*36th EPS Conference on Plasma Phys. Sofia, June 29 - July 3, 2009 ECA Vol.**33E**, P-4.127 (2009)*

Calculation of the magnetic surface function gradient and associated quantities in stellarators with broken stellarator symmetry*

V. V. Nemov[1,2], S. V. Kasilov[1,2], W. Kernbichler[2], B. Seiwald[2]

[1] *Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Akademicheskaya Str. 1, 61108 Kharkov, Ukraine*

[2] *Association EURATOM-ÖAW, Institut für Theoretische Physik - Computational Physics, TU Graz, Petersgasse 16, A–8010 Graz, Austria*

Introduction

The computation of the gradient of the magnetic surface function, $\nabla\psi$, plays an essential role in plasma physics, e.g. for investigations of plasma equilibrium currents or transport fluxes in stellarators. To evaluate $\nabla\psi$ for real magnetic configurations given in real space coordinates a convenient technique has been presented in [1]. The computation of $\nabla\psi$ becomes more complicated if the magnetic field **B** does not exhibit stellarator symmetry. For such conditions preliminary computations of the corresponding magnetic surface are necessary. Here, a scheme for computation of $\nabla\psi$ for magnetic configurations which do not show stellarator symmetry is presented. This new technique for $\nabla\psi$ computations is applied to Uragan-2M [2]. Taking into account the influence of current-feeds and detachable joints of the helical winding the magnetic configuration does not exhibit stellarator symmetry. Computations of $\nabla\psi$, the effective ripple $\varepsilon_{\rm eff}$ and the geometrical factor λ_b for the bootstrap current in the $1/\nu$ transport regime are performed.

Basic equations

In the proposed technique gradients of different integrals of the magnetic field line equations are analyzed using the integration along the magnetic field line. The distribution of $\nabla\psi$ along the magnetic field line can be expressed as linear combination of gradients of two conveniently chosen independent integrals of the magnetic field line equation, $\nabla\theta_1$ and $\nabla\theta_2$, respectively:

$$\nabla\psi = \nabla\theta_1 + \alpha\nabla\theta_2 \qquad (1)$$

To find $\nabla\theta_1$ and $\nabla\theta_2$ the equations

$$\frac{dQ_{1,i}}{ds} = -\frac{1}{B}\frac{\partial B^j}{\partial\xi^i}Q_{1,j}\ , \qquad (2)$$

$$\frac{dQ_{2,i}}{ds} = -\frac{1}{B}\frac{\partial B^j}{\partial\xi^i}Q_{2,j}\ , \qquad (3)$$

can be used. Here, B^j are the contravariant components of **B** in real space coordinates ξ^i, whereas $Q_{1,j} = \partial\theta_1/\partial\xi^j$ and $Q_{2,j} = \partial\theta_2/\partial\xi^j$ are the covariant components of $\mathbf{Q_1} \equiv \nabla\theta_1$ and $\mathbf{Q_2} \equiv \nabla\theta_2$, respectively. Equations (2) and (3) are a consequence of the equation $\mathbf{B}\cdot\nabla\theta_{1,2}$=0 (see in [1]). The quantity α in Eq. (1) can be determined as follows. Preliminary computations of any magnetic surface have to be performed using the field line integration technique. The integration of the magnetic field line as well as the integration of Eqs. (2) and (3) have to be

*This work, supported by the European Communities under the contract of Association between EURATOM and the Austrian Academy of Sciences, was carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission. Additional funding is provided by the Austrian Science Foundation, FWF, under contract number P16797-N08.

performed for a sufficiently large integration interval corresponding to a sufficiently large number of turns around the torus. During this integration all intersections of the field line with the initial cross-section of the magnetic surface are considered. The intersection, which is the nearest to the starting point of the integration $\mathbf{r}_{st}$, is chosen as the final integration point $\mathbf{r}_{fin}$. Since $\mathbf{r}_{fin}$ is close to $\mathbf{r}_{st}$, the quantities $\nabla\psi_{st}$ and $\nabla\psi_{fin}$ should be also close each to other. Here $\nabla\psi_{st}$ and $\nabla\psi_{fin}$ are $\nabla\psi$ at $\mathbf{r}_{st}$ and $\mathbf{r}_{fin}$, respectively. Therefore, the quantity $f(\alpha) = (\nabla\psi_{fin} - \nabla\psi_{st})^2$, or

$$f(\alpha) = (\nabla\theta_{1,fin} + \alpha\nabla\theta_{2,fin} - \nabla\theta_{1,st} - \alpha\nabla\theta_{2,st})^2, \qquad (4)$$

should be small. To find the correct value of α one has to minimize $f(\alpha)$. Equating the derivative of $f(\alpha)$ with respect to α to zero one finds

$$\alpha = -\frac{b}{a} \qquad (5)$$

with

$$\begin{aligned} a &= (\nabla\theta_{2,fin} - \nabla\theta_{2,st})^2 , \\ b &= (\nabla\theta_{1,fin} - \nabla\theta_{1,st}) \cdot (\nabla\theta_{2,fin} - \nabla\theta_{2,st}). \end{aligned} \qquad (6)$$

First, for any considered magnetic surface preliminary computations of $\nabla\theta_1$ and $\nabla\theta_2$ have to be performed for a sufficiently large integration interval to find α. Then the computation of $\nabla\psi$ and associated quantities can be done.

Computational results

To demonstrate the capabilities of the proposed approach computations of the bootstrap current and the effective ripple, ε_{eff}, are performed for the magnetic configuration of U-2M [2] (l=2 torsatron) where the influences of current-feeds and detachable joints of the helical winding are taken into account (R=170 cm, n_p=4, R is a big radius of the torus, n_p is a number of the helical field periods along the torus). Because of the non-symmetric arrangement of these elements of the magnetic system the stellarator symmetry of the resulting magnetic field of U-2M is violated and there exists no cross-sections of magnetic surfaces with an up-down symmetry. The resulting magnetic field and its spatial derivatives can be computed using the Biot-Savart law code [3] which has been developed in order to take into account the above elements of the magnetic system. To minimize the computer time expenses for this application the Biot-Savart law code is not used for the calculations. The magnetic field is represented as superposition of a finite number of toroidal harmonic functions containing the associated Legendre functions (158 harmonics). The expansion coefficients of the decomposition are obtained with preliminary computations of the magnetic field using the Biot-Savart code. The magnetic field exhibits magnetic surfaces which are well centered with respect to vacuum chamber and the rotational transform ι is within 1/3< ι <1/2 (k_φ=0.31, see [2,3]). Comparative computations of some magnetic surfaces have shown that in case of the magnetic field representation with toroidal harmonics the shapes of the magnetic surfaces as well as the rotational transform are in good agreement with those obtained with help of the Biot-Savart code.

For studies of the bootstrap current the geometrical factor λ_b is computed in real space coordinates using a field line following code based on equations obtained in [4]. For any magnetic surface such a computation is performed in three stages. In the first stage a point corresponding to the global maximum of B, s_m, has to be found by integration of the magnetic field line. In the second stage the parameter α (5) has to be determined by using s_m as initial point for the integration. The geometrical factor λ_b is calculated in the third stage using the integration along the magnetic field line. Simultaneously, $\nabla\psi$ is computed with help of Eqs. (1), (2) and (3). Computational results for $\lambda_{bb} = \lambda_b \langle B^2 \rangle / B_0^2$ as a function of the mean radius of the magnetic surface are presented in Fig. 1 (see curves 1 and 2; $\langle\rangle$ denote the average over the volume of a thin layer between the neighboring magnetic surfaces, B_0 is the reference magnetic field). It follows from [5] that an additional factor enters into the expression of the bootstrap current if the fraction of trapped particles is not very small. This factor is approximately equal to $1/f_c$ with f_c being the fraction of the circulating particles. This factor is also shown in Fig. 1 (curve 3).

For the $1/\nu$ neoclassical transport study the quantity $\varepsilon_{\text{eff}}^{3/2}$ is computed using a field line following code based on equations elaborated in [6]. The obtained results for $\varepsilon_{\text{eff}}^{3/2}$, displayed as a function of the mean radius of the magnetic surface, are presented in Fig. 2 (curve 1). Also the approach, presented in [3], for calculation of $\nabla\psi$ is used for comparison of $\varepsilon_{\text{eff}}^{3/2}$ (curve 2, open circles). Fig. 2 (rhombi) shows analogous results obtained in [3] where the Biot-Savart law code has been used and where $\nabla\psi$ has been calculated using a different approach then presented here. It follows from the comparison that the results of [3] only slightly differ from those obtained here.

The last fact allows to state that for the $1/\nu$ transport study calculation of $\nabla\psi$ using the scheme, presented in [3], is admissible. This cannot be stated, however, with respect to bootstrap current calculations. This is illustrated in Fig. 3, which shows a behavior of $\lambda_b(L)$ along the magnetic field line for a near-boundary magnetic surface (L is the current integration interval). The integration starts from s=s_m and corresponds to 500 turns around the main axis of the torus. It can be clearly seen that $\lambda_b(L)$ (curve 1) is oscillating with decreasing amplitude and that it converges to the final value of λ_b (line 2). For comparison, curve 3 in Fig. 3 shows $\lambda_b(L)$ where the approach of [3] is used for the $\nabla\psi$ computation. It is seen that $\lambda_b(L)$ does not converge to any final value in this case. The reason of such a phenomenon is the insufficient accuracy of the $\nabla\psi$ computation with the approach of [3] for the same integration intervals of the preliminary magnetic surface computations.

Summary

In the proposed technique $\nabla\psi$ is presented as a linear combination of gradients of two conveniently chosen, independent, not single valued integrals of the magnetic field line equations. The corresponding coefficients of this combination can be determined in a preliminary computation of these gradients using the integration along the magnetic field line for a sufficiently large integration interval. Therefore, the requirement that $\nabla\psi$ is single valued should be fulfilled on this interval. After this preliminary computations, $\nabla\psi$ and associated quantities can be calculated using the obtained coefficient α. The proposed technique is applied to U-2M to study the $1/\nu$ neoclassical transport as well as the bootstrap current for the $1/\nu$ regime, accompanied by comparison with the approach of [3] for the $\nabla\psi$ computation. For the same integration intervals the comparison has demonstrated the advantage of the proposed technique for the equilibrium current studies. At the same time it is found that the technique applied in [3] for the $1/\nu$ transport computation is admissible.

References

[1] V. V. Nemov, Nucl.Fusion **28**, 1727 (1988).

[2] O. S.Pavlichenko for the U-2M group, First results from the "URAGAN-2M" torsatron, *Plasma Phys. Control. Fusion* **35**, B223-B230 (1993).

[3] V. V. Nemov, V. N. Kalyuzhnyj, S. V. Kasilov, W. Kernbichler, G. G. Lesnyakov, B. Seiwald, N. T. Besedin, Proc. 34th EPS Conf. on Plasma Phys., Warsaw, Poland, 2-6 July 2007, ECA Vol.31F, P-1.077 (2007)

[4] S. V. Kasilov, V. V. Nemov, W. Kernbichler, M. F. Heyn, in 27th EPS Conference on Controlled Fusion and Plasma Physics (Editors: K.Szego, T.N.Todd and S.Zoletnik), 12-16 June 2000, Budapest, Hungary (Report P2.066)

[5] K. C. Shaing and J. D. Callen, Phys. Fluids **26**, 3315 (1983).

[6] V. V. Nemov, S. V. Kasilov, W. Kernbichler and M. F. Heyn, Phys. Plasmas, 6, p 4622, 1999.

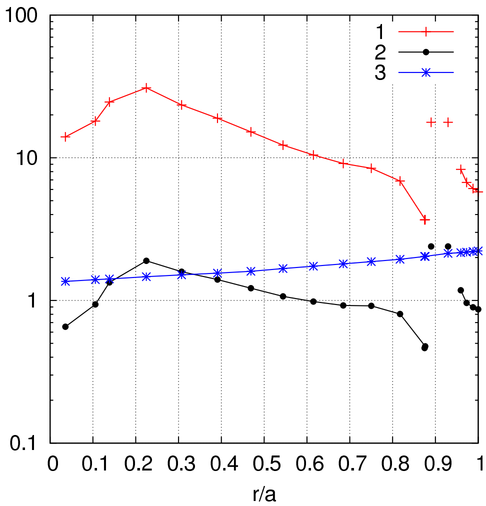

Fig.1. Parameters λ_{bb} (curve 1) and $1/f_c$ (curve 3). Curve 2 shows a normalized λ_{bb} value, λ_{bbn}, determined as a ratio of λ_{bb} to the corresponding parameter of an equivalent tokamak; r is the mean radius of the magnetic surface, a is a mean radius of the outermost magnetic surface inside the vacuum chamber. Enhanced values of λ_{bb} and λ_{bbn} for $r/a \approx 0.9$ correspond to island magnetic surfaces.

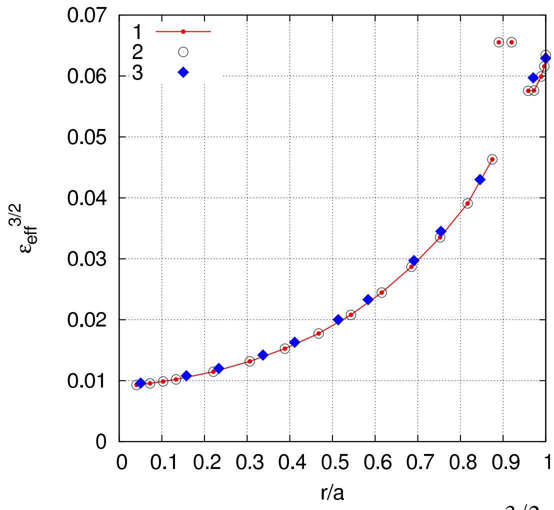

Fig.2. Computational results for $\varepsilon_{\mathrm{eff}}^{3/2}$; 1: for the proposed technique of the $\nabla\psi$ calculation, 2: (open circles) for the $\nabla\psi$ calculation using approach [3], 3: (rhombi) corresponding results from [3]; r and a are the same as in Fig. 1; enhanced values of $\varepsilon_{\mathrm{eff}}^{3/2}$ for $r/a \approx 0.9$ correspond to island magnetic surfaces.

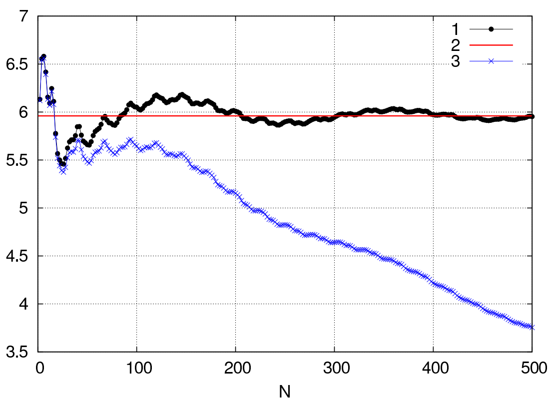

Fig.3. Determination of λ_b for some near-boundary magnetic surface; 1: the $\lambda_b(L)$ behavior along the magnetic field line, 2: corresponding value of λ_b, 3: the $\lambda_b(L)$ behavior in case of the $\nabla\psi$ computation using approach [3], N is the number of turns around the main axis of the torus.