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Modelling of the turbulence wave number spectra reconstruction from the radial correlation reflectometry data.

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Fluctuation reflectometry is widely used technique providing information on the tokamak plasma micro turbulence. In order to improve the fluctuation reflectometry wave number selectivity a more sophisticated radial correlation reflectometry (RCR), using simultaneously different frequencies for probing was proposed and developed at numerous magnetic fusion devices. The coherence decay of two scattering signals with growing difference of probing frequencies is studied in this diagnostic and applied for estimation of the turbulence radial correlation length in a very straightforward manner. Namely, it is assumed that the distance between cut offs at which the correlation of two reflectometry signals is suppressed is equal to the turbulence correlation length.

However in a number of numerical and analytical papers based on both 1D and 2D approach utilizing numerical Born approximation and full-wave modeling a role of small angle scattering was shown, reducing the diagnostic spatial resolution and leading to a very slow decay of coherence in RCR [1-4]. Recently [5] an analytical integral formula expressing the RCR cross-correlation function (CCF) in terms of turbulence radial wave number spectrum has been obtained and a procedure of its correct inversion for the spectrum determination from the CCF was proposed [6]. This procedure feasibility check was started in the framework of 1D numerical modeling in [6] where the possibility of reconstruction of multi Gaussian spectra parameters was shown.

In the present paper the method possibilities are studied in conditions relevant for experiments. The procedure accuracy dependence on probing frequency range and resolution as well as on the poor statistics and presence of noise is investigated.

The turbulence wave number spectrum reconstruction approach.

We treat the RCR problem using 1D model describing the O-mode probing by equation

$$\left\{ \frac{d^2}{dx^2} + \frac{\omega^2}{c^2} - \frac{4\pi e^2 \left[n(x) + \delta n(x) \right]}{m_e c^2} \right\} E_z(x,\omega) = 0 \qquad (1)$$

where $n(x) = n_c(\omega_1)\, x / x_c(\omega_1)$ is the background density profile supposed linear in this paper and $\delta n(x)$ stands for turbulent fluctuations assumed statistically homogeneous. As it was shown in [6], the turbulence spectrum can be expressed in the case the Born approximation is valid in terms of the RCR cross correlation function (CCF) using the following expression

$$\frac{\delta n^2}{n_c^2(\omega_2)}\tilde{n}_\kappa^2 = \frac{\sqrt{i}}{2\sqrt{\pi}}\frac{c^2}{x_c(\omega_2)\omega_2^2}\frac{|\kappa|}{F\left[\sqrt{\kappa x_c(\omega_2)}\right]}\int CCF e^{-i\kappa\Delta} d\Delta \tag{2}$$

where $\Delta = x_c(\omega_1) - x_c(\omega_2)$ is the separation of cutoff positions for the reference and signal frequencies; $F(s) = \int_0^s \exp(i\varsigma^2)\, d\varsigma$ is a Fresnel integral and the CCF of reflectometry signals

$$A_s = \frac{i\omega\sqrt{S_i}}{16\pi}\int_0^\infty \frac{\delta n(x)}{n_c} E_0^2(\omega, x)\, dx \tag{3}$$

is determined as

$$CCF = \frac{\left\langle \left[A_s(\omega_2) - \langle A_s(\omega_2)\rangle\right]\left[A_s^*(\omega_1) - \langle A_s^*(\omega_1)\rangle\right]\right\rangle}{\sqrt{\left\langle \left|A_s(\omega_2) - \langle A_s(\omega_2)\rangle\right|^2\right\rangle \left\langle \left|A_s(\omega_1) - \langle A_s(\omega_1)\rangle\right|^2\right\rangle}} \tag{4}$$

The electric field $E_0^2(\omega, x)$ entering (3) is a solution of (1) calculated in the absence of density fluctuations and normalized to the unit probing power flux density.

Numerical reconstruction of the turbulence spectrum and CCF.

Here we shall analyze the accuracy of the proposed inversion procedure using the CCF computed numerically from (1) in the frame of Born approximation (3). The superposition of $m=10^4$ harmonics $\delta n(x) = \delta n_0 \sum_{j=1}^{j=m} \cos(jqx + \varphi_j)\sqrt{\frac{2q}{\pi}\tilde{n}_{jq}^2}$ possessing wave numbers jq, random phases φ_j, and amplitude distributed in accordance with the turbulence spectrum $\tilde{n}_\kappa^2$ is used in analysis. The calculation parameters are as follows: $x_c = 40cm$; correlation length $l_c = 2cm$; $\omega_1 = 6\cdot 10^{11} c^{-1}$. The averaging is performed over ensemble of typically 500 random phase samples. In the case of exponential spectrum $\tilde{n}_\kappa^2 = 0.5 l_c e^{-|\kappa| l_c}$, suppressed at small wave numbers, shown in fig. 1a by red curve, the CCF calculated in the interval $-20l_c < \Delta < 20l_c$ is shown in fig. 1b by the blue curve. It is much broader than turbulence Gaussian correlation function (red curve in fig. 1b), asymmetric and possesses small, but finite imaginary part, shown by green line. Accordingly, the CCF spectrum obtained after extrapolation of the CCF to higher Δ values is very peaked around the zero wave number, unlike the initial spectrum. However after been treated in agreement with (4) its real part takes a form similar to the turbulence spectrum (see black curve fig. 1a). The oscillations of the reconstructed real part of the spectrum around the initial one are produced by discontinuities of the extrapolation procedure at $\Delta = \pm 20 l_c$. A smaller imaginary part of the reconstructed spectrum (shown by

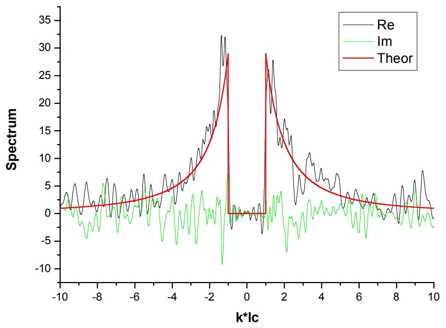

Fig. 1a. The reconstructed spectrum versus normalized wave number.

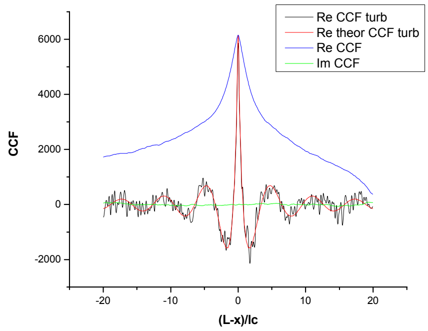

Fig. 1b. The signal and reconstructed turbulence CCF.

green line in fig. 1a) is oscillating around the zero line. It is important to note that these oscillations originated by extrapolation procedure could be removed to the matching region ($\Delta = \pm 20 l_c$ in the present computation) by performing Fourier transform of the reconstructed spectrum providing the turbulence CCF. The result of this transformation in the case of spectrum of fig. 1a is shown by black curve in fig. 1b. As it is seen there, the reconstructed real part of the turbulence CCF fits perfectly the initial turbulence CCF describing not only the kernel of the CCF at $\Delta < 2l_c$, but also oscillations caused by the spectrum discontinuity. In Fig. 1 we have performed reconstruction based on the signal CCF computed in a very wide signal frequency range corresponding to $|\Delta| \le 20 l_c$ not possible in the experiment. In Fig. 2 the reconstructed Gaussian turbulence spectra and CCF are shown for the realistic case $|\Delta| \le 2l_c$. As it is seen there, in spite of the fact the reconstructed spectrum (black curve) is different from the initial Gaussian spectrum (red curve) the obtained turbulence CCF fits the Gaussian well. The previous evaluation of the CCF used for the reconstruction in Fig.1

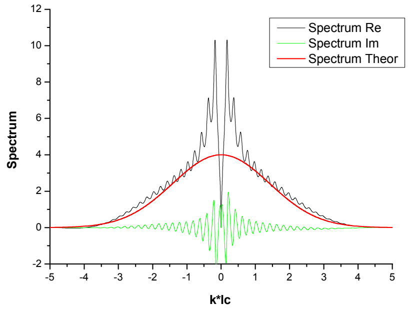

Fig. 2a. The reconstructed turbulence spectrum.

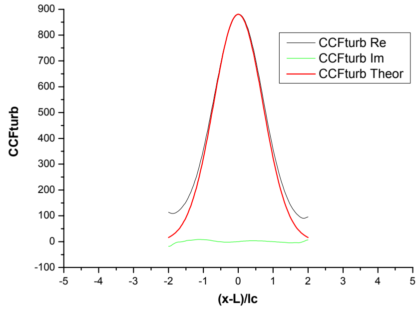

Fig. 2b. The reconstructed turbulence CCF.

was performed with fine spatial resolution in a wide region (10000 points in the $\left[-20l_c, 20l_c\right]$ interval), which corresponds to probing with the very detailed frequency resolution and in a very wide range not always possible in the experiment. In more realistic conditions of only 6 RCR measurements the reconstruction of the turbulence CCF is also possible, as we show in Fig.3 based on the RCR data obtained at $|\Delta| < 2l_c$ with the signal frequency cut off step $|\delta\Delta| = 0.08l_c$ and $|\delta\Delta| = 0.64l_c$.

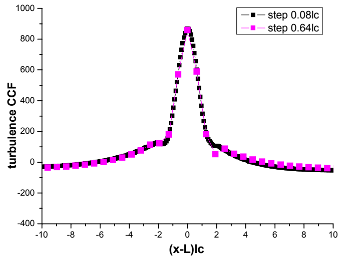

Fig. 3. The turbulence CCF reconstructed at $|\Delta| \leq 2l_c$ *and* $|\delta\Delta| = 0.08l_c$ *and* $|\delta\Delta| = 0.64l_c$.

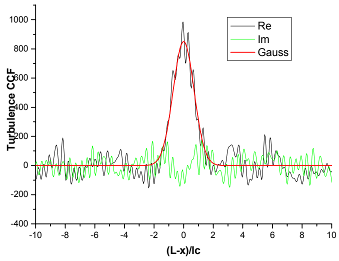

Fig. 4a. The turbulence CCF reconstruction at 10% SNR, 500 samples.

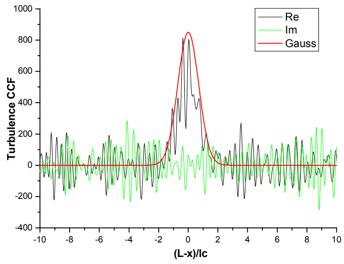

Fig. 4b. The turbulence CCF reconstruction at 100% SNR, 10000 samples.

Very important for the feasibility of the proposed procedure is its weak sensitivity to the experimental noise. As it is seen in Fig.4a and 4b, in the case of 10% SNR the reconstruction of the Gaussian turbulence CCF is possible with averaging performed over only 500 random turbulence samples, whereas in the case of 100% SNR 10000 samples are needed.

Conclusion.

It is worth to underline that application of the proposed procedure to the turbulence spectrum and CCF reconstruction from the RCR data in numerical modeling have led to very promising results in conditions relevant for experiments. The demonstrated possibility of fine reconstruction, at least in 1D geometry, is proving the procedure feasibility and appealing for further optimization and tests in 2D numerical modeling.

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