

ANALYTICAL EXPRESSIONS FOR RADIATIVE PROPERTIES OF LOW Z PLASMAS

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1. Introduction.

The accurate computation of radiative opacities is needed in several research fields such as astrophysics, magnetic fusion or ICF target physics analysis, in which the radiation transport is an important feature to determine in detail. When simulating ICF targets, radiation hydrodynamics codes need the use of thousands of spectrally resolved opacity points for each temperature and density mesh point and to determine them takes a large calculation time. A usual approach consists of weighting these opacities in a small number of multigroups opacities or even only one group, using in this case, the Rosseland and Planck mean opacities. Even in this case, the accurate computation of these quantities is too long to be used in-line with the hydrodynamic code. For this reason, analytical formulas for giving mean radiative properties versus temperature and density of the plasma seem to be a useful tool. In this path, our main goal is to obtain analytical expressions for the Rosseland and Planck mean radiative opacities of several low Z plasmas in a wide range of temperature and densities. These formulas are obtained fitting the analytical expression proposed to mean opacities data computed by using the code ABAKO/ RAPCAL [1]. This code compute the radiative properties of plasmas such as the spectrally resolved and mean opacities and emissivities and the intensity, both in LTE and NLTE conditions, under the detailed-level-accounting approach. The code has been successfully tested comparing its results with other proven codes in the fourth and fifth Kinetic Code Comparison Workshops [2, 3]. It also has demonstrated to be useful for diagnostic of experiments [4].

2. Method of opacity calculation

In this work we have performed a detailed analysis of the mean opacity for Lithium, Beryllium and Boron plasma, using high quality atomic data obtained using the FAC code [5]. This code provides atomic magnitudes calculated into the DLA approach, using appropriate coupling schemes and including configuration interaction. The calculation of ionic charge state distributions and level populations are performed by solving a collisional-radiative steady-state model (CRSS). The level populations are computed with a reasonable accuracy

for plasmas of any element in a wide range of conditions embracing non-LTE, LTE and CE situations. The spectrally resolved opacity is obtained as a sum of the bound-bound, bound free, free-free and scattering contributions

$$\kappa(\nu) = \frac{1}{\rho} (\mu_{bb}(\nu) + \mu_{bf}(\nu) + \mu_{ff}(\nu) + \mu_{scatt}(\nu)) \quad (1)$$

The bound-bound spectrum includes all the allowed transitions in the dipole approximation along with all the detailed atomic levels considered. Line profiles incorporate Doppler, natural and electron-impact widths. The bound-free and free-free processes are calculated using the Kramer's semiclassical expressions. The Rosseland and Planck mean opacities are then computed by:

$$\frac{1}{\kappa_{Rosseland}} = \int_0^{\infty} d\nu B'(\nu, T) / \kappa(\nu) \quad \kappa_{Planck} = \int_0^{\infty} d\nu B(\nu, T) [\kappa(\nu) - \kappa_{scatt}(\nu)] \quad (2)$$

where κ_{scatt} is the absorption coefficient contribution by scattering (Thomson cross section), and $B(\nu, T)$ and $B'(\nu, T)$ are the Planck weighting function and its derivative.

3. Results and discussion

Analytical expressions can be found in the literature [6–8] for the opacity of low-Z plasmas in a wide range of temperatures and densities. The expression used as reference to model Rosseland and Planck mean opacities is a power law depending on temperature and density,

$$k_{R,P} = e^a \rho^b T^c \quad (3)$$

where T is the temperature (eV) and ρ is the mass density (gcm^{-3}). The definition of the parameter a can change depending on the author. These expressions are usually fitted to match LTE data but there are no values for a wide range of density and temperature conditions where NLTE assumptions are important. In a logarithmic representation, eq. (3) is a plane surface over the entire range of temperatures and densities. This geometrical form can give a good fit for highly ionized plasmas, but it fails at low temperatures where the atomic structure leads to local modulations of the mean opacity.

In order to extend its range of application, we have modified this formula including a multiplicative term depending on the temperature and the electron density, thus,

$$\mu_{R,P} (\text{cm}^{-1}) = \rho \kappa_{R,P} = e^{a_0} T^{a_1} \rho^{a_2} f(x, y) \quad (7)$$

with

$$f(x, y) = \left[\exp a_3 xy + a_4 x^2 + a_5 y^2 \right] \quad (8)$$

where $x = \log(T)$ and $y = \log(\rho)$. We have fit the mass absorption coefficient instead off the

radiative opacity because it has a smoother behavior. This new expression is a quadratic fit to surface which gives more flexibility in relation to a linear one.

The variation of Planck and Rosseland mean absorption coefficient in cm^{-1} for Beryllium plasma in the range 1 to 1000 eV of temperature and 10^{-10} to 10^{-1} g cm^{-2} of density is shown in figure 1a and 1b. As expected, both opacities decrease as the temperature increases and rises as the density increases. In figure 1c and 1d we show a piecewise fit for both opacities. It can be appreciate how to perform a linear fit over the entire range of temperature and densities would be a crude approximation, and the formula (7) follows better the oscillating behavior exhibit by the opacity at low temperatures.

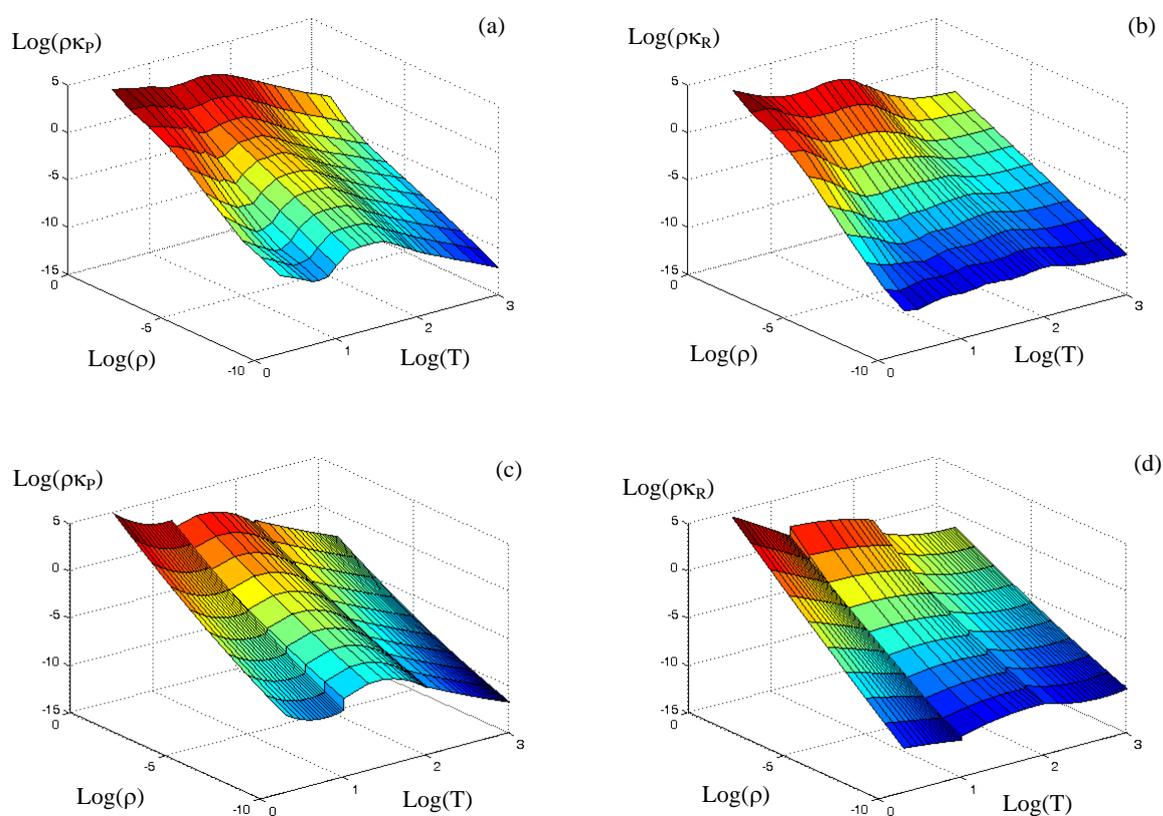


Figure 1. Planck (a) and Rosseland absorption coefficients in cm^{-1} for a Be plasma computed by ABACO/RAPCAL code. (c), (d) Same quantities computed using equations (7)-(8).

Values of the parameters obtained for several low Z elements are given in Table 1. These results should be used with caution because the accuracy of the fit increases with the temperature, thus for the range $10^2 - 10^3$ eV we find local deviations around 10-20 % in the opacities but in the range 1 -10 eV they can deviate by factor 5 from the calculated values. Nevertheless this fit could be a useful tool in the study of phenomena in hydrodynamics involving radiative transport, when these local deviations can be assumed.

Table 1. Fitting coefficients.

	Planck			Rosseland		
	Z=3					
	$1 \leq T \leq 10 \text{ eV}$	$10 < T \leq 10^2 \text{ eV}$	$10^2 < T \leq 10^3 \text{ eV}$	$1 \leq T \leq 10 \text{ eV}$	$10 < T \leq 10^2 \text{ eV}$	$10^2 < T \leq 10^3 \text{ eV}$
a₀	2.1195E+01	9.3986E+00	2.7807E+01	1.9424E+01	1.5594E+01	2.4996E+01
a₁	-9.5624E+00	4.2568E+00	-5.1315E+00	-7.7632E+00	1.5201E+00	-7.6016E+00
a₂	2.1810E+00	1.5861E+00	1.7701E+00	2.2338E+00	2.8647E+00	1.4683E+00
a₃	-3.8400E-01	2.8961E-02	-3.7040E-03	-1.0297E-01	-2.4069E-01	-4.3706E-02
a₄	2.4094E+00	-1.0845E+00	1.0344E-01	2.0365E+00	-8.6555E-01	5.5563E-01
a₅	1.2152E-02	2.3232E-02	2.5038E-02	1.7773E-02	2.5594E-02	7.0353E-03
	Z=4					
	$1 \leq T \leq 10 \text{ eV}$	$10 < T \leq 10^2 \text{ eV}$	$10^2 < T \leq 10^3 \text{ eV}$	$1 \leq T \leq 10 \text{ eV}$	$10 < T \leq 10^2 \text{ eV}$	$10^2 < T \leq 10^3 \text{ eV}$
a₀	2.5021E+01	-1.3335E+01	3.1069E+01	2.1479E+01	1.0988E+01	4.2232E+01
a₁	-1.1426E+01	1.5940E+01	-6.1691E+00	-5.2281E+00	1.8834E+00	-1.2768E+01
a₂	1.9024E+00	1.4265E+00	1.4141E+00	2.0529E+00	2.6111E+00	1.5055E+00
a₃	-8.7274E-02	-2.4457E-02	-4.7942E-03	-1.9886E-01	-1.3254E-01	-4.8302E-02
a₄	2.6164E+00	-2.5665E+00	1.8157E-01	-1.5990E-01	-5.5587E-01	9.4243E-01
a₅	1.5436E-02	1.1775E-02	1.4057E-02	4.6793E-03	3.0221E-02	7.5590E-03
	Z=5					
	$1 \leq T \leq 20 \text{ eV}$	$20 < T \leq 2 \times 10^2 \text{ eV}$	$2 \times 10^2 < T \leq 10^3 \text{ eV}$	$1 \leq T \leq 10 \text{ eV}$	$20 < T \leq 2 \times 10^2 \text{ eV}$	$2 \times 10^2 < T \leq 10^3 \text{ eV}$
a₀	1.6109E+01	-2.0046E+01	3.0288E+01	1.7740E+01	8.6753E+00	5.2951E+01
a₁	-8.8648E-01	1.6572E+01	-5.6685E+00	-5.7576E-01	2.1382E+00	-1.5937E+01
a₂	1.4934E+00	1.2575E+00	1.1586E+00	1.9185E+00	2.5659E+00	1.1639E+00
a₃	4.5994E-02	-2.7165E-02	-2.0608E-03	-1.2073E-01	-1.3522E-01	-1.2894E-02
a₄	-2.8942E-01	-2.3022E+00	1.2952E-01	-9.2101E-01	-5.0343E-01	1.1695E+00
a₅	1.5892E-02	5.6522E-03	6.2114E-03	3.6289E-03	2.8486E-02	3.1661E-03

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