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A Self-Organized Criticality Model for Ion Temperature Gradient (ITG) Mode Driven Turbulence in Confined Plasma

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Introduction

Self-Organised Criticality (SOC) is a possible state of complex, spatially extended systems that are systematically driven and that have mechanisms to develop local instabilities and to relax them. SOC is characterised by intermittent transport events that range from very small size up to system size, so called avalanches. SOC models are usually constructed in the form of Cellular Automata (CA), *i.e.* by using a discrete grid and rules for the evolution of the system, with proto-type the sand-pile model of [1].

In studies of confined plasmas and the related transport phenomena, evidence has been collected that the plasma might well be in the state of SOC, e.g. the fluctuations in density and electron temperature show avalanche-like characteristics, such as intermittency and power-spectra of power-law shape (see the recent discussion in [2]). Several SOC models have been suggested for fusion plasmas that address different aspects of turbulent transport and that are able to reproduce a number of observed phenomena (see references in [2]). These are all CA models, namely of the sand-pile type, and they basically are variants of the original SOC model of [1].

Here, we introduce a new CA model for ion temperature gradient (ITG) mode driven turbulence, constructed in such a way that the system is able to reach the state of SOC. Our particular aim is that the elements of the model are consistently interpretable in the usual physical way, e.g. the usual physical variables are used, and the temporal evolution of the CA, which necessarily is in the form of rules, mimics actual physical processes as they are considered to be active in ITG mode driven turbulence.

The application we make here is to the ion temperature evolution in a toroidally confined plasma, specifically the one seen in the JET experiment. The physical system meets the three prerequisites for a system to be able to reach the SOC state: it is systematically driven, namely heated, the ITG mode instability is threshold dependent, and there is a process that relaxes the instabilities, given the fact that ITG mode driven turbulence reaches a saturated stationary state.

The model

We consider a one dimensional grid with L grid-points and grid-spacing $\Delta x = 2sa/(L-1)$ (considered to be finite), which extends along the minor radius a and covers the core region $[-sa, sa]$ in a toroidal confinement device ($s < 1$). The basic scalar grid variable at the grid sites $x_k \in [-sa, sa]$ (for $k = 1, \dots, L$) is the local ion temperature $T_k \equiv T(x_k)$, considered a positive real number. The system evolves in equal, discrete time-steps of duration 1.

The ITG mode instability has a critical dependence on the normalized scale length R/L_T, where R is the major radius and L_T is the temperature-gradient scale-length, $1/L_T := |\nabla T|/T$, in the sense that ITG mode instabilities are triggered if the condition $R|\nabla T|/T > R/L_{\text{crit}}$ is fulfilled, and on defining $\delta_c := 1/L_{\text{crit}}$ the instability criterion takes the form $|\nabla T|/T > \delta_c$. In order to calculate the gradient, we interpolate T_k locally in the neighbourhood $(k-1, k, k+1)$ with a second order polynomial and we differentiate the polynomial with respect to x at x_k. The grid site k is then considered to be unstable if

$$|\delta_k| = \frac{1}{2\Delta x} \frac{|T_{k+1} - T_{k-1}|}{T_k} > \delta_c. \tag{1}$$

We assume a diffusion process to be triggered around the grid site where an instability occurs, localized around the unstable site and of normal nature. In the CA, the local diffusion process is formulated in terms of a set of *redistribution rules*, which describe how the temperature in the local neighbourhood is redistributed to relax the instability. In the formulation of these rules we demand partial relaxation, $|\delta_i^+| < \delta_c$ (the superscript "+" denotes variables after relaxation).

The asymptotic state of the diffusive process is characterised by $\partial_x^2 T = 0$, *i.e.* a linear local profile, T_i^+ thus equals the mean temperature after the relaxation, and, due to temperature conservation, it also equals the mean temperature prior to redistribution,

$$T_i^+ = \frac{1}{3}\left(T_{i-1} + T_i + T_{i+1}\right) \equiv T_i - \frac{2}{3}\tau_i, \tag{2}$$

where $\tau_i := T_i - (T_{i-1} + T_{i+1})/2$. For practical reasons we add $\tau_i/3$ to each of the neighbours,

$$T_{i\pm1}^+ = T_{i\pm1} + \frac{1}{3}\tau_i + A_\pm. \tag{3}$$

The additional terms A_+ and A_- take into account local anisotropies in the relaxation process and they are compatible with energy conservation provided that $-A_- = A_+ =: A$. The role of A becomes clear when calculating the slope of the relaxed temperature profile, which is $T_{i+1}^+ - T_i^+ = (T_{i+1} - T_{i-1})\,/\,2 + A$. For $A = 0$, the slope of the relaxed profile equals a kind of mean slope of the unstable profile. It is thus natural to define A such as to control the slope of the relaxed profile, $A := -\sigma\,(T_{i+1} - T_{i-1})\,/\,2$, so that the slope of the relaxed profile writes as

$$T_{i+1}^+ - T_i^+ = \frac{1-\sigma}{2}\left(T_{i+1} - T_{i-1}\right). \tag{4}$$

After all, the relaxation rules are

$$
\begin{aligned}
T_i^+ &= T_i - \frac{2}{3}\tau_i \\
T_{i\pm1}^+ &= T_{i\pm1} + \frac{1}{3}\tau_i + \frac{\sigma}{2}(T_{i\mp1} - T_{i\pm1}),
\end{aligned} \tag{5}
$$

with a free parameter to control the slope of the relaxed profile, $\sigma \in [0,1)$. The range of σ is further restricted in order the relaxation process to fulfil the additional requirements that heat must not be transported from a colder to a hotter site, and that the relaxation process should indeed relax the instability.

A constant value T_b of the temperature is applied outside the domain, at the two external "ghost" grid-sites indexed by 0 and $L+1$, *i.e.* we assume $T_0 = T_{L+1} = T_b$.

The system undergoes a driving (heating) process, in which the temperature is increased, simulating the effect of heat injection. The heating process has two temporal variants, (a) continuous heating in every time-step, and (b) heating only in stable configurations, *i.e.* in the absence of instabilities. Also, different spatial heating patterns are applied, following experimentally applied heating profiles. The latter case includes heating only in the central region, reminiscent of Ohmic heating, heating only in a region off-axis, as in the case of rf heating, and a combination of the two. In any case, the temperature at each site that is heated is increased by a predefined amount ΔS_j with probability q_0.

Results

The parameters of the model are chosen to coincide with those of the JET experiment, with major radius $R = 2.95\ m$, and minor radius $a = 1.25\ m$. Since the ITG instabilities are considered to dominate turbulence only in the core, our domain is restricted to $[R-0.8a, R+0.8a]$. The grid size Δx is assumed of the order of the typical ion Larmor radius, $\rho_i \simeq 0.49\ cm$, yielding a grid of $L = 401$ sites, with grid-spacing $\Delta x = 0.5\ cm$. The external and the initial temperature is set to $T_b = 500\ eV$. The threshold R/L_{crit} of the ITG mode instability is found from experiments at JET to lie in the range $[3.5, 5]$.

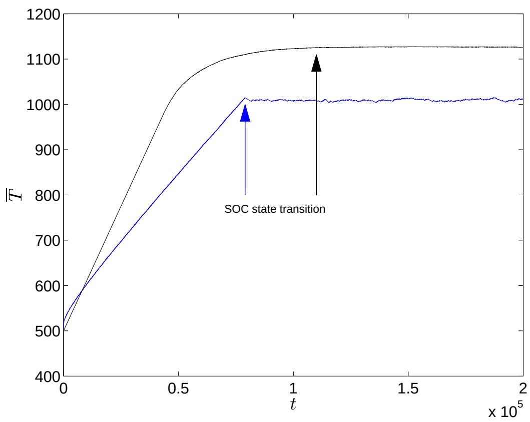

Figure 1: Spatial mean of the temperature as a function of time for loading only in stable states (lower curve, $q_0 = 0.1,\ \Delta S_j = 10$) and for continuous loading (upper curve, $q_0 = 0.1,\ \Delta S_j = 0.1$).

The transition of the system to the SOC state is illustrated in Fig. 1 by the time-series of the spatial mean of the temperature $\overline{T}(t)$. There is a transient phase, where $\overline{T}$ grows, and an asymptotic state, where it fluctuates around an average value. The behaviour of the mean absolute normalized scale-length $\overline{|R/L_T|}$ is similar. Once the SOC state is reached, avalanches of widely varying sizes start to appear.

The temperature profiles in the SOC state are shown in Fig. 2 for different values of the instability threshold. In all cases, the profiles are of exponential shape. The threshold, together with the value T_b of the temperature at the boundaries, determine the maximum value that is reached in the centre. We also find that the temperature profiles are independent of the applied spatial heating pattern, in particular central heating and off-axis heating yield the same temperature profiles. Last, we note that the profiles yielded by the model are very similar to those observed at JET in the core region (for L- as well as for H-mode).

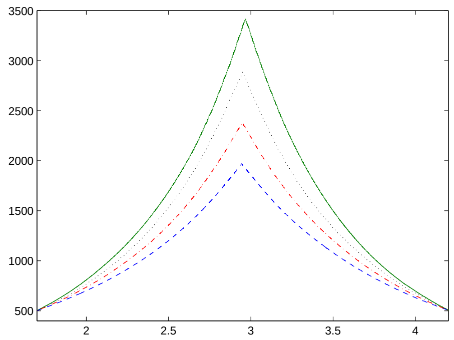

Figure 2: Temperature profiles during the SOC state for different values of the instability threshold $R/L_{\rm crit}$: 3.5 (dashed), 4 (dot-dashed), 4.5 (dotted) and 5 (solid).

Conclusion

We introduced a new CA model for ITG mode driven turbulence. The model yields symmetric temperature profiles of exponential shape, which exhibit very high stiffness, in that they are basically independent of the applied heating pattern (central and off-axis heating yield the same profile). This implies that there is anomalous heat transport (“uphill” heat transport, against the driving gradient) present in the system. The temperature profiles yielded by the model are in good agreement with those observed at JET in the core region.

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