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Ion plasma waves in a weakly ionized plasma with ion flow

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In this paper we analyze the dispersion relation of ion plasma waves in a weakly ionized plasma with ion flow driven by an electric field. The dispersion relation is derived using the kinetic equation with an ion-neutral collision term of the Bhatnagar-Gross-Krook form; electrons are treated as a homogeneous neutralizing background. The analysis reveals an instability which occurs when the ratio of the ion flow velocity to the thermal velocity of neutrals is greater than approx. 8 and the ratio of the ion-neutral collision frequency to the ion plasma frequency is less than approx. 0.2.

Let us consider a homogeneous weakly ionized plasma with ion flow driven by an electric field $\mathbf{E}_0$. To investigate ion plasma waves in this plasma we employ the kinetic approach with an ion-neutral collision term of the Bhatnagar-Gross-Krook form [1]. Then the ion plasma perturbations are governed by

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} + \frac{e}{m}\left(\mathbf{E}_0 - \frac{\partial \phi}{\partial \mathbf{r}}\right)\cdot\frac{\partial f}{\partial \mathbf{v}} = -\nu f + \nu \Phi_{\mathrm{M}} \int f(\mathbf{r},\mathbf{v}')\,d\mathbf{v}', \tag{1}$$

$$-\frac{\partial^2 \phi}{\partial \mathbf{r}^2} = 4\pi e\left(\int f\,d\mathbf{v} - n_0\right), \tag{2}$$

where f is the ion distribution function, $e > 0$ is the elementary charge (ions are assumed to be singly ionized), m is the ion mass, ν is the ion-neutral collision frequency,

$$\Phi_{\mathrm{M}} = \frac{1}{(2\pi \upsilon_{\mathrm{tn}}^2)^{3/2}} \exp\left[-\frac{|\mathbf{v}|^2}{2\upsilon_{\mathrm{tn}}^2}\right] \tag{3}$$

is the normalized Maxwellian velocity distribution of neutrals, υ_{tn} is the thermal velocity of neutrals, $n_0 = \int f_0\,d\mathbf{v}$ is the ion number density in the unperturbed state, f_0 is the ion distribution function in the unperturbed state, ϕ is the electrostatic potential of ion plasma perturbations.

The ion distribution function in the unperturbed state is found self-consistently from Eq. (1) to be

$$f_0 = \frac{n_0}{(2\pi \upsilon_{\mathrm{tn}}^2)^{3/2}} \int_0^\infty \exp\left[-\xi - \frac{|\mathbf{v}-\xi\mathbf{u}|^2}{2\upsilon_{\mathrm{tn}}^2}\right] d\xi, \tag{4}$$

where $\mathbf{u} = e\mathbf{E}_0/(m\nu)$. Equation (4) shows that the flow velocity $\int \mathbf{v} f_0\,d\mathbf{v}/n_0$ is exactly equal to $\mathbf{u}$.

The dispersion relation is derived by linearizing Eqs. (1) and (2) with respect to ϕ and $f - f_0$

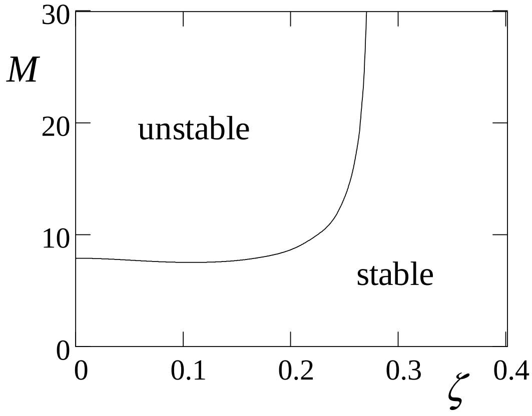

Figure 1: Stability diagram

to be

$$\begin{aligned}1+\frac{1}{k^2\lambda^2}\frac{B(\omega,\mathbf{k})}{1-A(\omega,\mathbf{k})} &= 0,\\ A(\omega,\mathbf{k}) &= \frac{\zeta}{k\lambda}\int_0^\infty \exp[-\Psi(\omega,\mathbf{k},\tau)]\,d\tau,\\ B(\omega,\mathbf{k}) &= \int_0^\infty \frac{\tau\exp[-\Psi(\omega,\mathbf{k},\tau)]}{1+i(\mathbf{k}\cdot\mathbf{e}/k)M\tau}\,d\tau,\\ \Psi(\omega,\mathbf{k},\tau) &= \frac{\zeta}{k\lambda}\tau - i\frac{\omega/\omega_{\rm pi}}{k\lambda}\tau+\frac{1}{2}\tau^2+\frac{i}{2}\frac{\mathbf{k}\cdot\mathbf{e}}{k}\frac{M\zeta}{k\lambda}\tau^2,\end{aligned} \tag{5}$$

where $M=|\mathbf{u}|/\upsilon_{\rm tn}$ is the thermal Mach number, $\zeta=\nu/\omega_{\rm pi}$ is the collision parameter, $\omega_{\rm pi}=\sqrt{4\pi n_0 e^2/m}$ is the ion plasma frequency, $\lambda=\upsilon_{\rm tn}/\omega_{\rm pi}$ is the ion Debye length, $\mathbf{e}=\mathbf{E}_0/|\mathbf{E}_0|$ is the unit vector along the field $\mathbf{E}_0$, $k=|\mathbf{k}|$, perturbations are $\propto \exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r})$ with complex ω and real $\mathbf{k}$ [2].

Note that in the limit $M=0$, $\zeta\to 0$ the dispersion relation (5) yields the well-known modes of a collisionless one-component Maxwellian plasma [3, 4].

The numerical analysis of the dispersion relation (5) yields the stability diagram shown in Fig. 1 [5].

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