*36th EPS Conference on Plasma Phys. Sofia, June 29 - July 3, 2009 ECA Vol.**33E**, P-4.116 (2009)*

Test-particle simulations of ion drift in stochastic magnetic fields

H. Isliker[1], A. Vogiannou[1], L. Vlahos[1], M. Negrea[2], I. Petrisor[2], B. Weyssow[3]

[1] *Section of Astrophysics, Astronomy and Mechanics, Department of Physics, University of Thessaloniki, Association Euratom-Hellenic Republic, GR 541 24 Thessaloniki, GREECE*

[2] *Department of Physics, Association Euratom-MEdC, Romania, University of Craiova, A.I. Cuza str. 13, Craiova, ROMANIA*

[3] *Physique Statistique - Plasma, Association Euratom - Etat Belge, Universite Libre de Bruxelles, Campus Plaine, Bd. du Triomphe, 1050 Bruxelles, BELGIUM*

Abstract

We study the influence of stochastic magnetic fields on ion diffusion, using a drift approximation in slab geometry, and applying a stationary stochastic magnetic field on top of a uniform background field. The stochastic magnetic field follows a Gaussian in its distribution function, and it obeys a prescribed auto-correlation function with given correlation length. The running diffusion coefficients of the ions are determined with the use of test-particle simulations in the three dimensional environment, for different levels of turbulence (varying Kubo number). The results of the test-particle simulations are compared to and used to validate the results as obtained for the same physical system by the semi-analytical Decorrelation Trajectory (DCT) method.

Introduction

Turbulence induced stochastic magnetic fields perturb the regularly nested magnetic field structures in toroidal confinement devices, they thus open new channels through which particles can potentially be transported and possibly give rise to enhanced or even anomalous particle diffusion. The resulting particle transport can be expected to depend on the level of the stochastic perturbations and on their spatial correlations. Here, we investigate the influence of the former on transport by performing test-particle simulations in numerically generated stochastic magnetic fields, from which we determine the running diffusion coefficients. The stochastic fields are generated (i) with prescribed Gaussian distribution of varying standard deviation, and (ii) with prescribed spatial auto-correlation of Gaussian shape and fixed correlation length, and they are superimposed on a strong and uniform background magnetic field.

The results of the test-particle simulations are also compared to and used to validate the results as obtained for the same physical system by the semi-analytical Decorrelation Trajectory (DCT) method.

The set-up

We consider a slab geometry, where the magnetic field $\mathbf{B}$ is assumed to have a strong background component B_0 in the Z-direction (in Cartesian coordinates X, Y and Z), and stochastic components in the perpendicular direction,

$$\mathbf{B}(\mathbf{X};Z) = B_0 \left\{ \mathbf{e}_Z + \beta b_X(\mathbf{X};Z)\,\mathbf{e}_X + \beta b_Y(\mathbf{X};Z)\,\mathbf{e}_Y \right\}, \tag{1}$$

with the dimensionless β determining the strength of the perturbations, b_X and b_Y the normalized stochastic fields, and $\mathbf{X} = (X,Y)$.

In the parallel direction, we assume the particles to move as

$$\frac{dZ}{dt} = V_{||}, \tag{2}$$

where $V_{||}$ is the parallel velocity, and, for simplicity, we assume $V_{||}$ to be constant and to equal the thermal velocity V_{th}. The Z coordinate thus plays a dummy role, and we use it in the following instead of time t. We furthermore normalize the spatial coordinates with the correlation lengths λ_i, $i = X,Y,Z$ (defined below), $x := \frac{X}{\lambda_X}$, $y := \frac{Y}{\lambda_Y}$, $z := \frac{Z}{\lambda_Z}$.

For a stationary magnetic field, the linearized guiding center equations of motion in the perpendicular direction can be written as (see Refs. [1], [2])

$$\frac{dx(z)}{dz} = K_m b_x(\mathbf{x};z) - K_{dr}\frac{\partial b_y(\mathbf{x};z)}{\partial z}, \tag{3a}$$

$$\frac{dy(z)}{dz} = \Lambda K_m b_y(\mathbf{x};z) + \Lambda K_{dr}\frac{\partial b_x(\mathbf{x};z)}{\partial z}, \tag{3b}$$

where $K_m = \beta\frac{\lambda_X}{\lambda_Z}$ is the magnetic Kubo number, $\Lambda = \frac{\lambda_X}{\lambda_Y}$ the stochastic anisotropy parameter, $K_{dr} = \beta\frac{V_{th}}{\Omega\lambda_X}$ the drift or thermal Kubo number, and $\Omega = eB/mc$ the gyro-frequency. The system of equations is numerically integrated with a fourth order Runge Kutta, adaptive step-size scheme.

The running diffusion coefficients for the motion of the ions in the magnetic field are determined as

$$D_x(z) = \frac{\langle (x(z)-x(0))^2 \rangle}{2z}, \qquad D_y(z) = \frac{\langle (y(z)-y(0))^2 \rangle}{2z}, \tag{4}$$

where the averaging is taken over a large number of test particles.

The stochastic magnetic field

The spatial auto-correlation function M of the stochastic part $\mathbf{A}_S = (0,0,A_z)$ of the vector potential is assumed to factorize,

$$M(X,Y,Z) = M_X(X)M_Y(Y)M_Z(Z), \tag{5}$$

and to be of Gaussian shape for each of the coordinates,

$$M_X(X) \propto \exp\left(-\frac{X^2}{2\lambda_X^2}\right), \quad M_Y(Y) \propto \exp\left(-\frac{Y^2}{2\lambda_Y^2}\right), \quad M_Z(Z) \propto \exp\left(-\frac{Z^2}{2\lambda_Z^2}\right), \tag{6}$$

where λ_i, $i = X,Y,Z$, are the correlation lengths in the X, Y, and Z direction, respectively.

To construct the vector potential A_z itself, we make use of the Wiener-Khinchine theorem. We first Fourier transform $M(X,Y,Z)$, which yields$\hat{M}(k_X,k_Y,k_Z)$, and the Fourier transform $\hat{A}_Z$ of A_Z is then given as

$$\hat{A}_Z = \left|\hat{M}(k_X,k_Y,k_Z)\right|^{1/2} \exp(i\varphi_{k_X,k_Y,k_Z}), \tag{7}$$

with the phases φ_{k_X,k_Y,k_Z} chosen uniformly random in $[0,2\pi]$, and from which A_Z is determined by Fourier back-transformation. Derivatives of A_Z are also calculated via Fourier space, e.g. $\partial_X A_Z$ is calculated as the back-transform of $\widehat{\partial_X A_Z} = ik_X\hat{A}_Z$, and likewise for higher order derivatives.

In this way, the magnetic field $(b_X,b_Y) = (\partial_Y A_Z, -\partial_X A_Z)$ and its derivative with respect to Z are determined on a 3-dimensional grid. We use natural dimensional coordinates (X,Y,Z) in the construction of the grid, so that e.g. $b_x(x,y,z) = b_X(\lambda_X x, \lambda_Y y, \lambda_Z z)$. The grid-size in each direction is such that it contains several correlation lengths. The values of $b_X\,(X,Y,Z)$ and $b_Y\,(X,Y,Z)$ for points (X,Y,Z) in-between the grid-sites are calculated by linearly interpolating the magnetic field components at the nearest grid-sites.

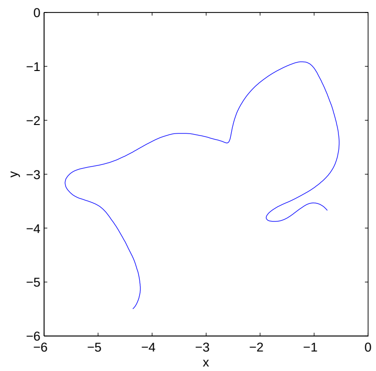

Figure 1: A typical ion trajectory, projected into the x-y plane.

The magnetic field components obey Gaussian distributions, as a consequence of the central limit theorem, and their standard deviations, σ_{b_X} and σ_{b_Y}, are enforced to be equal to one.

By construction, the magnetic field is periodic in all three directions, and particles leaving the simulation box are re-injected at the plane opposite to the one through which they leave.

Results

The parameter values we use are $V_{th}/\Omega = 0.3\,$m for the Larmor radius, $\beta = 10^{-2}$ for the strength of the magnetic perturbations, and for the values of the correlation lengths we assume $\lambda_X = \lambda_Y = 10^{-2}\,$m and $\lambda_Z = 1\,$m, so that $K_m = 1$, $\Lambda = 1$, and K_{dr} is varied in the range $[0\,,0.6]$. The stochastic magnetic field is generated on a grid with 64^3 grid-points, with a grid-spacing such that the grid-size equals 9 correlation lengths λ_i in each direction.

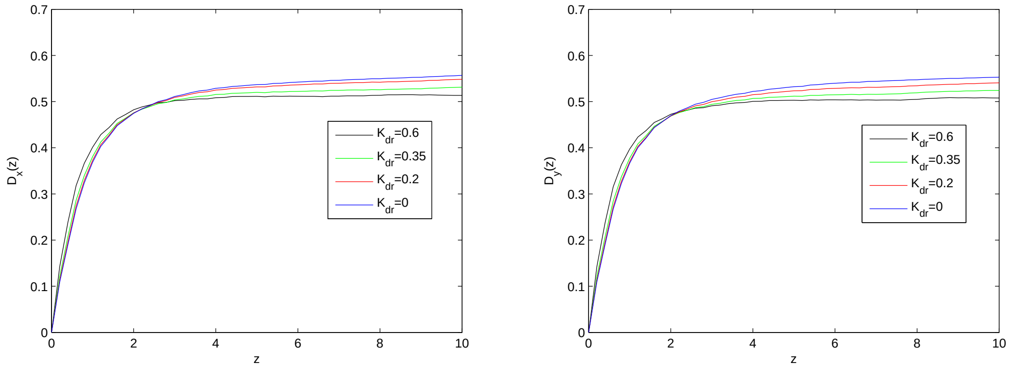

Figure 2: The running diffusion coefficients $D_x(z)$ (left panel) and $D_y(z)$ (right panel) as a function of z, for different values of K_{dr}.

A typical trajectory is shown in Fig. 1. The running diffusion coefficients are calculated according to Eq. (4), and they are shown as a function of z and for different values of K_{dr} in Fig. 2 (for 10^6 test particles in 10^3 different samples of the magnetic field.). They decrease from 0.55 to 0.5 when K_{dr} increases from 0 to 0.6, the diffusive process thus slows down when either the Larmor radius or the strength of the magnetic perturbation increases. With the DCT method, the respective diffusion coefficients are found to lie between 0.6 and 0.7, they though increase with increasing K_{dr}, and they reach the asymptotic state earlier, approximately at $z = 3$ (see [1]).

Conclusion

We considered ion diffusion in a magneto-static, perturbed magnetic field environment, and we find that diffusion is of normal nature. The diffusive process slows down with both, increasing strength of the magnetic perturbation, and increasing mass of the ions considered, respectively. The values of the diffusion coefficients coincide within 20% with the results yielded by the DCT method, we find though a different scaling of the diffusion coefficient with the thermal Kubo number.

Acknowledgment: Work performed under the Contract of Association Euratom-Hellenic Republic. It is the sole responsibility of the authors and does not necessarily represent the views or opinions of any of the sponsors, which do not bear any responsibility for its contents.

References

[1] Negrea, M., Petrisor, I., and Balescu, R., Phys. Rev. E **70**, 046409 (2004).

[2] Vlad, M., Spineanu, F., Misguich, J.H., and Balescu, R., Phys. Rev. E **53**, 5302 (1996).