*36th EPS Conference on Plasma Phys. Sofia, June 29 - July 3, 2009 ECA Vol.**33E**, P-5.086 (2009)*

Wave propagation in a counterstreaming electron-positron plasma

M.W. Verdon[1], D.B. Melrose[1]

[1] ***School of Physics, The University of Sydney, NSW 2006, Australia***

An oscillating model for the magnetospheres of pulsars suggests the presence of a counter-streaming electron-positron pair plasma, where the Lorentz factor of the streams varies over several orders of magnitude. The relativistically correct response of the plasma is calculated analytically, and numeric calculations of the dispersion functions performed. We present here the structure of the wave modes in this plasma for some different parameters, including a cold plasma and the case of parallel propagation. We discuss some of the implications for the generation and escape of pulsar radio emissions.

Introduction

We consider wave dispersion in a strongly magnetized pulsar plasma in which the electrons and positrons have bulk 4-velocities that are oppositely directed and may be very large. This type of distribution arises in an oscillating model for pair creation in the pulsar magnetosphere. In this [1] oscillating model, the oppositely-directed electrons and positrons oscillate, with the peak value of the oscillating $\boldsymbol{E}_{\|}$ determined by the threshold for effective pair creation. The relative motion is highly relativistic except for two brief periods per oscillation around the phases where the relative velocity changes sign. The oscillation period is several orders of magnitude longer than the radio-wave period.

The oscillations may be treated as large amplitude electrostatic waves [2], with a nett plasma outflow. Here results are derived in a frame co-moving with the nett flow.

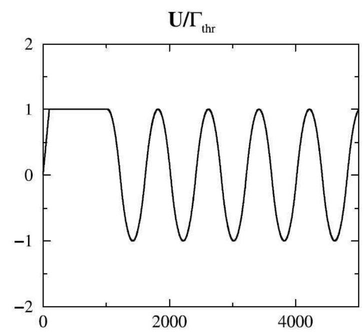

Figure 1: A diagram of the normalized relativistic bulk velocity ( ) of plasma vs. time in the oscillating model.

Cold, counterstreaming plasma, parallel propagation

The dispersion properties are first analysed assuming the plasma is cold and strongly magnetized. Even in this simple limit there is a rich variety of behaviour [3]. The dispersion curves for propagation parallel to the magnetic field at a non-relativistic value of $\beta = 0.3$ are shown in Figure 2. At parallel propagation, the dispersion equation is factorized into longitudinal and transverse parts. There are two longitudinal modes – the upper Langmuir-like mode has a cutoff at the plasma frequency but does not exhibit any instability. The lower frequency branch implies a streaming instability, with intrinsically growing solutions for $\omega_\parallel^2 < \omega_p^2$, $\omega_\parallel = \boldsymbol{k}_\parallel \beta \boldsymbol{c}$. The solution for ω in this regime is purely imaginary. The maximum growth rate, for $\omega_\parallel^2 = 3\omega_p^2/8$, is

$$\Gamma_{\max} = \left(\frac{1}{8}\right)^{1/2} \frac{\omega_p}{\gamma}, \qquad \boldsymbol{k} = \left(\frac{3}{8}\right)^{1/2} \frac{\omega_p}{\gamma\beta\boldsymbol{c}}. \tag{1}$$

It follows that the maximum growth is smaller the higher the Lorentz factor of the flow, so that maximum growth is when the flow is nonrelativistic or mildly relativistic, $\gamma \approx 1$.

There are four transverse modes at parallel propagation. The four modes may be regarded as split versions of the ordinary and extraordinary modes of the pure pair plasma, which are degenerate for parallel propagation. The upper-frequency branches correspond to a streaming-induced splitting of the degenerate upper-frequency branch of the degenerate modes, and the lower-frequency branches may be regarded similarly [3].

Instability is also present in the lowest-frequency branch of the transverse modes in two ranges of $\boldsymbol{k}$: for $\boldsymbol{k} = 0$ and $\boldsymbol{k} = \boldsymbol{k}_-$, and for $\boldsymbol{k}_+ < \boldsymbol{k} < \Omega/\beta\boldsymbol{c}$. For $\boldsymbol{k}_- < \boldsymbol{k} < \boldsymbol{k}_+$ and $\boldsymbol{k} > \Omega/\beta\boldsymbol{c}$ the mode is real. Here

$$\boldsymbol{k}_\pm \boldsymbol{c} = \frac{\Omega}{2\beta\gamma} \pm \frac{\Omega}{2\beta\gamma}\left(1 - \frac{4\omega_p^2\beta^2\gamma^2}{\Omega^2}\right)^{\frac{1}{2}} .$$

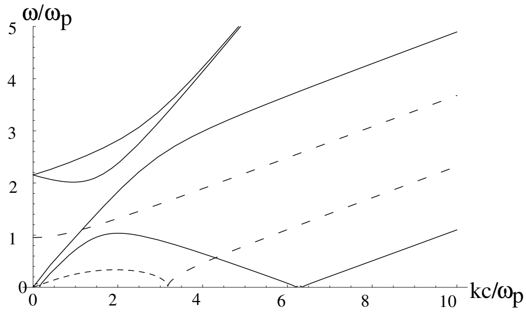

Figure 2: Dispersion curves, vs. k, in normalized units for parallel propagation in a cold counterstreaming plasma with $= 0.3$, $\Omega_e = 2$ $_p$. Dotted lines show imaginary parts, dashed lines show longitudinal real parts and solid lines show transverse real parts.

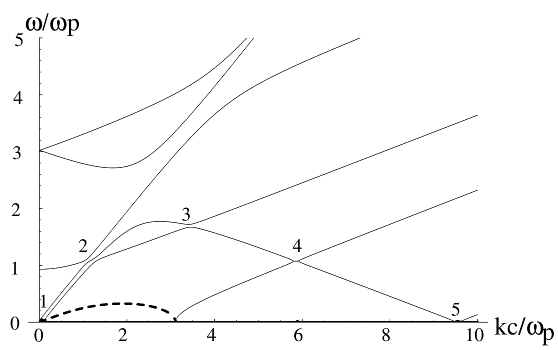

Figure 3: Dispersion curves for nearly parallel propagation ($= 0.1$ rad) with $= 0.3$, where $\Omega_e = 3$ $_p$. Dashed lines are imaginary parts and solid lines are real parts. Points 2, 3 and 4 are locations at which mode coupling is likely [3].

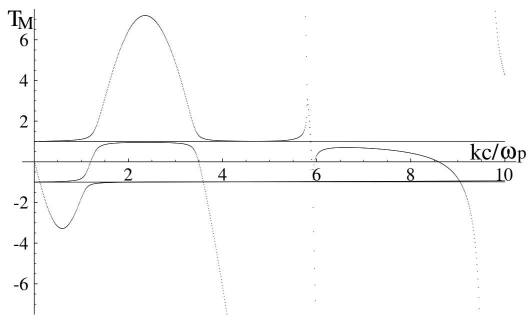

Figure 4: Plot of axial ratio of polarization vs. k in normalized units for nearly parallel propagation ($= 0.1$ rad) in a cold counterstreaming plasma with $= 0.3$, where $\Omega_e = 3$ $_p$. Mode coupling is predicted where the polarization changes rapidly.

Oblique propagation

When longitudinal and transverse modes cross in the limit of parallel propagation, they reconnect for oblique propagation, with the polarization changing rapidly from nearly longitudinal to nearly transverse, or vice versa.

In the presence of streaming an alternative type of reconnection is possible: two real modes that cross in the parallel limit can separate in $\boldsymbol{k}$ with the frequency becoming complex.

This type of instability is characterized by two real modes, for instance those corresponding to the lowest transverse and the lower longitudinal modes, merging to become a pair of modes with one growing and the other decaying.

Relativistic thermal counterstreaming plasma

Using the results for the cold plasma in a covariant formalism, thermal effects were included by integrating over a 1-D Jüttner distribution. There are three plasma dispersion functions appearing: following Godfrey et al. [4] defining

$$\boldsymbol{T}(\boldsymbol{z},\rho) = \int_{-1}^{1} \boldsymbol{d}\beta' \frac{\boldsymbol{e}^{-\rho\gamma}}{\beta' - \boldsymbol{z}} \ , \qquad (2)$$

the functions are $\boldsymbol{T}'(\boldsymbol{z},\rho)$, $\boldsymbol{T}(\boldsymbol{a}_{\pm},\rho)$. $\boldsymbol{z}$ and $\boldsymbol{a}_{\pm}$ are functions of ω, $\boldsymbol{k}$, the streaming speed β and the cyclotron frequency Ω [3]. The thermal parameter ρ is the ratio of the electron rest energy to the temperature, $\rho = \boldsymbol{mc}^2/\boldsymbol{kT}$. Dispersion curves are obtained by numeric solution of the resulting dispersion equations with Mathematica. Attention is restricted here to the case of parallel propagation.

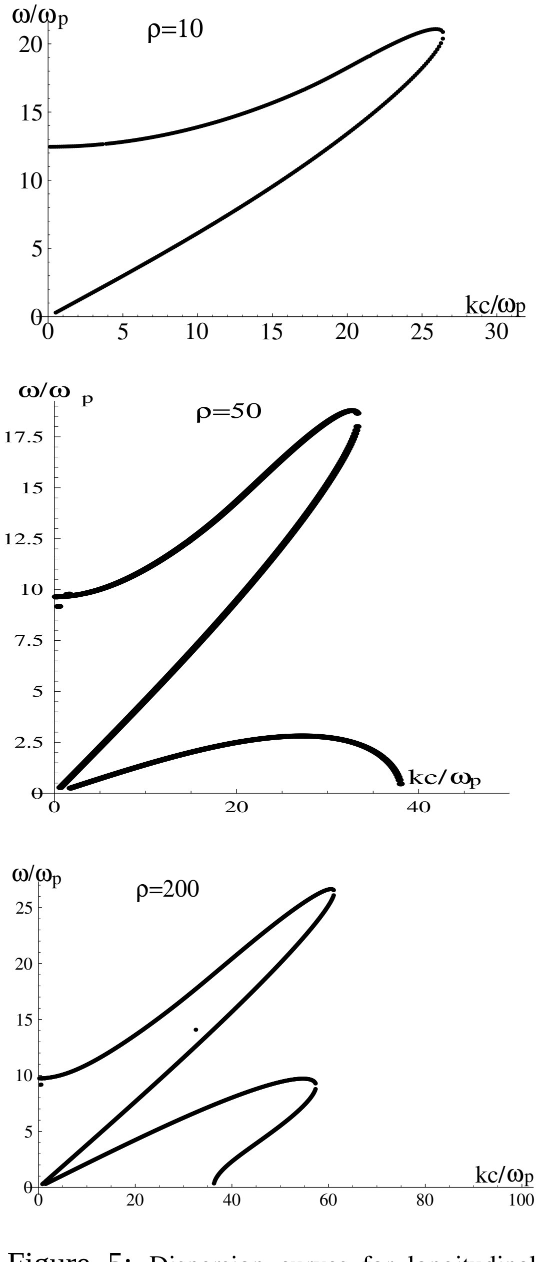

Figure 5: Dispersion curves for longitudinal modes at parallel propagation for three different temperature parameters.

Parallel longitudinal modes

Figure 5 shows the dispersion curves for longitudinal modes for parallel propagation in the thermal counterstreaming plasma, all for $\beta = 0.3$ but with varying temperature. The plots show the effect of decreasing temperature, with

$\rho = 10, 50, 200$ respectively. The upper Langmuir branch of the dispersion curve exists at all temperatures, but is cut off at a lower value of $\boldsymbol{k}$ for higher $\boldsymbol{T}$. The undersection of the curve after it has 'turned around' is very strongly damped and of no physical interest. As the temperature is decreased, the lower branch which owes its existence to the counterstreaming emerges, at first strongly damped and then finally with a component relatively free to propagate. The sections of the curves corresponding to the cold plasma case can readily be identified and the other sections are very strongly damped and not physical.

Applications and conclusion

Wave dispersion in oscillating plasma has properties that may be important in understanding pulsar radio emissions. To explain the observed radiation one must identify a coherent emission mechanism along with mode coupling across the cyclotron resonance. A counterstreaming plasma is predicted by an oscillating model, and there are strong physical arguments that the magnetosphere must be unstable. A streaming instability for longitudinal waves is a possible mechanism for the generation of radio emission. In the time-dependent system the frequency of the radiation changes as a function of time, making mode coupling difficult to treat. There is no known theory for mode coupling in an intrinsically time-dependent magnetized medium. However, the polarization swings imply that mode coupling should occur at several different points. The time-dependence of the counterstreaming plasma shifts the cyclotron resonance by a factor of up to 10^6 at different phases of the oscillation, as the Lorentz factor of the bulk plasma components changes. The cyclotron frequency which, in a pulsar, is initially far above the plasma frequency at which the radiation is generated, becomes comparable with this frequency at later phases. The cyclotron resonance can therefore interact with the radiation. This is important for the generation of the observed orthogonally polarized modes.

Wave propagation even in a cold counterstreaming plasma displays many interesting features. Different types of possible mode coupling points exist for the almost-longitudinal and almost-transverse cases. The extension to a relativistically thermal plasma suppresses the dispersion features introduced by the streaming motion, and cuts off the modes at some wavelength. A solid understanding of this system may help to provide some insight into pulsar emission.

References

[1] A. Levinson, D.B. Melrose, A. Judge, and Q. Luo. *Astrophys. J*, 631:456–465, 2005.

[2] Q. Luo,and D.B. Melrose. *Mon. Not. R. Astron. Soc.*, 378:1481–1490, 2007.

[3] M.W. Verdon and D.B. Melrose. *Physical Review E*, 77, 4:046403, 2008.

[4] B.B. Godfrey, B.S. Newberger, and K.A. Taggart. *IEEE Transact. Plasma Sci.*, PS-3, 2:60–67, 1975.