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# PIC Simulation of the Neoclassical Tearing Mode Threshold

H.Qu, H. R. Wilson

*Department of Physics, University of York, Heslington, YO10 5DD, UK*

## 1. Introduction

The neoclassical tearing mode (NTM) plays an important role in determining tokamak plasma pressure limits, and can lead to disruption [1,2,3]. The basic process of the tearing instability is that anti-parallel magnetic field lines reconnect in the plasma and form magnetic islands. The result is that the radial particle and energy flux is enhanced in the region of magnetic islands and then cause a degradation of confinement.

Because of its importance in tokamak plasma research, and ITER in particular, many have worked to understand the neoclassical tearing mode [4,5]. Nevertheless, the theory of the threshold of the neoclassical tearing instability is still short of being predictive. When the nonlinear effects associated with the magnetic island are taken into account, two important neoclassical mechanisms need to be investigated carefully [3]. First, the bootstrap current inside the magnetic island can provide an additional drive for the island in tokamak plasma when $dp/dq < 0$, where $p$ is the plasma pressure and $q$ is the safety factor. Second, the polarization drift may have a stabilizing effect on small magnetic islands and thus provide a threshold to the neoclassical tearing mode. These two mechanisms have attracted significant interest and been investigated on JET, DIII-D and ASDEX upgrade [6,7,8], for example. They have also been discussed analytically in full toroidal geometry by assuming that the width of the island is much larger than the ion banana width [3]. The theory suggests that the bootstrap current and polarization drift effects may play dominant roles in determining the threshold of the NTM when the magnetic island width is comparable to the ion banana width. However, when the island is this small, the ion orbit width effects cannot be treated perturbatively, and an analytical treatment seems unlikely to be tractable. Therefore, to explain the seed NTM threshold, complicated numerical techniques, such as particle simulation, are required.

Numerical particle-in-cell (PIC) simulation has proven to be a powerful tool in understanding the kinetic physics of various fundamental plasma processes. In fact, drift-kinetic and gyrokinetic PIC simulations have already been adopted to address the linear growth and nonlinear saturation of the tearing instability [9,10]. In these works, the particle simulation was performed in 2D slab geometry for simplicity. This means that the bootstrap current and the neoclassical polarization drift cannot be investigated. Full 3D simulations in toroidal geometry [11] have been developed but such an approach is computationally expensive and parameter scans are limited.

The purpose of this work is to analytically reduce the full toroidal system to provide a 2D computation code for simulating the physics of the NTM close to threshold, including the effects of bootstrap current and the neoclassical polarization drift. In the simulation model, the electrons will be treated analytically for simplicity, following [3]. Such an approach is valid because the electron banana width is assumed to be much less than the magnetic island width. Ions are more complicated and will be treated numerically using PIC simulation. A coordinate system is employed in which the ion distribution function is independent of poloidal angle $\theta$, thus reducing the simulation system to 2D. This paper describes this analytic reduction.

## 2. Simulation model

The ion distribution function can be decomposed into an adiabatic and a non-adiabatic part as follows,

$$f_i = \left(1 - \frac{q_i\Phi}{T_i}\right) F_M + g_i \,.$$

There are two fields associated with a magnetic island. First is the parallel component of vector potential $A_\parallel$, which provides the magnetic geometry of the island. Second is the electrostatic potential, $\Phi$, created by the differing ion and electron responses to the small magnetic island. The non-adiabatic response, $g_i$, is given by the drift-kinetic equation [3]

$$\frac{\partial g_i}{\partial t} + v_\parallel \mathbf{b}\cdot\nabla g_i + \left(\mathbf{v}_d + \frac{c\mathbf{B}\times\nabla\Phi}{B^2}\right)\cdot\nabla g_i - \frac{q_i}{m_i}(\mathbf{v}_d\cdot\nabla\Phi)\frac{\partial g_i}{\partial\varepsilon} = C(g_i)$$
$$-\frac{q_i F_M}{T_i}\left[(\omega-\omega_{*i}^T)\left(\left.\frac{\partial\Phi}{\partial\xi}\right|_\chi - \frac{v_\parallel\partial A_\parallel}{c\partial\xi}\right) - \frac{q_i\Phi}{T_i}\mathbf{v}_d\cdot\nabla\Phi\right] - \left(1-\frac{q_i\Phi}{T_i}\right)\frac{Iv_\parallel}{Rq}\frac{\partial}{\partial\vartheta}\left(\frac{v_\parallel}{\omega_{cj}}\right)\frac{F_M}{n}\frac{dn}{d\chi}\frac{\omega_{*i}^T}{\omega_{*i}}$$

Here, $\xi = m(\theta - \phi/q)$ is the helical angle, $\chi$ is the poloidal flux, $I(\chi) = RB_\phi$, $v_\parallel$ is the ion parallel velocity, $\omega_{ci}$ is the ion cyclotron frequency, $\theta$ and $\phi$ are the poloidal and toroidal angles. It is conventional to use the $(\chi, \vartheta, \xi)$ coordinate system, but this does not exploit the close proximity to toroidal symmetry when the island width is small. In fact, when the island width is comparable to the ion banana width, analytic theory indicates that to leading order the problem can be solved in a 2D spatial coordinate system $(p, \xi)$ where $p = \chi - I(\chi)v_{\|}/\omega_{ci}$. We expand $g_i = g_i^{(0)} + (w/r)\cdot g_i^{(1)} + \cdots$, where the island width $w$ is assumed to be small compared to the radius of the rational surface, $r$. In addition, we define $h_i = g_i^{(0)} + F_{Mi}$. To leading order, the drift kinetic equation provides $\left(\frac{\partial h_i}{\partial \vartheta}\right)_{p,\xi} = 0$, which means $h_i$ is independent of $\theta$ in $(p, \vartheta, \xi)$ coordinate system. Proceeding to the next order, we have

$$\begin{aligned}
&\frac{\partial h_i}{\partial t} + \frac{v_{\|}}{Rq}\left(\frac{\partial g_i^{(1)}}{\partial \vartheta}\right)_{p,\xi} + k_{\|}v_{\|}\left(\frac{\partial h_i}{\partial \xi}\right)_{\Omega,\vartheta} + (\mathbf{v}_d \cdot \nabla\vartheta)\left(\frac{\partial h_i}{\partial \vartheta}\right)_{\chi,\xi} + (\mathbf{v}_d \cdot \nabla\xi)\left(\frac{\partial h_i}{\partial \xi}\right)_{\chi,\vartheta} \\
&\quad + \frac{c(\mathbf{B}\times\nabla\Phi)}{B^2}\cdot\nabla h_i - \frac{q_i}{m_i}v_{d\chi}\frac{\partial\Phi}{\partial\chi}\frac{\partial h_i}{v\partial v} \\
&= -\frac{q_i F_{Mi}}{T_i}\left\{(\omega - \omega_{*i}^T)\left[\left(\frac{\partial\Phi}{\partial\xi}\right)_{\chi} - \frac{v_{\|}}{c}\frac{\partial A_{\|}}{\partial\xi}\right] - \frac{q_i\Phi}{T_i}v_{d\chi}\left(\frac{\partial\Phi}{\partial\chi}\right)_{\vartheta,\xi}\right\} \\
&\quad + \frac{q_i\Phi}{T_i}v_{d\chi}\frac{\partial F_{Mi}}{\partial\chi} + k_{\|}v_{\|}\left(\frac{\partial F_{Mi}}{\partial\xi}\right)_{\Omega,\vartheta} + \frac{c(\mathbf{B}\times\nabla\Phi)}{B^2}\cdot\nabla F_{Mi} - \frac{q_i}{m_i}v_{d\chi}\frac{\partial\Phi}{\partial\chi}\frac{\partial F_{Mi}}{v\partial v} + C(h_i)
\end{aligned} \tag{1}$$

It can be expressed simply in the form

$$\frac{\partial h_i}{\partial t} + \frac{v_{\|}}{Rq}\left(\frac{\partial g_i^{(1)}}{\partial\vartheta}\right)_{p,\xi} + \left\{V_p\frac{\partial}{\partial p} + V_\xi\frac{\partial}{\partial\xi} + V_\varepsilon\frac{\partial}{\partial\varepsilon}\right\}h_i = \sum_{\nu=1}^{8} S_\nu \tag{2}$$

where

$$V_p = -k_{\|}v_{\|}\frac{w_\chi^2}{4\chi}\sin\xi - (\mathbf{v}_d\cdot\nabla\vartheta)\frac{\partial}{\partial\vartheta}\left(\frac{Iv_{\|}}{\omega_{ci}}\right) + \frac{c(\mathbf{B}\times\nabla\Phi)\cdot\nabla p}{B^2} \tag{3}$$

$$V_\xi = k_{\|}v_{\|} + (\mathbf{v}_d\cdot\nabla\xi) + \frac{c(\mathbf{B}\times\nabla\Phi)\cdot\nabla\xi}{B^2} \tag{4}$$

$$V_\varepsilon = -\frac{q_i}{m_i}v_{d\chi}\frac{\partial\Phi}{\partial\chi} \tag{5}$$

Here, $\varepsilon = v^2/2$, $V_p, V_\xi, V_\varepsilon$ are the equation of motion, $S_\nu$ indicates the eight source terms in the right side of Eq.(1). $V_p, V_\xi, V_\varepsilon, S_\nu$ are functions of $(p, \vartheta, \xi)$. In eq.(2), the term involving $g_i^{(1)}$ and the $\theta$-dependence of the other terms can be annihilated by performing a $\theta$-average. For passing particles, the annihilation operator is

$$\hat{L}_u \cdots = \frac{1}{2\pi}\int_0^{2\pi} \frac{Rq}{v_\parallel}(\cdots) d\vartheta \equiv < \frac{Rq}{v_\parallel}(\cdots) >_\vartheta$$

and for trapped particles

$$\hat{L}_t \cdots = \sum_{\sigma=\pm 1} \int_{-\vartheta_T}^{\vartheta_T} \frac{Rq}{|v_\parallel|}(\cdots) d\vartheta \equiv < \frac{Rq}{|v_\parallel|} \cdots >_\vartheta \quad ,$$

where $\sigma = v_\parallel / \left| v_\parallel \right|$ and $\theta_T$ is the angle of the bounce point. These represent averages over the ion orbits, and must be performed at fixed $p$. Thus, the path of integration depends on the value of $v_\parallel$ of particles. The result is an equation for the leading order distribution function, $h_i$, that depends on only two spatial variables $p$ and $\xi$.

## 3. Future plan

Having obtained the kinetic equation and equation of motion, we are now developing the 2D PIC simulation code in $(p,\xi)$ geometry. In the first stage, we will compare our simulation results with previous analytical results [3] to provide a benchmark. Then we shall investigate the evolution of small-size magnetic islands and quantify the roles of the bootstrap current and the polarization drift in the threshold of NTMs.

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