Relativistic Theory of Radial Electric Field $E_r(r)$ in non-periphery Tokamak Plasma

Romannikov A. N.

Nuclear Fusion Institute, Russian Research Center Kurchatov Institute, pl. Kurchatova I., 1, Moscow 123182, Russia

The approach for studying velocities of plasma flows and radial electric field $E_r(r)$ on magnetic surfaces is based on the following simple relation:

$$E_r(r) \cong \frac{V_t(r)\cdot B_p(r)}{c} + \frac{1}{|e|\cdot n_i(r)}\cdot\frac{dP_i(r)}{dr} - \frac{V_p(r)\cdot B_t}{c} \quad \mathbf{(1)}.$$

We assume for our purposes that the velocity of poloidal rotation can be taken from experiments or from the neoclassical theory. Unfortunately, eq. (1) is insufficient for calculating $E_r(r)$ in axisymmetric configurations of a tokamak due to the indeterminacy of velocity $V_t(r)$**.** The ambipolarity equation for radial flows cannot be used since ambipolarity emerges automatically in the configuration symmetric in the toroidal direction considered here. It should be emphasized that a relation of type (1) always holds when the plasma is in the equilibrium stationary state and with not so fast $V_t(r)$. It is well known that the Poisson equation is not used for determining the radial electric field in quasi neutral plasma with $n_e \cong n_i$. Nevertheless, in the quasi cylindrical case considered here, we have: $\frac{1}{r}\cdot\frac{d[E_r(r)\cdot r]}{dr} \cong 4\pi\cdot q(r)$ **(2)**; and the $E_r(r)$ in the tokamak is produced precisely by redistribution of charge density $q(r)$. Distribution $q(r)$ can be quite complex. Substituting $E_r(r)$ from eq. (1) into eq. (2), we can unambiguously calculate $q(r)$ in equilibrium case (1). We will use this substitution to estimate the value of $q(r)$ for characteristic parameters of ohmic and L modes with ion–cyclotron heating in the tokamak. Estimates show that $|q(r)|_{\max} \approx 10^{-3}$ CGSE units. This is the characteristic scale of deviation of the plasma from neutrality.

Is it possible to calculate q for a moving quasi-neutral current-carrying independently from first principles? Yes, see, for instance, [1]. We have for the tokamak plasma:

$$q = |e|\left(n_i^0\cdot\frac{1}{\sqrt{1-\frac{V_i^2}{c^2}}} - n_e^0\cdot\frac{1}{\sqrt{1-\frac{V_e^2}{c^2}}}\right) \cong -\frac{j^2(r)}{2\cdot c^2\cdot|e|n_e(r)} + \frac{j(r)\cdot V_t(r)}{c^2} + |e|(n_i^0(r) - n_e^0(r)) \quad \mathbf{(3)}.$$

The first two terms on the right-hand side of expression (3) relate to relativistic contraction of volumes in electron and ion flows, while the last term is relates to motion of specific electrons and ions (diffusion, convection, etc.), which changes the number of particles on magnetic surfaces. General expression (3) derived for $q(r)$ in the above approximations makes it possible to calculate the value of $E_r(r)$ in the tokamak from Poisson equation (2) independently of relation (1). We can calculate the radial velocity profile for equilibrium toroidal rotation of bulk ions (and, hence, plasma on the whole) and the radial profile for equilibrium $E_r(r)$ using the procedure described in [2]. We have:

$$V_t(r) \cong V_p^n(r)\frac{B_t}{B_p(r)} - \frac{c}{|e|\cdot n_i(r)\cdot B_p(r)}\frac{dP_i(r)}{dr} - \left[V_p^n(r)\frac{B_t}{B_p(r)} - \frac{c}{|e|\cdot n_i(r)\cdot B_p(r)}\cdot\frac{dP_i(r)}{dr}\right]_{r=a} - \\ -\int_r^a\left[V_p^n(\xi)\cdot\frac{B_t}{B_p(\xi)} - \frac{c}{|e|\cdot n_i(\xi)\cdot B_p(\xi)}\cdot\frac{dP_i(\xi)}{d\xi} - \frac{j(\xi)}{2\cdot|e|n_e(\xi)} + |e|(n_i^0(\xi)-n_e^0(\xi))\cdot\frac{c^2}{j(\xi)}\right]\times \\ \times\left(\frac{1}{\xi}+\frac{1}{B_p(\xi)}\frac{dB_p(\xi)}{d\xi}\right)d\xi \quad \textbf{(4);}$$

$$E_r(r) \cong -\frac{B_p(r)}{c}\cdot\int_r^a\left[V_p^n(\xi)\cdot\frac{B_t}{B_p(\xi)} - \frac{c}{|e|\cdot n_i(\xi)\cdot B_p(\xi)}\cdot\frac{dP_i(\xi)}{d\xi} - \frac{j(\xi)}{2\cdot|e|n_e(\xi)} + |e|(n_i^0(\xi)-n_e^0(\xi))\cdot\frac{c^2}{j(\xi)}\right]\times \\ \times\left(\frac{1}{\xi}+\frac{1}{B_p(\xi)}\frac{dB_p(\xi)}{d\xi}\right)d\xi + \left[\frac{V_p^n(r)\cdot B_t}{c} - \frac{1}{|e|\cdot n_i(r)}\cdot\frac{dP_i(r)}{dr}\right]_{r=a} \quad \textbf{(5).}$$

The eq. (4), shows that there are conditions for compensation of relativistic volume charge density $-\frac{j^2}{2\cdot c^2\cdot|e|n_e}$ (associated with electron flow) by term $|e|(n_i^0(r)-n_e^0(r))$ (associated with ion and electron flows across the magnetic field), see Fig.1. If term $-\frac{j^2}{2\cdot c^2\cdot|e|n_e}$ cannot be compensated ($n_i^0(r)=n_e^0(r)$), logarithmic uncertainness will always appear in real current profiles at the center. Such a beam cannot exist in the plasma, which is probably one of the reasons for the anomalous behavior of electrons. So, we can expect that $-\frac{j^2(r)}{2\cdot c^2\cdot|e|n_e(r)} + |e|(n_i^0(r)-n_e^0(r)) \cong 0$ **(6)**, in non-periphery plasma. It means that the second term in eq. (3) (so-called Lorentzian) may play a decisive role for those regimes. Let us calculate $E_r(r)$ from Poisson equation (2) using only the Lorentzian term in relation (3). This gives: $E_{r,2}(r) \cong \frac{B_p(r)\cdot V_t(r)}{c} - \frac{1}{r\cdot c}\int_0^r B_p(\xi)\cdot\frac{dV_t(\xi)}{d\xi}\xi\cdot d\xi$ **(7)**.

It can be seen that the volume charge density inside surface of radius r is not screened and affects the formation of $E_{r,2}(r)$. This result is contradicted traditional approach to $E_r(r)$ in plasma. On the Fig.2, quantitative comparison of experimental measurement $E_r(r)$ and calculated by eq. (7) is presented. The other example of quantitative application of this approach is comparison of the $E_{r,2}(r)$ profile calculated from eq. (7) with the results of experiments on measuring the profile of the radial electric field in the so-called locked mode. In this case, plasma stops rotating in toroidal directions on all magnetic surfaces, and the experimental radial electric field becomes close to zero everywhere. The equality to zero of the radial electric field in the absence of toroidal rotation follows from eq. (7) automatically.

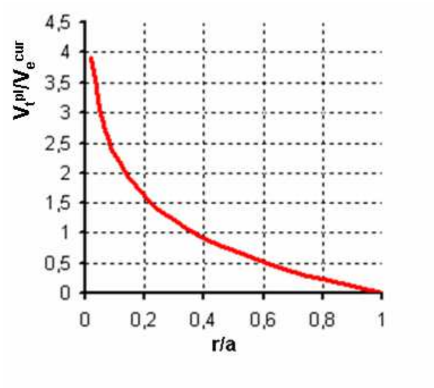

Fig.1 The ratio of plasma toroidal rotation velocity (eq. (5)) to the current electron velocity versus of effective minor radius.

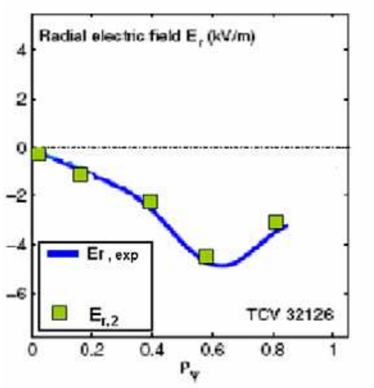

Fig.2 An estimation of radial electric field profile $E_r(r)$ in a typical ohmic discharge in the TCV plasma [3]: the $E_{r,\exp}$ is experimental profile in the discharge, the $E_{r,2}$ is calculated by eq. (7).

It can be found using eqs.(2, 3, 4, 5, 7) a lot of quantitative and qualitative correlations and direct explanations real tokamak experiments. For example:

1) The sign and characteristic value of core toroidal rotation velocities for main discharges, when the velocity of toroidal rotation is opposite to the plasma current, are correctly described by eqs. (4, 7) for real ion pressure profiles; 2) An important conclusion of eq.(6) means that there exists a powerful anomalous electron loss channel associated with $n_e^0(r)$ in the expression $|e|(n_i^0(r) - n_e^0(r))$, which makes it possible to compensate volume charge density $-\dfrac{j^2}{2 \cdot c^2 \cdot |e| n_e}$; 3) If the sum of the first and the third terms on the right-hand side of the eq. (3), becomes smaller than zero at the plasma periphery (in fact, this often indicates suppression of electron loss at the periphery), central plasma regions begin to rotate on the direction of the current, which correlates with experiments; 4) An important feature of the eq.

(4) is the fact that plasma confinement is better in the regime with co-current rotation against counter-current rotation if two regimes exist with close parameters of the plasma, which differ only in the direction of the velocity of toroidal rotation. This is confirmed, for example, by experiments performed in [4]; 5) A specific feature of the dependence of equilibrium field $E_r(r)$ (eq. (5)) on the volume charge density in the tokamak is the fact that the negative total charge density in the first and third term in formula (3) corresponds to positive equilibrium field $E_r(r)$, which correlates with some experiments; 6) In view of the integral nature of the relation between plasma parameters, eqs (4, 5) in the general case, a local variation of the plasma parameters in some region (e.g., at the periphery) leads to a change in the velocity of toroidal rotation and in field $E_r(r)$ in the entire plasma in the direction from the perturbation to the center. All magnetic surfaces will "instantaneously" perceive this local perturbation to a certain extent. Such a nondiffusive penetration of perturbations is observed in experiments (see, for example, [5]). Additional examples we can find in [1].

The first term in the right hand of eq. (3) can particularly change of the tokamak chamber electric potential to the ground earth, Δφ. The example of the first preliminary result of electric potential measurement for T-11M tokamak plasmas ($\langle n_e \rangle \le 10^{13}$ cm^{-3}, Ip ~50 kA) is shown on Fig.3 [6]. Unfortunately, RC at the chamber was less then 10 ms (R≈4 MOm). Nevertheless, it is necessary to emphasize that there were some correlation of measured Δφ(t) with $Ip^2/\langle n_e \rangle$. That parametric dependence is expected from (3). In addition, expected maximum amplitude of Δφ from (3), RC and Ip(t) in those experiments was minus 30÷70 V. Presented measurement means that a conductor with current may look like charged.

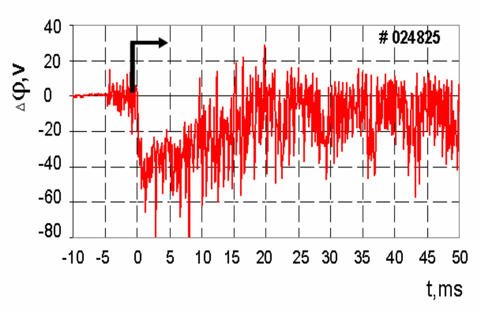

Fig.3 Time dependence of the T-11M tokamak chamber electric potential during the discharge.

[1] A. Romannikov, JETP (2009), Vol. 108, pp.340-348; [2] Romannikov A., PPCF 49, 641 (2007); [3] B. P. Duval, A. et al, PPCF 49, B195 (2007); [4] H. Shirai, et al, Nucl. Fusion 39, 1713 (1999); [5]. S. Neudatchin, et al, Nucl. Fusion 44, 945 (2004); [6] A. Romannikov, E. Kuznetsov, private communication (2009).