EBCD calculation in the TJ-II using different models

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Introduction

Plasma waves in the range of the electron cyclotron frequency have been widely studied theoretically and experimentally for current generation in plasmas. This wave-induced current plays an important role in the plasma stability and confinement in stellarators, counteracting deleterious plasma currents or allowing the rotational transform profile modification. It is also expected to overcome the pulsed operation in tokamaks.

The Electron Bernstein Waves (EBW) heating system of the TJ-II stellarator [1](O-X-B1 conversion, 1$^\text{st}$ harmonic, 28 GHz and 300 kW) motivates this work, where a calculation of the current by means of EBW (EBCD) is carried out using different models. This methods have been coupled to the ray tracing code TRUBA, currently in use to study the EBW propagation and absorption properties in TJ-II [2].

The models

Due to the lack of density cutoffs and its strong cyclotron damping, EBW are commonly used for an efficient heating of overdense plasmas. Its electrostatic character makes them achieve a parallel refractive index greater than one, and therefore extends the resonance scenario both for heating and current generation. This current can be generated through the preferential heating of resonant electrons in one particular direction along the magnetic field lines, which produces an asymmetric modification of the electron resistivity (Fisch-Boozer mechanism [3]), and through the diffusion of electrons from the passing to the trapping region in momentum space, leading to an asymmetry in the number of current-carrying electrons (Okhawa mechanism [4]).

The techniques used for the calculation of the current efficiency in this work includes on the one hand, the response function approach in its relativistic formulation [5], taking into account the Okhawa mechanism, which provides a modified current efficiency in momentum space $\eta_T(\textbf{u})$ [6], being $\textbf{u} = p/mc$, $m$ the electron mass, and $c$ the speed of light. This model assumes the high speed limit (hsl) approximation for the linearised electron self-collision operator, and integrates the kinetic equation for the perturbed distribution function through the equivalent set of Langevin equations for the trajectories. It includes the trapped particles influ-
ence by removing the trapped electrons from the current density integral, i.e. those with $\lambda \geq 1$, where $\lambda = u_\perp / u_b$, $b = B / B_{\text{max}}$, $B$ the local magnetic field, and $B_{\text{max}}$ the maximum magnetic field in a flux surface.

On the other hand, under the adjoint approach formalism, three different choices for Spitzer function $K(u)$, thus for the response (Green’s) function $\chi(u)$, have been used for the current efficiency calculation. Assuming the hsl approximation in the non relativistic limit, the response function is expressed as [7]

$$\chi_{\text{Taguchi}}(u) = \frac{f_c}{Z_{\text{eff}} + 1} u^4$$

where $Z_{\text{eff}}$ is the effective ions charge, and $f_c$ is the effective circulating particle fraction in the neoclassical transport theory,

$$f_c = \frac{3}{4} \langle b^2 \rangle \int_0^1 \frac{\lambda d\lambda}{\langle \sqrt{1 - \lambda b} \rangle},$$

where $\langle \ldots \rangle$ denotes flux surface average. The expression for $f_c$ is taken into account in the fmfp-regime ($\nu_e \ll \tau_e^{-1}$; $\nu_e$ is the electron collision frequency and $\tau_e$ the electron bounce time). In the collisional limit ($\nu_e \gg \tau_e^{-1}$), $f_c = 1$ and no trapped particle effects are included. Both limits stablish the lower and upper bounds respectively for the current estimation.

The expression for $\chi_{\text{Taguchi}}$ represents the non-relativistic limit of the relativistic Spitzer function in the hsl-approach proposed in Ref. [8],

$$\chi_{\text{Lin-Liu}}(u) = \left( \gamma + \frac{1}{\gamma - 1} \right) \rho^{\rho/2} \int_0^u du' \left( \frac{u'}{\gamma'} \right) \left( \frac{\gamma' - 1}{\gamma' + 1} \right)^{\rho/2}, \rho = \frac{Z_{\text{eff}} + 1}{f_c},$$

being $\gamma = \sqrt{1 + u^2}$ the Lorentz factor. Due to the lack of momentum conservation of the hsl based models, a third response function $\chi_{\text{mc}}(u)$, obtained from a variational principle [9], with momentum conservation in the like-particle collisions and weakly relativistic case is present in the calculations. An extension of this model for arbitrary collisional regimes, apart from the mentioned weakly relativistic limit, can be found in Ref. [10]. The range of validity of the so call mc-model is $u/\nu_th < 4$.

Finally, the current efficiency $\eta_{\text{cd}}$ in terms of the response function is expressed as,

$$\eta_{\text{cd}} = \frac{\langle j \rangle}{P_{\text{th}}} = \frac{e v_{\text{th}} \langle b \rangle \int_{u_{\perp}} du ||D_{q||} \hat{A}(f_c) \hat{\chi}(\chi)||}{v_e m_e c^2 \int_{u_{\perp}} du ||D_{q||} \hat{A}(f_c)||}$$

where $e$ is the electron charge, $v_{\text{th}}$ is the thermal velocity, $f_e = \frac{\mu}{2K} e^{-\mu \gamma}$ the relativistic Maxwellian, $\mu = m_e c^2 / T_e$, and $D_{q||} = u_{\perp}^2 ||\Pi_s||^2$ is the normalized quasi-linear Kennel-Engelmann diffusion coefficient for the harmonic $s$, with the polarization factor $\Pi_s = e^{-J_{s-1} (k_{\perp} \rho_e)} + e^J_{s+1} (k_{\perp} \rho_e) + e^E (u_{\parallel} / u_{\perp}) J_s (k_{\perp} \rho_e)$, $\rho_e$ the electron Larmor radius, $k_{\perp}$ the perpendicular wave vector, and $J_n$ the n-order Bessel function.
Figure 1: In the left y-axis the current efficiency $\eta_{CD}$ for each model under different collisional regimes, where $FL$ denotes the Fisch model using the Langevin equations approach, $T$ denotes adjoint approach proposed by Taguchi, Lin-Liu, and $mc$ the momentum conservation model. In the right y-axis, the effective radius $\rho$, parallel refractive index $N_{\parallel}$ and $Y_s$ for the first harmonic are represented. All quantities are represented along the ray length, and considering central densities and temperatures of $6.75 \times 10^{19}$ m$^{-3}$ and 1.6 keV (a), and $5.0 \times 10^{19}$ m$^{-3}$ and 0.2 keV (b).

**Numerical results**

The calculation of the current efficiency $\eta_{CD}$ and toroidal current density $j_\phi$ has been carried out coupling the output of the ray tracing code TRUBA with the routines for the calculation of the response function. Electron temperature and density analytical profiles are expressed as $n(\rho) = n_0[1 - (\rho^2)^7]$ and $T(\rho) = T_0[1 - (\rho^2)^7]^{10}$, where $n_0$ and $T_0$ are its central values. The position of the heating system mirror which optimizes the OXB transmission efficiency is fixed for all the calculations here presented.

Figure 1 shows the efficiency for $n_0 = 6.75 \times 10^{19}$ m$^{-3}$ and $T_0 = 1.6$ keV (a), and $5.0 \times 10^{19}$ m$^{-3}$ and 0.2 keV (b). The former shows a worse agreement between $\eta_{CD}^{Lin-Liu}$ and $\eta_{CD}^{Taguchi}$, due to the higher $T_0$, and the subsequent stronger relativistic effects. Since $\eta_{CD}^{FL}$ does not take into account the friction of the circulating particles with the trapped ones, this approach should be close to the $\eta_{CD}^{Lin-Liu}$, which in fact happens. The reason of their disagreement lays on the fact that the former considers the Okhawa mechanism, and assumes the strict high velocity limit, which drops the energy diffusion term in the linearised self collision operator. On the other hand, in both subfigures, $\eta_{CD}^{mc}$ does not lean exclusively towards any of the other approaches, which, apart from its self-consistency, positions itself as the most appropriate method for the current estimation.

Nevertheless, and oppositely to the electromagnetic case, the wide range of $N_{\parallel}$ and $Y_s$ may displace the resonance condition out of the validity range of the mc-model. This can happen just in the onset of the resonance condition fulfilment. This can be observed in both figures.
Finally Fig. 2 shows the toroidal current density corresponding to the parameters of the figure 1.a, for different number of rays per beam and under the mc-model. This plot shows the need of using multi-ray simulation for the Bersteins mode, in order to reach the current density profile convergence. The thin lines represent the current density produced per ray in each case, and the thick one the sum of them.

Conclusions

The implementation of different methods for the EBCD calculation using ray tracing techniques in TJ-II has been successfully done. Due to its self-consistency and the fact that the current is driven by electrons with $u/u_{th} < 4$, the mc-model is better positioned than the hsl approaches for the current estimation. A special care has to be taken with EBW, since its absorption can step out its validity range, and a hybrid use of the models in each region of the momentum space could avoid wrong results in some critical points. Furthermore, the beam requires multiple-ray modelling, in order to obtain an acceptable current density profile.

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References