TEST PARTICLE SIMULATIONS FOR IMPURITY TRANSPORT
IN FUSION NON MAXWELLIAN PLASMA

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The modification of transport coefficients of impurity ions caused by the transition from the background plasma Maxwellian to non Maxwellian distribution functions is studied by means of test particle approach. For this purpose the set of monoenergetic neon test impurities are followed in a toroidal plasma consisting of deuterons and electrons. The collisional interaction of impurities with main plasma is modeled by a discritized collision operator derived for an arbitrary distribution functions. The non Maxwellian distribution is obtained by adding a fraction of energetic particles of the same species as plasma. It is demonstrated that a change of collision frequencies of impurities takes place in presence of this energetic fraction of particles leading to a change of impurity neoclassical transport regime.

EQUATIONS OF MOTION AND COLLISION OPERATOR

To simulate the motion of the test particle of mass $m$ and charge $Ze$ in toroidal magnetic field $B$, taking into account the effect of the electric field $E$, we solve the guiding center equations in general vector form

$$v_g = v_\parallel \frac{B}{B^2} + \frac{c}{B^2} E \times B + \frac{mc}{2ZeB^3} (2v_\parallel^2 + v_\perp^2) B \times \nabla B + \frac{mcv_\perp^2}{ZeB^4} (B \times \text{rot}B) \times B,$$

(1)

taking care of the energy conservation law $W = m(v_\parallel^2 + v_\perp^2)/2 + Ze\Phi = const$ together with the conservation of the invariant of motion $\mu = v_\perp^2/B = const$. Coulomb collisions are introduced through the Monte Carlo equivalent of the collision operator, which reads

$$F_n = F_o + \langle d\langle F\rangle/dt \rangle \Delta \tau \pm \sqrt{\langle d\sigma_k^2/dt \rangle} \Delta \tau,$$

(2)

where $\Delta \tau$ is the integration time step. The variable $F$ represents one of the quantities, i.e. the kinetic energy $K = mv^2/2$, the pitch angle $\lambda = v_\parallel/v$ or the gyro angle $\phi$. The sign $\pm$ is chosen randomly but with the equal probability $[1, 2]$. The time derivatives of expected value and square of standard deviation of quantity $F$ are determined through the time derivative of
test particle distribution function $f_d^a$, which could be obtained from the Fokker-Planck collision operator

$$
\frac{df_d^a}{dt} = \frac{v_{d1}}{2} \frac{\partial}{\partial \lambda} \left[ (1 - \lambda^2) \frac{\partial f_d^a}{\partial \lambda} \right] + \frac{v_{d2}}{2} (1 - \lambda^2) \frac{\partial f_d^a}{\partial \phi} + \frac{1}{2} \left( v_{d3} \frac{\partial f_d^a}{\partial \theta} + v_{d4} \frac{\partial^2 f_d^a}{\partial \phi^2} \right) +
\frac{1}{v^2} \frac{\partial}{\partial v} \left[ v^3 \left( \frac{m_a}{m_a + m_b} v_S f_d^a + \frac{v}{2} v_1 \frac{\partial f_d^a}{\partial v} \right) \right],
$$

(3)

where $v_{d1} = -\frac{2L_{ab}}{v^3} \frac{\partial \Psi_b}{\partial v}$, $v_{d2} = -\frac{2L_{ab}}{v^3} \frac{\partial^2 \Psi_b}{\partial v \partial \lambda}$, $v_{d3} = -\frac{2L_{ab}}{v^3} \frac{1}{1 - \lambda^2} \frac{\partial^2 \Psi_b}{\partial v \partial \phi}$ and $v_{d4} = v_{d1} \frac{1}{1 - \lambda^2}$ are deflection frequencies in pitch and gyro spaces. Quantities $v_S = L_{ab} (1 + m_a/m_b) (\partial \Phi_b/\partial v)/v$ and $v_{||} = -2L_{ab} (\partial^2 \Psi_b/\partial v^2)/v^2$ are slowing down and parallel velocity diffusion frequencies respectively. $L_{ab} = (4\pi Z_a Z_b e^2/m_a)^2 \ln \Lambda$ is the function of Coulomb logarithm. The Rosenbluth potentials $\Phi_b(v) = -1/4\pi \int f_b(v')/u' d^3v'$ and $\Psi_b(v) = -1/8\pi \int u f_b(v') d^3v'$ are functions of the relative velocity of particles $u = |v - v'|$ and background plasma distribution function $f_b$. The first deflection frequency $v_{d1}$ represents diversion of the velocity vector in velocity space for the case of isotropic background plasma distribution [3]. It plays the key role in our simulations as soon as our model is restricted to isotropic distribution functions.

**PLASMA DISTRIBUTION FUNCTION AND COLLISION FREQUENCIES**

For the simulation we choose the simple TOKAMAK toroidal field model with the major radius $R_0=300$cm, the minor radius $r_a=130$cm and on-axis magnetic field $B_0=3$T. The model plasma profiles are displayed in figure 1. We consider two plasma distribution functions: one is pure Maxwellian distribution and the other is Maxwellian together with the energetic particle fraction presented as $f_b(v) = n_i/\left(\pi^{3/2} v_i^3\right) e^{-\left(v/v_i\right)^2} + n_2/\left(\pi^{3/2} v_2^3\right) e^{-\left(v-v_i\right)^2}$.

where $v_i = \sqrt{2T_i/m_p}$ (see figure 2). Both distribution functions are isotropic in velocity space.

In the case of neon ions colliding with the pure Maxwellian plasma we can easily estimate the values of the deflection frequency $v_{d1}$ relying upon the chosen plasma profiles. The values of $v_{d1}$ for 1keV and 50keV neon ions are plotted versus the minor radius on figure 3.

If we “inject” in plasma the energetic deuterium fraction with the mean velocity $V$ related to the energy 3keV and temperature 100eV we observe the modification of collision frequencies [3]. The variation of $v_{d1}$ due to different energetic fraction density values $n_2$, which vary from 2% up to 10% of initial plasma density $n_1$, is presented on figure 4.
**IMPURITY DIFFUSION COEFFICIENTS**

The mean values of running monoenergetic diffusion coefficients of neon impurities are plotted on figures 5a and 5b. In case of Maxwellian plasma the collision frequencies vary in the ranges $5350 < \nu < 7390$ and $70 < \nu < 110$ for 1keV and 50keV neon ions respectively (see figure 3). These collision rates are pointed in figures 5a and 5b with dark yellow ovals. For the non Maxwellian distribution we still stay within the same neoclassical transport model but with modified collision frequencies, which increase following the increase of the fraction of energetic particles (see figure 4). This modification leads to a shift between different neoclassical transport regimes. On figure 5a with the red arrow the shift of regime for 1keV neon impurities in the plasma with 2% density of energetic fraction is shown. Further increasing of density $n_2$ leads to the more significant shift towards high...
Fig 5. Mean values of monoenergetic diffusion coefficient (blue dots) of neon ion with different values of kinetic energy (1keV and 50 keV on figures a) and b) respectively) versus the deflection frequency. The standard neoclassical values of diffusion coefficient are plotted in green and pink lines for the possible smallest and largest values of neon Larmor radius respectively. The collision rates 1ν and 2ν are related to the boundaries between Galeev-Sagdeev, plateau and Pfirsch-Schlüter regimes in standard neoclassical theory.

rates of Pfirsch-Schlüter regime. The same tendency is observed for the neon impurities with the energy 50keV (see figure 5b).

CONCLUSIONS

A test particle simulation for the neoclassical transport of neon impurity in plasma consisting of deuterons and electrons was performed. Two scenarios of plasma distribution function, i.e. pure Maxwellian and Maxwellian with the energetic deuterium fraction were considered.

It is shown that in presence of the energetic fraction the diffusion coefficients of the impurities vary due to modification of collision frequencies. This modification which depends on the impurity energy and the parameters of the energetic particle fraction brings the impurities in different neoclassical transport regimes.

The phenomenon demonstrated by our simulations plays an important role for the impurity transport problem in non Maxwellian plasma. Such plasma is observed in modern fusion devices due to magnetic field lines reconnection, particle injection and plasma heating.

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