Self-consistent Determination of Magnetic Islands Frequency in ν and 1/ν Neoclassical Viscous Regimes

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Abstract

In this work well established results of neoclassical kinetic theory are used in a fluid model to show that in low collisionality regimes (banana ν and banana-drift 1/ ν) the propagation velocity. of NTM magnetic islands of sufficient width is determined self-consistently by the neoclassical toroidal viscosity (NTV) appearing because of broken symmetry. At the same time this affects the role of the neoclassical ion polarization current on neoclassical tearing modes (NTM) stability.

I-Effects of symmetry breaking.

Although an ideal tokamak plasma equilibrium is axisymmetric, but in reality axisymmetry is broken by various helical perturbations. These may be due to external error fields with helical components, driving magnetic reconnection at rational q =m/n surfaces, or by spontaneous reconnection of tearing modes forming magnetic islands in classical or neoclassical collisional regimes. The growth and propagation speed of these dangerous perturbations is still an open problem. Here we focus attention on the fact that driven or spontaneous magnetic reconnection introduces a helical perturbation that leads to the appearance of a toroidal viscous drag that would be identically zero in axisymmetry. This allows to consider the question of the "natural propagation speed" of NTM magnetic islands from a novel point of view.

The local modulation of the magnetic field strength associated with the an (m,n) NTM island $B \approx B_0\left(1-\frac{r_s}{R_0}\cos\theta-\frac{r_s}{R_0}\frac{W}{qL_s}\cos(m\theta-n\phi)\right)$ (here $L_s = Rq_s^2/q_s' r_s$), breaks the axisymmetry causing a radial current and a toroidal braking force,i.e. $F_\phi = -J_r B_\theta$ (in coord. (r,θ,φ)) proportional to the island width. In a fluid description the magnetic field toroidal non-uniformity $\partial B/\partial\phi \neq 0$ is associated with a neoclassical toroidal viscous force (NTV)

$\mathbf{B}_\phi \nabla\cdot \underline{\underline{\Pi}}_{//} \approx -m_i n V_\phi (\partial B/\partial\phi)$, that damps the local flow [1]. Thus NTV due to breaking of axisymmetry hinders self-consistently the toroidal motion of the plasma in the island region [2]. The neoclassical parallel stress tensor in non-axisymmetric configurations is provided by kinetic theory and depends on the collisionality regimes [3,4].

II-Neoclassical model of magnetic island rotation

In a perturbed magnetic configuration described by $\mathbf{B} = R_0 B_{0\phi}\nabla\phi + \nabla\Psi_0 \times\nabla\phi + R_0\tilde{B}_{1\phi}\nabla\phi + \nabla\Psi^* \times\nabla\phi$ a nonlinear magnetic island at the rational surface $q_s \equiv \mathbf{B}\cdot\nabla\phi/\mathbf{B}\cdot\nabla\theta = m/n$ is represented by the constant helical flux contours $\Psi^*(r,\xi,t) = \left(\frac{B_0}{2L_s}\right)x^2 - \tilde{\psi}_s(t)\cos\xi(t)$ where $x = r - r_s; \xi(t) = m\theta - n\phi - \int^t \omega(t')dt'$, is the mode's phase and $\tilde{\psi}_s(t)$ is the helical flux reconnected at the rational surface. The island phase ξ(t) and width $W(t) = 4\left(L_s\tilde{\psi}_s(t)/B_0\right)^{1/2}$ evolve with nonlinear rates derived from the cosξ and sinξ moments of the parallel current [1]: $\tilde{\psi}_s\Delta'_c(W) = \frac{4}{c}\int_{-\infty}^{\infty} dx \oint d\xi J_{//}\cos\xi$ and $\tilde{\psi}_s\Delta'_s(W) = \frac{4}{c}\int_{-\infty}^{\infty} dx \oint d\xi J_{//}\sin\xi$. While the first determines the island growth rate [1] . The latter is proportional to the torque applied to the island region and requiring it to vanish is the cardinal mechanical condition to obtain the island propagation speed. In a neoclassical regime where axisymmetry is broken by sufficiently large magnetic islands, such propagation speed is determined self-consistently by the bumpy ride in the modulated confining B field. By the change of variables from (x,ξ) to (Ψ*,ξ), using the current closure condition and the parallel momentum balance the expression (3) can be written in the useful form [2]:

$$\tilde{\psi}_s\Delta'_s = \frac{4}{c}\int_{-\infty}^{\infty} dx \int d\xi J_{//}\sin\xi = \frac{4}{B^2}\frac{L_s}{k_\theta R\tilde{\psi}_s}\int_1^{\infty} d\Psi^* \int_{\xi_-}^{\xi_+} d\xi \mathbf{e}_r\cdot\nabla\times\left(\frac{f\left\langle \mathbf{B}\cdot\nabla\cdot\underline{\underline{\Pi}}_{//a}\right\rangle}{\left\langle\nabla_{//}B\right\rangle}\mathbf{B}\right) \qquad (1)$$

where <f>=O(1) and $\xi_\pm$ correspond to the island's contours turning points. It is apparent that for an arbitrary island the torque vanishes if the parallel neoclassical viscous stress vanishes [3]. The neoclassical viscous force (from the divergence of the stress in CGL form) generalized to non-axisymmetric cases, is linearly related to the poloidal particles and heat flows by the general viscous matrix obtained in Ref .[5] for various collisionality regimes labeled by λ:ν,1/ν,P (banana, banana-drift, plateau):

$$\begin{pmatrix} \langle \mathbf{B}\cdot\nabla\cdot\underline{\underline{\Pi}}_\alpha \rangle \\ \langle \mathbf{B}\cdot\nabla\cdot\underline{\underline{\Theta}}_\alpha \rangle \end{pmatrix} = \langle B^2 \rangle \underline{\underline{M}}^\lambda_a \cdot \begin{pmatrix} u_{\theta,\alpha} \\ -(2/5p_a)q_{\theta,\alpha} \end{pmatrix} \; ; \; \begin{matrix} u_{\theta,\alpha} \equiv \langle \mathbf{V}_\alpha\cdot\nabla\theta \rangle / \langle \mathbf{B}\cdot\nabla B \rangle \\ q_{\theta,\alpha} \equiv \langle \mathbf{q}_\alpha\cdot\nabla\theta \rangle / \langle \mathbf{B}\cdot\nabla B \rangle \end{matrix} \qquad (2)$$

The condition for vanishing torque on an island is $\Delta'_s = 0$ and from eq.3 this is equivalent to the condition $M_1^\lambda u_{\theta i} + M_2^\lambda \frac{2q_{\theta i}}{5p} = 0$. To determine the (ion) poloidal flow velocity the corresponding heat flow can be provided by the friction-flow relation [3] $\langle \mathbf{B}\cdot\nabla\cdot\underline{\underline{\Theta}}_i \rangle = \langle \mathbf{B}\cdot\mathbf{F}_{2,i} \rangle$

$$u_{\theta i} \cong -\frac{M_{2i}^\lambda (l_{22}^i/n_i m_i)}{\left| M_{1i}^\lambda M_{3i}^\lambda - \left(M_{2i}^\lambda\right)^2 \right| + M_{2i}^\lambda (l_{22}^i/n_i m_i)} \frac{\omega_{*Ti} R}{B} \quad \text{with} \quad \omega_{*Ti} = \left(\frac{k_\theta}{B}\frac{cT_i}{Ze}\right)\frac{T_i'}{T_i} \qquad (3)$$

The radial force balance equation relates the flows within a surface *f* e to gradients of flux surface quantities (the thermodynamic driving forces): $u_{\theta i} \cong \frac{\langle Bu_{//} \rangle}{\langle B^2 \rangle} + \frac{R}{B}\left[\frac{c\Phi'}{B} + \frac{cp_i'}{eZm_i B}\right]$ (4)

In steady state, the electrostatic component of the parallel electric field around the magnetic island is almost balanced by the inductive electric field, and this introduces a dependence of the electrostatic potential with the island phase velocity

$$0 = E_{//} = -c^{-1}\frac{\partial \Psi}{\partial t} - \nabla_{//}\Phi, \quad \Rightarrow \Phi = B_0\left(ck_\theta\right)^{-1}\left(\omega - \omega_E\right)\left[x - \lambda(\Psi)\right]$$

where λ(Ψ) is profile function asymptotically ~ x. From the expressions. (3,4) an equation for the island frequency can be obtained for all collisional regimes spanned by a parameter s 0<s<1/n. and s>1: $\omega - \omega_E = -\omega_{pi} - \frac{M_{2i}^\lambda (l_{22}^i/n_i m_i)\omega_{*Ti}}{\left| M_{1i}^\lambda M_{3i}^\lambda - \left(M_{2i}^\lambda\right)^2 \right| + M_{2i}^\lambda (l_{22}^i/n_i m_i)}\omega_{*Ti} + \frac{k_\theta \langle Bu_{//} \rangle}{B}$ (5)

where the coefficients are those of Refs.[4,5]. It is interesting to consider In absence of a bulk plasma parallel velocity it is interesting to consider the variation of the island frequency with the collisionality parameter. The coefficients of eq.5 can be represented as lowest order Pade' approximants in a collisionality parameter $s = \omega_E/(\nu_i/\varepsilon)$ .The transition *equation* for the island normalized frequency $\overline{\omega}_N = (\omega - \omega_E)/\omega_E$ is then:

$$\overline{\omega}_N \omega_E = -\left(\omega_{*pi} + g_i(\overline{\omega}_N, s)\omega_{*Ti}\right)\left(1 + s^2\right)\left[1 + \frac{\left(\omega_{*pi} + g_i(\overline{\omega}_N, s)\omega_{*Ti}\right)}{\left(\omega_{*pe} + g_e(\overline{\omega}_N, s)\omega_{*Te}\right)}s^2\right]^{-1} \qquad (6)$$

with

$$g_i(\varpi_N,s)=\left(\frac{-0.177}{0.7}\right)\frac{\varpi_N^2 s^2+(0.7/13.708)G}{\varpi_N^2 s^2+(-0.177/32.444)G},$$

$$g_e(\varpi_N,s)=\left(\frac{-0.677}{1.707}\right)\frac{\varpi_N^2 s^2+(1.707/6.39)G}{\varpi_N^2 s^2+(-0.677/15.592)G}, \qquad G\propto (w/r_s)^2$$

The example of Fig.1 for ITER-like parameters show that the island frequency (full thick line) changes from $\omega_{*pe}+g_e\omega_{*Te}$ (dash–dot line) to $\omega_{*pi}+g_i\omega_{*Ti}$ (dashed line) as s spans the collisional regimes. The thin dashed and dotted lines are ω_{*pi} (=-ω_{*pe})and ω_{*Ti} .As s increases the (neoclassical) ion polarization current changes sign as shown by the thick line of Fig.2, losing its stabilising role, and eventually vanishes as the natural island speed (dotted line) changes from the banana (ν) regime to the banana drift (1/ν) regime (thin line). This behaviour in different collisionality regime may be associated with trigger less onset of NTMs [6].

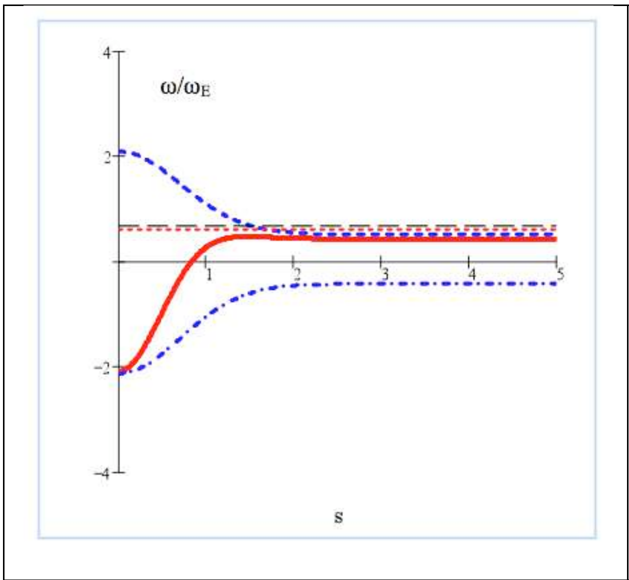

Fig.1

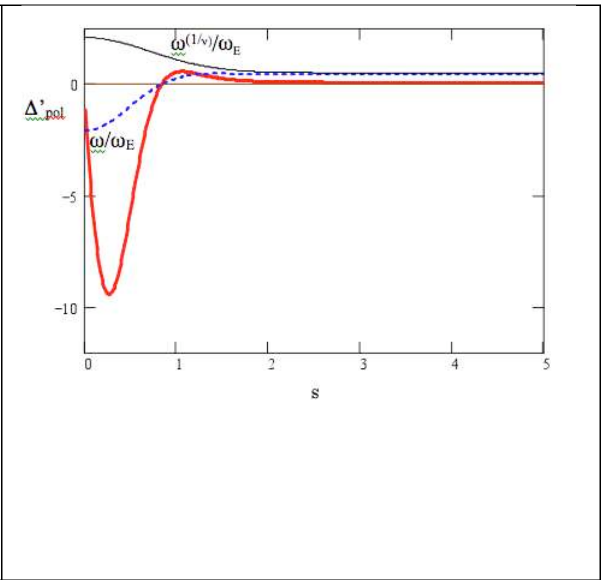

Fig.2

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