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Fractal Properties of Plasma Discharge Current Fluctuations

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In nature, two different types of multifractality in time series exist: (1) Multifractality due to a broad probability density function for the values of the time series. In this case the multifractality cannot be removed by shuffling the series. (2) Multifractality due to different long-range (time) correlations of fluctuations that surrogate of time series (to make it Gaussian) can not affect the multifractality. If both kinds of multifractality are present, the shuffled series will show weaker multifractality than the original series. We have studied the discharge current fluctuations (Figure 1) by the multifractal detrended fluctuation analysis (MF-DFA) and Fourierdetrended fluctuation analysis (F-DFA) methods. Using the method proposed in [1, 2].

Experimental data are often affected by non-stationarities, such as trends (which must be welldistinguished from the intrinsic fluctuations of the series) in order to determine their correct scaling behaviour. The MF-DFA is a method for determining the scaling behaviour of noisy data in the presence of trends [3, 4]. The MF-DFA method follows four steps as below[5, 6]:

1-Computing the profile of underlying data series:

$$Y(i) \equiv \sum_{k=1}^{i} \left[x_k - \langle x \rangle\right], \qquad i = 1, \ldots, N \tag{1}$$

2- Divide the profile $Y(i)$ into $N_s \equiv \mathrm{int}(N/s)$ nonoverlapping segments of equal lengths s, and then computing the fluctuation function for each segment.

$$F^2(s,\nu) \equiv \frac{1}{s}\sum_{i=1}^{s} \left\{Y[N-(\nu-N_s)s+i] - y_\nu(i)\right\}^2 \tag{2}$$

where $y_\nu(i)$ is a fitting polynomial in segmentν. Usually, a linear function is selected for fitting the function. If there do not exist any trends in the data, a zerothorder fitting function might be enough [3, 4].

3-Averaging the local fluctuation function over all the part, given by

$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} \left[F^2(s,\nu)\right]^{q/2} \right\}^{1/q} \tag{3}$$

The scaling behaviour of the fluctuation functions by analyzing log-log plots of,

$$F_q(s) \sim s^{h(q)} \tag{4}$$

The classical multifractal scaling exponents $\tau(q)$, defined by the standard partition function based formalism, discussed in literature [6,7], and the generalized multifractal dimensions $D(q)$ are related to the generalized hurst exponent via the MF-DFA as below:

$$\tau(q) = qh(q) - 1 \tag{5}$$

$$D(q) \equiv \frac{\tau(q)}{q-1} = \frac{qh(q)-1}{q-1} \tag{6}$$

Singularity spectrum $f(\alpha)$ and α is related to $h(q)$ such as: $f(\alpha) = q[\alpha h(q)] + 1$ In order to study the stochastic nature of the discharge current fluctuations, we constructed an experimental setup the same as lower panel of Figure 1. The discharge glass tube has two copper ends, 80 mm in diameter and 110 cm in length. One end is the anode electrode (a flat copper plate as a positive pole), while the other end is the cathode (tungsten filament as a negative pole and electron propagator). The discharge tube is evacuated to a base pressure of 0.1 up to 0.8 torr under a voltage of 400 - 900V and filled with Helium as the working gas. The pressure, voltage and current should be optimal for ensuring the stability of the plasma. The discharge current fluctuations were monitored using a resistor which was connected to an operational amplifier impedance converter. It has been studied in fix pressure that how the statistical properties of plasma changing under variation of the current, namely, 50, 60, 100, 120, 140, 180, and 210 mA.

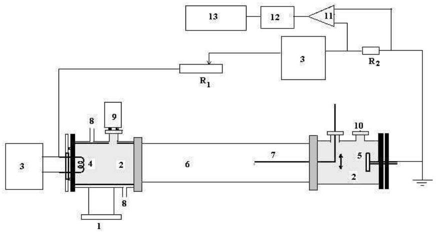

Figure 1: Sketch of experimental setup to record discharge current fluctuations in the tube filling by Helium gas. The scheme of the measurement setup and the experimental discharge tube, 1) to vacuum pumps, 2) copper cylinder , 3) power supply 4) hot cathode, 5) anode plate , 6) glass tube, 7) single Langmuir probe, 8) water cooling, 9) Pirani pressure gage, 10) gas inlet, 11) $OP-Amp$, 12) low pass filter, 13) A/D card and PC.

The result is shown in Figure 2(left panel). The generalized Hurst exponent, the classical multifractal scaling exponents and the singularity spectrum for the data after the elimination of sinusoidal trends are illustrated in Figures 2(middle and right panel respectively). The clean data is a multifractal process, as indicated by the strong q-dependence of the generalized Hurst exponents [9].

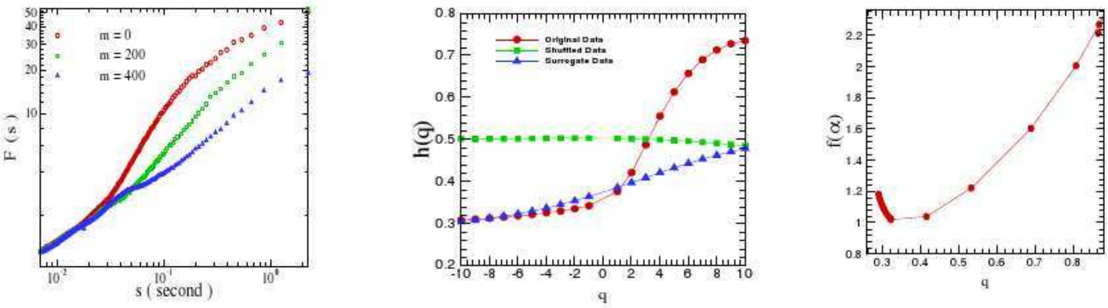

Figure 2: Left-Crossover behaviour of log-log plot $F(s)$ versus s for original time series for $q = 2.0$. There are some crossover time scales in plot of $F(s)$. middle-generalized Hurst exponent versus q for original, surrogate and shuffled series without sinusoidal trend. Right- correspond to classical multifractal scaling exponent as a function of q for current 50 mA without sinusoidal trend.

sample	γ	β
50mA	1.16 ±0.02	-0.16 ±0.02
60mA	1.10 ±0.02	-0.10 ±0.02
100mA	1.26 ±0.02	-0.26 ±0.02
120mA	1.24 ±0.02	-0.24 ±0.02
140mA	1.18 ±0.02	-0.18 ±0.02
160mA	0.82 ±0.02	0.18 ±0.02
180mA	0.10 ±0.02	-0.10 ±0.02
210mA	1.04 ±0.02	-0.04 ±0.02

Table 2- The values of correlation and power spectrum exponents for original data set in different electrical currents obtained by MF-DFA.

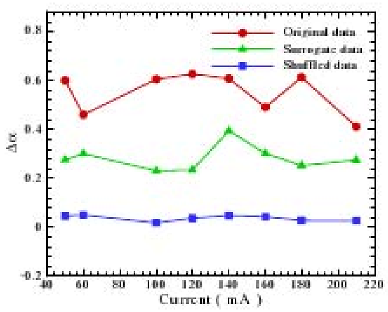

Figure 3: The width of singularity strength, $\Delta\alpha$, for original, surrogate and shuffled of data set in various electrical discharge currents.

sample		H(q=2)	τ
	Original	0.42 ± 0.01	-0.16 ±0.02
50mA	Surrogate	0.40 ± 0.01	-0.21 ±0.02
	Shuffled	0.50 ± 0.01	-0.004 ±0.02
	Original	0.45 ± 0.01	-0.10 ±0.02
60 mA	Surrogate	0.42 ± 0.01	-0.16 ±0.02
	Shuffled	0.50 ± 0.01	0.002 ±0.02
	Original	0.37 ± 0.01	-0.25 ±0.02
100 mA	Surrogate	0.36 ± 0.01	-0.28 ±0.02
	Shuffled	0.49 ± 0.01	0.00 ±0.02
	Original	0.38 ± 0.01	-0.23 ±0.02
120 mA	Surrogate	0.36 ± 0.01	-0.28 ±0.02
	Shuffled	0.50 ± 0.01	0.00 ±0.02
	Original	0.41 ± 0.01	-0.17 ±0.02
140 mA	Surrogate	0.40 ± 0.01	-0.21 ±0.02
	Shuffled	0.00 ±0.02	0.00 ±0.02
	Original	-0.09 ±0.02	-0.09 ±0.02
180 mA	Surrogate	-0.20 ±0.02	-0.20 ±0.02
	Shuffled	-0.003 ±0.02	-0.003 ±0.02
	Original	-0.04 ±0.02	-0.04 ±0.02
210 mA	Surrogate	-0.13 ±0.02	-0.13 ±0.02
	Shuffled	0.01 ±0.02	0.01 ±0.02

Table1- The values of H=h(q=2) and classical multifractal scaling exponents for q=2.0 for original, surrogate and Shuffled of data set in different electrical currents obtained by MF-DFA.

The easiest way to distinguish the type of multifractality, is by analyzing the corresponding shuffled and surrogate time series. If the multifractality only belongs to the long range correlation, we should find $h_{\mathrm{shuf}}(q) = 0.5$. Thus the corresponding shuffled time series exhibit monofractal scaling, since all long-range correlations are destroyed by the shuffling procedure. We have shown that multifractality due to the correlation has more contribution than the broadness of the probability density function. This finding also has been confirmed by width of singularity strength, $\Delta\alpha$, shown in Figure 3.

References

[1] K. Hu, P. Ch. Ivanov, Z. Chen, P. Carpena and H. E. Stanley, Phys. Rev. E **64**, 011114(2001).

[2] Zhi Chen, Plamen Ch. Ivanov, Kun Hu, H. Eugene Stanley,Phys. Rev. E **65**, 041107 (2002).

[3] C. K. Peng, S. V. Buldyrev, S. Havlin, M. Simons, H. E. Stanley and A. L. Goldberger,Phys. Rev. E **49**, 1685 (1994).

[4] S. M. Ossadnik, S. B. Buldyrev, A. L. Goldberger, S. Havlin, R. N. Mantegna, C. K. Peng, M. Simons and H. E. Stanley, Biophys. J.**67**, 64(1994).

[5] M. S. Taqqu, V. Teverovsky and W. Willinger, Fractals **3**, 785(1995).

[6] J. Feder, Fractals (Plenum Press, New York)(1988).

[7] A. Eke, P. Herman, L. Kocsis and l. R. Kozak, Physiol. Meas. **23**, R1-R38(2002).

[8] M. Sadegh Movahed and Evalds Hermanis, Physica A, **387**,915(2008).

[9] J. W. Cooley and J.W. Tukey , An algorithm for the machine computation of complex Fourier series, Mathematics of Computation, **19**, 297(1965).

[10] J. W. Kantelhardt, S. A. Zschiegner, E. Koscielny-Bunde, A. Bunde, S. Havlin and H. E. Stanley, Physica A **316**, 87 (2002).