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STATISTICAL MODELING OF THE ERROR IN THE DETERMINATION OF THE ELECTRON TEMPERATURE IN JET BY A NOVEL THOMSON SCATTERING LIDAR APPROACH

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**See the Appendix of F. Romanelli et al., Proceedings of the 22nd IAEA Fusion Energy Conference, Geneva, Switzerland, 2008*

1. Introduction. Profiles of the plasma electron temperature T_e and density n_e are routinely determined on JET with the LIDAR Thomson scattering (TS) system[1,2]. Although the system is robust and produces data for all plasma discharges independent of plasma parameters (e.g. field, current, density) the signal to noise performance is below modern expectations. The approach used so far for T_e and n_e measurement is based on log-linear or non-linear fit of the experimentally-obtained, relativistically-thermally-broadened lidar-return spectra to the corresponding theoretical expression. Recently we developed two novel complimentary approaches for determination of T_e on the basis of an analysis of the relativistic TS spectrum and estimated analytically their potential accuracies[3]. The methods are based on the unambiguous temperature dependence, respectively, of the "center-of-mass wavelength" (CMW) of the LIDAR-return spectrum and of the ratio of the signal powers of two spectral regions. The main purpose of the present work is to perform thorough simulations of the determination of electron temperature by the CMW method and to estimate accurately the measurement error as a function of T_e and the signal-to-noise ratio (SNR).

2. Theoretical background. The simulations performed are based on reproducing statistically realizations of the TS LIDAR return signal and the parasitic background plasma light. If we consider M receiving spectral channels with wavelength intervals $[\lambda_{s1k},\lambda_{s2k}]$ (k=1,..., M) (see Fig.1), the mean return signal in each of them, in number of photoelectrons per second, can be written as

$$N_k[\lambda_{s1k},\lambda_{s2k},R(t)] = \frac{N_0 c}{2}\Delta\Omega[R(t)]r_0^2 n_e[R(t)]\int_{\lambda_{s1k}}^{\lambda_{s2k}} K_n(\lambda_i,\lambda_s)\eta(\lambda_s,R(t))\beta[\lambda_i,\lambda_s,v_{th}(R(t))]d\lambda_s\ , \quad (1)$$

λ_i and λ_s are the wavelengths of the emitted and the backscattered radiation, $R(t)$ is the line of sight (LOS) position of the scattering volume corresponding to the moment t after the pulse emission, N_0 is number of photons in the sensing laser pulse, $\Delta\Omega[(R(t)]$ is the solid angle of collection, $v_{th}(R)=[2k_B T_e(R)/m_e]^{1/2}$ is the rms thermal velocity of the electrons, m_e is the electron

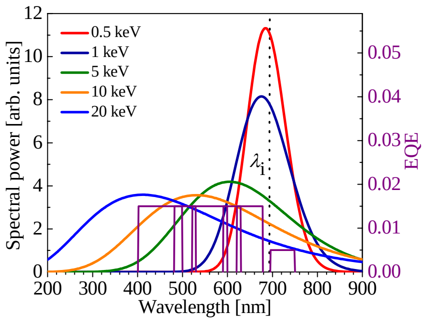

Fig.1. Thomson backscattering spectrum at λ_i=694 nm. The receiving spectral intervals used in the simulations are also shown.

rest mass, r_0 is the classical electron radius, $K_n(\lambda_i,\lambda_s)=K_t(\lambda_i)K_t(\lambda_s)K_f(\lambda_s)EQE(\lambda_s)\lambda_s/\lambda_i$, $K_t(\lambda_i)$ and $K_t(\lambda_s)$ are the optical transmittances of the irradiating and collecting paths, $K_f(\lambda_s)$ is the receiver filter spectral characteristic, $EQE(\lambda_s)$ is the effective quantum efficiency of photon detection, $\eta(\lambda_s,R)$ is the receiving efficiency including in general the vignetting effect,

$$\beta[\lambda_i,\lambda_s,v_{th}(R)]=\frac{c}{\sqrt{\pi}\lambda_i v_{th}(R)}\left(1+\frac{15}{16}\frac{v_{th}^2(R)}{c^2}+\frac{105}{512}\frac{v_{th}^4(R)}{c^4}\right)^{-1}\left[\frac{(\lambda_i/\lambda_s)^4}{(1+\lambda_i/\lambda_s)}\right]\exp\left[-q(R)+\frac{2c^2}{v_{th}^2(R)}\right][1+\delta(q)]^{[4]},$$

$\delta(q)=2e^q[E_3(q)-3E_5(q)]$, $q(R)=m_ec^2(\sqrt{\lambda_s/\lambda_i}+\sqrt{\lambda_i/\lambda_s})/[2k_BT_e(R)]$, and $E_n(q)$ is the exponential integral of the n-th order. The parasitic background due to plasma light penetrating into the k-th receiving spectral channel is characterized by the following photoelectron rate[3]:

$$N_{\rm bk}(\lambda_{s1k},\lambda_{s2k})=6.25\times10^{-21}E\int_R dRn_e^2(R)\left[k_BT_e(R)\right]^{-1/2}\int_{\lambda_{s1k}}^{\lambda_{s2k}}d\lambda_s K_t(\lambda_s)K_f(\lambda_s)EQE(\lambda_s)\lambda_s^{-1}\ln\left[k_BT_e(z)/(13.6h^2c^2/\lambda_s^2)^{1/3}\right], \quad (2)$$

where E is the detector's etendue and the quantities k_BT_e and hc/λ_s are in eV. The center-of-mass wavelength λ_{CM} is defined as

$$\lambda_{CM}=\left(\sum_k\lambda_kN_k\right)\Big/\left(\sum_kN_k\right)=f(T_e)\ , \quad (3)$$

where $\lambda_k=(\lambda_{s1k}+\lambda_{s2k})/2$. The unambiguous dependence of λ_{CM} on the electron temperature (see also Fig.1) is the essence of the method. The linear error propagation approach leads to the following expression of the rms error δT_e in the determination of T_e on the basis of the dependence $\lambda_{CM}=f(T_e)$:

$$\delta T_e=\left|d\ln f(T_e)/dT_e\right|^{-1}\left(\sum_kN_k\tau_d\right)^{-1}\left\{\sum_{k=1}\left(\frac{\lambda_k-\lambda_{CM}}{\lambda_{CM}}\right)^2N_k\tau_d(1+N_{bk}/N_k)\right\}^{1/2}. \quad (4)$$

Here τ_d is the system response time.

3. Simulations and discussion. The fusion plasma is supposed to occupy the LOS region between R=2 m and R=4 m. The sensing laser emits 300 ps long pulses of energy E_0=0.6 J at λ_i=694 nm. The solid angle of collection is from 0.005 sr, at R=2 m, to 0.007 sr at R=4 m. The detectors have diameters of 18 mm, the F# of detection is 1.5 and the corresponding value of detector's etendue is ~0.9 cm^{2}sr. EQE are 0.005 for channel 1 and 0.015 for the other channels (see Fig.1). The irradiating and collecting paths transmittances are 0.75 and 0.25, respectively. The vignetting factor is estimated to be 0.33. The accepted values of τ_d and the sampling interval are 800 ps and 250 ps, respectively. The described characteristic parameters are chosen to be close to those of the JET core TS LIDAR system.

The radial (LOS) distributions of T_e and n_e are modeled as parabolic ones. First, the reference function $\lambda_{CM}(T_e)$ is determined on the basis of the temperature dependence of the TS spectrum. The dependence of the relative rms error $\delta_r T_e(T_e)=\delta T_e(T_e)/T_e$ on the plasma electron temperature is analyzed on the basis of the results obtained at a fixed LOS position in the plasma. In this case T_e is varied with a step ΔT_e=0.1 keV. At known mean values $N_k(T_e)\tau_d$ and $N_{brk}(T_e)\tau_d$, by Poisson random-number generator we produce H realizations $\hat{N}_k^l(T_e)\tau_d$ and $\hat{N}_{brk}^l(T_e)\tau_d$ of the TS signal and the background photon counts, l=1…H. Afterwards we compose the quantities $\hat{N}_k^l(T_e)\tau_d+\hat{N}_{brk}^l(T_e)\tau_d-N_{brk}(T_e)\tau_d$ to use them, together with the reference function $\lambda_{CM}(T_e)$, for obtaining H estimates $\hat{T}_e^l$ of T_e. Then, an estimate $\hat{\delta T}_e$ of the measurement error is obtainable as $\hat{\delta T}_e=\left[H^{-1}\sum_{l=1}^{H}(\hat{T}_e^l-T_e)^2\right]^{1/2}$. Monte-Carlo estimates of $\delta_r T_e$, at different values of n_e (in fact, at different SNR), are compared in Fig.2 with the theoretical estimates obtained by Eq.(4) at R=3m. The error behavior is obviously consistent with that predicted analytically. Certainly, for lower values of n_e we obtain higher values of $\delta_r T_e$. In Fig.2 we have taken into account that the values of n_e=1x10^{19} m^{-3} are typical of the pedestal region where T_e is normally low. In Fig.3 the recovered temperature values from different realizations for T_e=5 keV are shown.

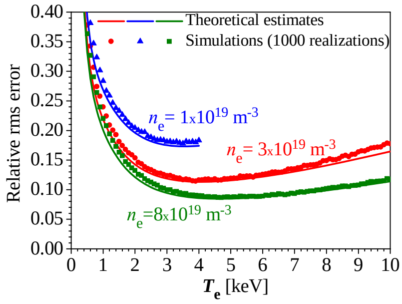

Fig.2. Comparison of the theoretically estimated relative rms errors and the results from the simulations.

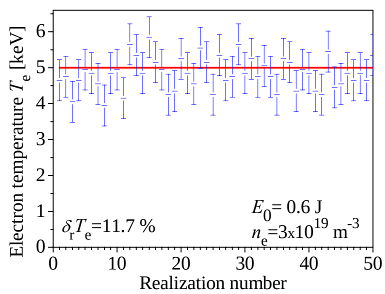

Fig.3. Recovered T_e values in the case of one temperature Monte-Carlo simulations.

In Fig.4 we present one restored LOS profile of T_e (at n_e=3x10^{19} m^{-3}) compared with the model distribution shown by the middle curve. In this case the fluctuating LOS profiles of the TS signal and plasma light background for each of the spectral channels are determined as above. The upper and lower curves denote the standard deviation limits in obtaining T_e. As seen, the retrieved points fit well in this interval.

4. JET data processing. The theoretical estimates of the rms error show that the CMW approach has a comparable accuracy with the fitting approach[3]. To prove this, we applied the

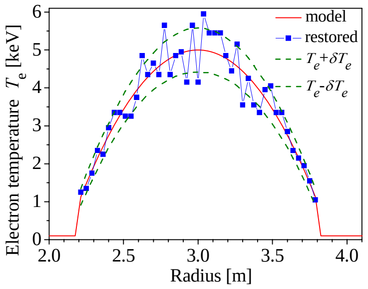

Fig.4. Input and restored electron temperature profiles.

CMW method to the same JET core LIDAR data as used in the intershot analysis code for determination of T_e, taking into account the real characteristics of the TS LIDAR system. The results obtained (Fig.5b) are compared with that determined on JET using the fitting procedure (Fig.5a). The shapes and magnitudes of both T_e profiles are similar and the reasons of some discrepancy should be further clarified.

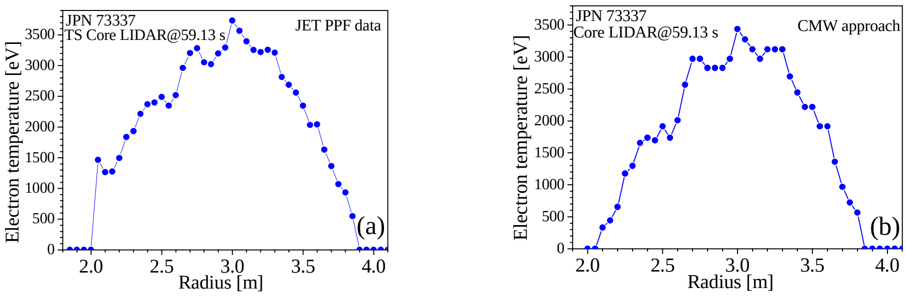

Fig.5. Electron temperature profiles determined using the fitting approach on JET (a) and CMW approach (b).

5. Conclusion. The reported simulations provide the authentic error in the determination of T_e in fusion plasma by the novel CMW TS LIDAR approach, depending on the temperature itself and SNR. The results obtained are consistent with the theoretical estimates and thus confirm the conclusion[3] that this approach and the fitting one have comparable efficiencies and can be used together for mutually validating the results obtained for T_e and for distinguishing real inhomogeneities in the recovered T_e profiles from spurious ones due to statistical fluctuations. This conclusion is also in agreement with the performed JET data processing using both the CMW and the fitting approaches. Because of some practical advantages, such as simple, clear and stable measurement procedures, without any additional considerations about the weight and the variance of the experimental data and the goodness of the fit, the new CMW approach is suitable for accurate determination of T_e profiles in tokamak plasma[5].

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