantity, starting from Eq. (1) with $\partial_t \equiv 0$ (stationary regime due to the large value of the speed of light) and decomposing the specific intensity, the source and loss terms in an average term plus a fluctuating one: $I=\langle I\rangle+\delta I;\quad \eta=\langle\eta\rangle+\delta\eta;\quad \chi=\langle\chi\rangle+\delta\chi$. The resulting transport equation for $<I>$ can formally be written in a simple form [4]

$$(\vec{n}.\vec{\nabla}+\chi_{eff})\langle I\rangle=\eta_{eff}\ , \tag{3}$$

where the effective coefficients χ_{eff} and η_{eff} are linear operators that depend on the statistical properties of the fluctuations. These quantities are defined by $\chi_{eff}=\langle\chi\rangle-LB_1(1-B_1+B_2)^{-1}L^{-1}\delta\chi$ and $\langle\eta\rangle-LB_1(1-B_1+B_2)^{-1}L^{-1}\delta\eta$, respectively, where $L=\vec{n}.\vec{\nabla}+\langle\chi\rangle$ is the averaged transport operator, the inverse L^{-1} is an integral operator and $B_1\ldots\equiv L^{-1}\langle\delta\chi\ldots\rangle$, $B_2\ldots\equiv L^{-1}\delta\chi\ldots$ A practical expression can be obtained for the effective coefficients in the case where turbulence is homogeneous with a correlation length l_{turb} much shorter than the photon mean free path. Namely, one has $\chi_{eff}=\langle\chi\rangle-\Delta\chi^2\left[1-\exp\left(-\tilde{\chi}s_{\max}\right)\right]/\tilde{\chi}$ and $\eta_{eff}=\langle\eta\rangle-\Delta\chi\Delta\eta\left[1-\exp\left(-\tilde{\chi}s_{\max}\right)\right]/\tilde{\chi}$ where s_{max} is of the order of the medium's size, $\tilde{\chi}=\langle\chi\rangle+1/l_{turb}$ and $\Delta\chi$, $\Delta\eta$ denote the RMS values of χ and η, respectively. In the limit where the fluctuations vanish, these terms tend to zero and the resulting effective coefficients are equal to the average values. In a similar way, the limit where l_{turb} equals zero also leads to a transport equation with $<\eta>$ and $<\chi>$ as effective coefficients.

4. Application to passive spectroscopy

We have applied the statistical formalism of the previous section to calculation of a spectral line shape observed in a medium satisfying the condition $<\chi>l_{turb} << 1$. Considering a radiation pencil of direction $\vec{n}$ coming to a detector of acquisition time much larger than t_{turb}, the measured signal is proportional to the averaged specific intensity $<I>$. The latter quantity is the solution of the transport equation Eq. (3), namely, if l_{los} denotes the length of the line of sight, $\langle I \rangle = (\eta_{eff} / \chi_{eff})(1 - \exp(-\chi_{eff} l_{los}))$. The Lyα line has been considered in conditions of tokamak edge plasmas. For the description of the line shape function ϕ, we have used three Voigt functions, which account for both the Zeeman-Doppler structure and the Stark effect [6]. The atomic state populations, which appear in the source and loss terms of the transfer equation, depend on the electron parameters mainly. For illustrative purposes, we have assumed that they satisfy Saha-Boltzmann equilibrium. Fig. 1 shows a plot of these populations in terms of the electron temperature for $N_{n=1} + N_{n=2} + N_e = N_{tot} = 5\times10^{14}$ cm^{-3}.

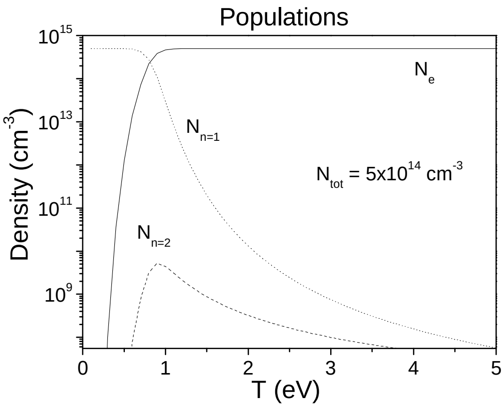

Fig. 1. Temperature dependence of the population density at Saha-Boltzmann equilibrium.

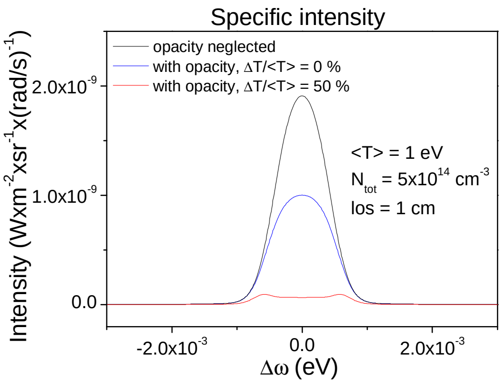

Fig. 2. Specific intensity of Lyα obtained when opacity is neglected (black line), when opacity is retained without turbulence (in blue), and with both opacity and turbulence (in red).

The measured signal has been calculated in the case of temperature fluctuations. We have assumed correlated fluctuations of T_e and T_{at} on a line of sight of 1 cm perpendicular to the magnetic field, and evaluated the averages $<\ldots> \equiv \int dT\, W(T)\ldots$ using a gamma model for the PDF $W(T)$. A value of 5 T has been considered for the magnetic field. Figure 2 shows the result obtained at $<T>$ = 1 eV without turbulence and neglecting opacity (i.e. setting $\chi_{eff} \rightarrow 0$ formally), compared to the case where opacity is retained but the plasma is not turbulent (i.e. $\Delta T/<T>$ = 0 %) and finally when the temperature fluctuates with a rate of 50 %. In the absence of fluctuations, the line is attenuated at centre due to opacity by about 50 %. On the other hand, the fluctuations lead to a dramatic additional attenuation of the intensity, in such a way that the line almost becomes unobservable. This strong attenuation is related to the modification of the effective emission and extinction coefficients η_{eff} and χ_{eff}, hence, qualitatively, of the average populations $<N_{n=2}>$ and $<N_{n=1}>$ when fluctuations are present. At $<T> = 1$ eV, a calculation shows that $<N_{n=2}(T)>/N_{n=2}(<T>) \approx 0.4$ and $<N_{n=1}(T)>/N_{n=1}(<T>) \approx 7$. This means that the emissivity should be reduced by the fluctuations whereas at the same time the opacity should be increased. This point has been confirmed quantitatively by calculations.

5. Conclusion

Radiative transfer in the presence of turbulence has been investigated for tokamak edge plasma conditions. Using a statistical formalism, we have derived a transport equation for photons that accounts for plasma fluctuations with a typical spatial scale smaller than the photon mean free path. An application to Lyα in the case of temperature fluctuations has shown a dramatic reduction of the specific intensity at low temperature. This result is closely related to the temperature dependence of the population densities of the upper and lower level of the transition. The present investigation is a preliminary one; a more detailed work in which the validity of the Saha-Boltzmann equilibrium assumption will be relaxed will be reported in a future communication. The problem of transport in the presence of turbulence also holds for neutrals in tokamak edge conditions. A paper on this issue presenting the status of calculations with the Monte-Carlo code EIRENE is currently under preparation [7].

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References

[1] H. A. Scott and M. L. Adams, Contrib. Plasma Phys. **44**, 51 (2004)

[2] D. Reiter, S. Wiesen, and M. Born, Plasma Phys. Cotrol. Fusion **44**, 1723 (2002)

[3] N. Bretz, Rev. Sci. Instrum. **68**, 2927 (1997)

[4] G. C. Pomraning, Linear Kinetic Theory and Particle Transport in Stochastic Mixtures (World Sci., 1991)

[5] D. Reiter, M. Baelmans, and P. Börner, Fusion Sci. Technol. **47**, 172 (2005); www.eirene.de

[6] J. Rosato et al., Phys. Rev. E **79**, 046408 (2009)

[7] Y. Marandet et al., 12th PET Workshop, Rostov, Russia, 2-4 Sept. 2009, to be published in Contrib. Plasma Phys.