Zonal flow-based interpretation of long-distance correlations in the edge shear layer of TJ-II

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A simple transition model for barrier formation was proposed in Ref. [1], in which a mean sheared flow is amplified by the Reynolds stress and turbulence is suppressed by shearing. The emergence of the plasma edge shear flow layer in TJ-II has been identified with the transition described by that model [2]. New experimental results in TJ-II [3] report the existence of long-range potential correlations in the toroidal direction above the critical point of the transition. The observed correlations correspond to non-zero frequencies below 30 kHz and thus cannot be explained by mean (i.e. surface averaged, zero-frequency) sheared flows. In this Conference Contribution we aim to show that the experimental findings of Ref. [3] can be understood in the context of the paradigm proposed in Ref. [1] if one appropriately incorporates the contribution of zonal flows. Here we use the term *zonal flow* in the sense of low frequency fluctuating flows with $k_\varphi = 0$ and small but non-zero k_θ. The reader is referred to Ref. [4] for further details.

The transition model

The model presented in this section is an extension of the one used in Ref. [2] to discuss the emergence of the plasma edge shear flow layer. This is a model formulated at a radial point. The dynamical variables are the fluctuation level envelope $E \propto \langle (\tilde{n}/n_0)^2 \rangle^{1/2}$, the mean flow shear $V \propto \partial_r \langle V_\theta \rangle$, the zonal flow amplitude shear, $V_{\rm ZF} \propto \partial_r \langle V_{\theta {\rm ZF}} \rangle$, and (minus) the normalized average pressure gradient $N \propto -a\partial_r \langle p \rangle / \langle p \rangle (0)$. Here a is the minor radius of the torus, $\langle \cdot \rangle$ stands for the average over angle coordinates, and $r = 0$ corresponds to the magnetic axis. The equations of the model are

$$\dot{E} = N^{2/3}E - N^{-1/2}E^2 - N^{-1/3}E(V^2 + V_{\rm ZF}^2), \quad (1a)$$

$$\dot{V} = a_1 N^{-4/3}E^2 V + a_2 N^{-2/3} V_{\rm ZF}^2 V - bV, \quad (1b)$$

$$\dot{V}_{\rm ZF} = \frac{a_1}{1 + N^{-1}V^2} N^{-4/3} E^2 V_{\rm ZF} + a_3 N^{-4/3} E^2 V - bV_{\rm ZF}, \quad (1c)$$

$$\dot{N} = \Gamma - DEN. \quad (1d)$$

where the dot stands for the time derivative. The structure of these equations is based on a quasi-linear approximation of resistive pressure-gradient driven turbulence (the resistive interchange mode, due to bad magnetic field line curvature, is assumed to be the basic instability at the edge of TJ-II). The form of the equation for the time evolution of zonal flows, Eq. (1c), coincides with the one proposed in Ref. [5], except for the term proportional to a_3, which is absent in the latter reference. It is worth commenting on the physical origin of that term. In the framework of the paradigm of shear flow generation by turbulence the Reynolds stress gives a non-zero contribution when the turbulent eddies are distorted by the presence of global shear flows. If only the mean flow is present the Reynolds stress gives the mean shear flow amplification term that we have discussed in the past [1]. When, in addition, zonal flows exist, the Reynolds stress gives two main contributions to the zonal flow equation. One comes from the coupling of the m and $-m+q$ components (both m and q denote poloidal wave-numbers) of the eigenfunctions distorted by the zonal flow (the first term on the rhs of Eq. (1c)). The other comes from a similar coupling but with the distortion induced by the mean flow (the second term on the rhs of Eq. (1c)). Here m is large and corresponds to the turbulent component of the flow, whereas q is the wave-number of the zonal flow. A detailed calculation of those terms will be provided elsewhere.

It is important to point out that there is a qualitative difference between $a_3 = 0$ and $a_3 \neq 0$. If $a_3 = 0$ the model exhibits a continuous transition between the state with $V = 0$ and the state with $V \neq 0$. In addition, the stable fixed points are such that $V_{\mathrm{ZF}} = 0$. However, if $a_3 \neq 0$ the transition is discontinuous and the stable, improved confinement state has both V and V_{ZF} non-vanishing. Since the toroidal correlations will be associated to the existence of stationary zonal flows, it seems that small but non-zero a_3 is required.

The toroidal correlation

In this subsection we will try to express the correlation of the potential fluctuations at two toroidal positions separated by a toroidal angle δ in terms of the variables of our model. The formula defining the correlation is

$$\mu = \frac{\Big\langle \Big(\Phi(r,\theta,\varphi,t) - \langle\Phi(r,\theta,\varphi,t)\rangle\Big)\Big(\Phi(r,\theta,\varphi+\delta,t) - \langle\Phi(r,\theta,\varphi+\delta,t)\rangle\Big)\Big\rangle}{\sqrt{\Big\langle\Big(\Phi(r,\theta,\varphi,t) - \langle\Phi(r,\theta,\varphi,t)\rangle\Big)^2\Big\rangle\Big\langle\Big(\Phi(r,\theta,\varphi+\delta,t) - \langle\Phi(r,\theta,\varphi+\delta,t)\rangle\Big)^2\Big\rangle}}. \tag{2}$$

Assume that a separation of time scales exists, so that one can write

$$\Phi(r,\theta,\varphi,t) - \langle\Phi(r,\theta,\varphi,t)\rangle = \Phi_{\mathrm{ZF}}(r,\theta,t) + \tilde{\Phi}(r,\theta,\varphi,t), \tag{3}$$

where $\Phi_{\mathrm{ZF}}(r,\theta,t)$ is related to the zonal flow and $\tilde{\Phi}(r,\theta,\varphi,t)$ to high frequency turbulent fluctuations. The high frequency fluctuations have short correlation length in the toroidal direction, except when the positions are aligned with the field lines. Here we assume that this is never the case. The zonal flows are characterized by a low frequency and not having a dependence on the toroidal angle. Now, take a field line on the magnetic surface labeled by r passing through (θ,φ) and $(\theta_\delta,\varphi+\delta)$. We are assuming that δ is such that $|r(\theta_\delta-\theta)|\gg l_\theta$, where l_θ is the poloidal correlation length of the high-k turbulence.

Let us finally write (2) in terms of the variables of the present model. Since the model equations can be derived from quasilinear calculations of a pressure-gradient-driven turbulence model (which in particular is a fluid model), we assume that the density perturbation is the result of the convection of the equilibrium density by the flow $\tilde{\mathbf{V}}=-\nabla\tilde{\Phi}\times\mathbf{B}/B^2$. After some manipulations (see Ref. [4] for details):

$$\mu=\left(1+\lambda\frac{E^2}{N^{2/3}V_{\rm ZF}^2}\right)^{-1},\quad \lambda>0. \tag{4}$$

This is the formula we were looking for. It gives the toroidal correlation of the electrostatic potential in terms of the variables of our model. In particular, it shows in a manifest way that the zonal flow is responsible for the appearance of toroidal correlations.

Comparison of model and experiment

Experiments were carried out in the TJ-II stellarator in Electron Cyclotron Resonance Heated plasmas ($P_{\rm ECRH}\leq 400$ kW, $B_T=1$ T, $\langle R\rangle=1.5$ m, $\langle a\rangle\leq 0.22$ m, $\iota(a)\in[1.5,1.9]$). The plasma density was varied in the range $[0.35\cdot 10^{19},1\cdot 10^{19}]\mathrm{m}^{-3}$. Different edge plasma parameters were simultaneously characterized in two different toroidal positions approximately 160° apart using two similar multi-Langmuir probes, installed on fast reciprocating drives (approximately 1 m/s). For details on the probe arrangement see Ref. [3]. It is important to note that the field line passing through one of the probes is approximately 100° poloidally apart when reaching the toroidal position of the other probe that is more than 5 m away.

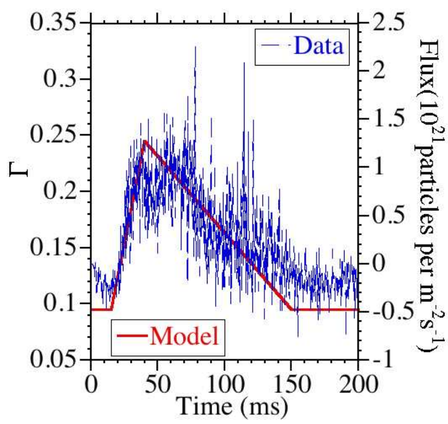

Figure 1: Flux function, $\Gamma(t)$, used in modeling the experimental data.

There are five parameters in the model and the λ parameter in the determination of the correlation; this is apart from the input function Γ. The two parameters b and D are directly related to the dissipation terms, viscosity and transport. We determine them from the expected values of those terms at the plasma edge. We take the flow damping rate to be $\nu_{ii}=1.7\cdot 10^4$ s^{-1} and the particle diffusivity $D_p=10^5$ cm^{2}s^{-1}. Therefore, we used $b=D=0.1$. The a_1 parameter is determined from the criticality condition and the experimental measure of the density profile at the critical point. The measured density profile at about the critical density in TJ-II is such that $N_c \approx 1$. Therefore, we take $a_1 = 0.1$.

In this model the long-range correlations are controlled essentially by the ratio a_3/λ. In order to have a reasonable level of correlation we need $a_3/\lambda \approx 1/3$. With the present data it is not possible to distinguish between the two parameters. Therefore, just for convenience,we have taken $a_3 = 0.01$ and $\lambda = 0.03$. Finally, the parameter a_2 has not a very visible impact on the comparison with the data and we have chosen $a_2 = 0.5$.

The input function required in modeling each discharge is the flux function $\Gamma(t)$. Since we are not doing a detailed modeling of data, but only a description of the main features, we have parameterized the flux using only linear dependences in time. A typical example of how the data are described by the model is shown in Figs. 1 and 2, corresponding to discharge 18229. This is a case with a ramp up and down where the plasma crosses the critical point twice, once in the way up and another in the way down. Similar results have been obtained for 10 discharges of TJ-II using the same set of values for the parameters. As one can see in Fig. 2, the agreement between the experimental data and the model description is quite satisfactory, especially noting the extreme simplicity of the latter.

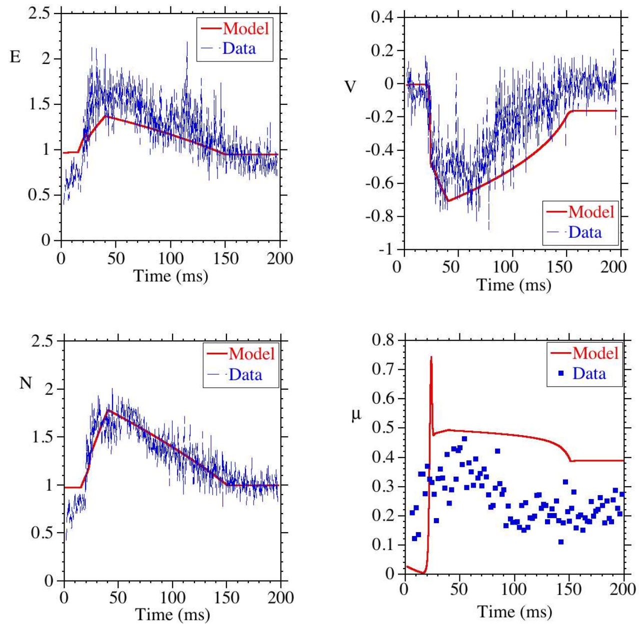

Figure 2: Comparison between the model, Eqs. (1), and the experimental data for the shot 18229.

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