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A COMPARISON METHOD BETWEEN FLUID MODEL SOLUTIONS AND PIC PLASMA SIMULATIONS

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1 INTRODUCTION

Theoretical models developed for magnetized plasma boundary and are often validated using computer simulations. One problem that can occur is the comparison of models dimensionless quantities, and the quantities yielded from the simulations. We made an attempt at solving this problem introducing an iterative method for normalizing analytical model results. In the following chapter we describe our simple fluid model and simulation principles. In the third chapter we present our comparison method and in the final chapter we draw some conclusions.

2 MODEL AND SIMULATIONS

In our model we considered a typical boundary layer problem with a quasineutral plasma shielded from a negative absorbing wall by a thin positive space charge layer. This layer characteristically spreads over a few Debye lengths λ_D and is much shorter than the usual extension L of the boundary layer disturbed by the presence of the limiting wall, which is usually called pre-sheath.

This is a one-dimensional, collisionless fluid model. Beside the normal electrostatic sheath mechanism a magnetic field is also applied, which results in the formation of a magnetized pre-sheath. The magnetic field can be applied at various angles and with various densities. Plasma is populated with singly charged ions and Boltzmann electrons. A similar model, but with collisions included, was also used by Zimmermann et al. [1]. The geometry of the model is shown in Fig.1. A

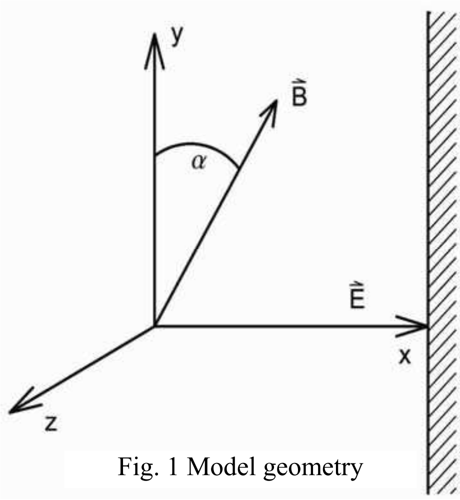

Fig. 1 Model geometry

large planar wall is set in the ***y-z*** plane with the positive ***x***-coordinate directed towards the wall. We consider isothermal ion flow $(\gamma = 1)$, therefore the Bohm criterion [2] stands:

$$c_s = \sqrt{\frac{k_B(T_i + T_e)}{m_i}}. \tag{1}$$

The equations for ion momentum and continuity are:

$$\begin{aligned} &m_i(\mathbf{v}\cdot\nabla)\mathbf{v} = e_0(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \tfrac{1}{n_i}\nabla p_i - m_i\mathbf{v}\nu_t \\ &\nabla\cdot(n_i\mathbf{v}) = n_e\nu_i, \end{aligned} \tag{2}$$

with ν_i being the ionisation rate and ν_t the total collision rate. From (2) by assuming presheath quasineutrality and using ion pressure and density equations we can derive the following dimensionless momentum equations by components and a dimensionless potential equation:

$$\begin{aligned} &\left(V_x - \tfrac{1}{V_x}\right)\tfrac{\partial V_x}{\partial X} = -1 - \omega K\cos(\alpha)V_z, \\ &V_x\tfrac{\partial V_y}{\partial X} = \omega K\sin(\alpha)V_z, \\ &V_x\tfrac{\partial V_z}{\partial X} = \omega K\left(\cos(\alpha)V_x - \sin(\alpha)V_y\right), \\ &V_x\tfrac{\partial \Phi}{\partial X} = -\tfrac{\partial V_x}{\partial X}. \end{aligned} \tag{3}$$

Here we used the following dimensionless variables:

$$\begin{aligned} &\mathbf{V} = \tfrac{\mathbf{v}}{c_s},\ X = \tfrac{x}{L},\ \Phi = -\tfrac{e_0 U}{k_B T_e}, \\ &K = \tfrac{L}{c_s},\ \omega K = \tfrac{e_0 B}{m_i}\tfrac{L}{c_s} = \tfrac{L}{r_{Li}}. \end{aligned} \tag{4}$$

We now have a system of 4 equations with 4 unknown functions of X with two independent parameters: magnetic field density, ωK and the angle at which it is applied, α. We can see that ωK gives us the number of Larmor radii that fit into the pre-sheath length scale ***L***. More details about the model can be found in [3]. The system of equations is integrated numerically to produce spatial profiles of functions. We reach sheath edge when V_x reaches unity, thus not being able to study profiles beyond this point. The geometry of the model is suitable for particle-in-cell simulation code. We have used the one-dimensional BIT1 code [4]. The length of the simulated system was 9***cm*** divided into 24000 cells with cold ions and the electrons being injected at $k_B T_e = 1eV$. The time step used was $5\cdot10^{-12}s$. We can obtain a number of quantities from the simulation, most importantly ion velocity perpendicular to the wall $v_x(x)$, ion velocity parallel to the magnetic field $v_{par}(x)$, potential profile $\phi(x)$ and density profiles $n_i(x)$ and $n_e(x)$.

3 RESULTS

The presented model was used to study the magnetic pre-sheath in [3]. We wanted to verify the model through the PIC simulation, but a problem occurred as we did not know the length scale $\boldsymbol{L}$ that corresponded to the simulation parameters. We improved the method that we had already used in [3] to determine the correct length scale of the system by also taking in account the length of the sheath. We proceeded in the following manner. Firstly, we determined the position of the edge of the sheath from the computer simulation. We did this using a Bohm criterion equation in the following form (5):

$$\frac{\mathrm{d}\boldsymbol{n}_i}{\mathrm{d}\phi} - \frac{\mathrm{d}\boldsymbol{n}_e}{\mathrm{d}\phi} \geq 0 \qquad (5)$$

The meaning of this equation is shown in Fig 2. It shows the difference of the numerically differentiated density profiles on the potential scale. The following case was made for a magnetic field being applied at $\alpha = 50°$ and its density being $\boldsymbol{B} = 0.05\boldsymbol{T}$. The sheath entrance is at approximately $\phi = 3.0\boldsymbol{V}$ or $\boldsymbol{x} = 0.04285\boldsymbol{m}$, where the difference is no longer zero. We obtained ion sound speed $\boldsymbol{c}_s$ from the spatial profile of $\boldsymbol{v}_x(\boldsymbol{x})$, $\boldsymbol{c}_s \cong 6450\boldsymbol{m}/\boldsymbol{s}$. Ion velocity parallel to the magnetic field on the sheath edge is $\boldsymbol{v}_{par}\big|_{\phi=3.0V} \cong 8100\boldsymbol{m}/\boldsymbol{s}$. Since we were making a comparison to a fluid model we could now discard all the values beyond the sheath entrance point. Both velocity profiles could now be normalised to the value of the $\boldsymbol{c}_s \cong 6450\boldsymbol{m}/\boldsymbol{s}$. Next step was to find the correct length scale $\boldsymbol{L}$ of the model and consequently the parameter $\omega\boldsymbol{K}$ that corresponds to the value of the magnetic field density used in the computer simulation. We did that using the following iterative method. We selected an initial value for $\omega\boldsymbol{K}$ $(\omega\boldsymbol{K} = 1)$ and solved the system (3) . $\boldsymbol{V}_x$ reaches unity in $\boldsymbol{X} = 0.6391$. We put this value into equation (4) and got a new approximation for $\omega\boldsymbol{K} = 52.2298$. This value was again used in solving the system (3), obtaining a new value for $\boldsymbol{X} = 0.5386$. The procedure converges rather quickly and after a few steps yields the following results: $\omega\boldsymbol{K} \cong 62$ and $\boldsymbol{X} = 0.5362$ or

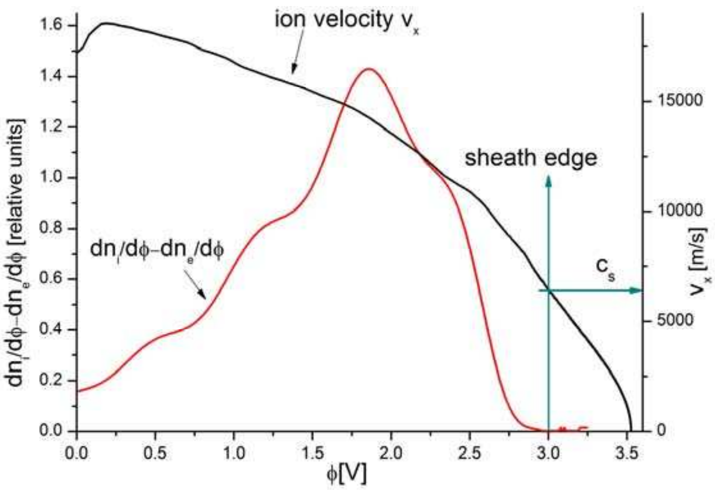

Fig. 2 Determining of the sheath edge and c_s

$\boldsymbol{L} \approx 0.08\boldsymbol{m}$ for the simulation parameters value: $\boldsymbol{m_i} = 1.67 \cdot 10^{-27} \boldsymbol{kg}$, $\boldsymbol{x} = 0.04285\boldsymbol{m}$, $\boldsymbol{B} = 0.05\boldsymbol{T}$ and $\boldsymbol{c_s} = 6450\boldsymbol{m}/\boldsymbol{s}$. We could now obtain the profiles from the equation system (3) and compare the results of the analytical model to the results from the simulation (Fig, 3). We found the results of this method to be qualitatively acceptable, with an important improvement considering the previous try [3]. The model is now more in compliance with the simulation results closer to the bulk plasma, while the discrepancy closer to the sheath edge is understandable with this being a relatively simple fluid model.

4 CONCLUSIONS

We presented an analytical fluid model that was used for studying the collisionless magnetized pre-sheath. A model operates with dimensionless quantities therefore we found it difficult to validate the model through comparison to a computer simulation. A method was developed to allow us the comparison. We found the results to be qualitatively very good, while the model lacks preciseness in the proximity of the sheath edge.

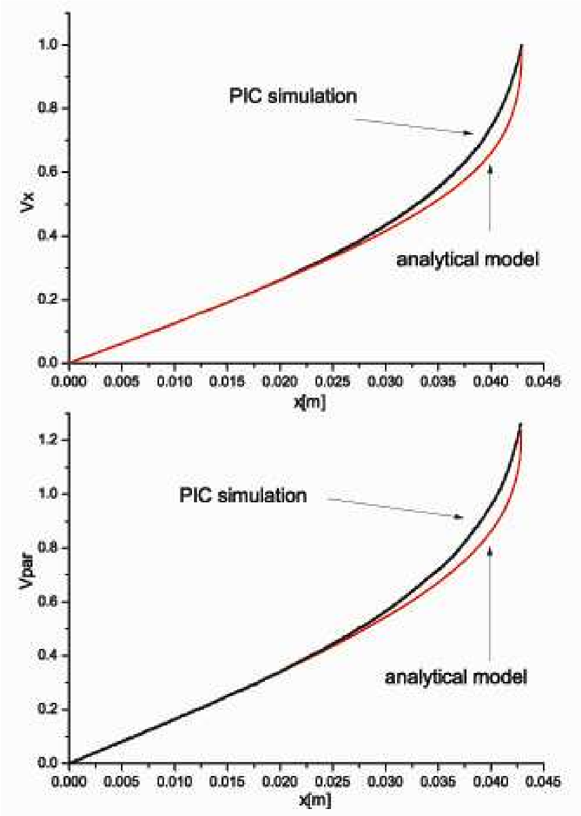

Fig. 3 Comparison of the analytical model results and the PIC simulation results

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