*36th EPS Conference on Plasma Phys. Sofia, June 29 - July 3, 2009 ECA Vol.**33E**, P-1.177 (2009)*

Resonant field amplification in tokamaks and reversed field pinches: experimental results and a linear model

V.D. Pustovitov

Russian Research Centre "Kurchatov Institute", Moscow 123182, Russia

1. Introduction. Experimentally observed effects of resonant field amplification (RFA) and Resistive Wall Mode (RWM) instability are compared with theory predictions based on the model [1] comprising the Maxwell equations, Ohm's law for the conducting wall, boundary conditions and assumption of linear plasma response to the external magnetic perturbations. The analysis includes the feedback rotation of RWM in the RFX-mod reversed field pinch [2], dependence of the RWM growth rate on the plasma-wall separation [3] and appearance of the slowly growing RWM precursors [4] in JT-60U tokamak. We show that the mentioned effects, seemingly different in different conditions, can be described on a common basis.

2. The model. To describe the magnetic perturbation $\mathbf{b}$ outside the plasma we use equation

$$\tau_w \partial B_m / \partial t = \Gamma_m B_m - \Gamma_m^0 B_m^{ext} \tag{1}$$

obtained in cylindrical geometry by integrating, with proper boundary conditions, the radial component of $\mu_0 \sigma \partial \mathbf{b} / \partial t = \nabla^2 \mathbf{b}$ through the wall considered as a thin shell. Here $\tau_w = \mu_0 \sigma r_w d$ with σ, r_w and d the conductivity, minor radius and thickness of the wall, $B_m \equiv b_{mn}(r_w)$ is the amplitude of (m,n) harmonic of $b_r \equiv \mathbf{b} \cdot \nabla r = \sum b_{mn}(r,t) \exp(im\theta - in\zeta)$ in cylindrical coordinates $r, \theta, z = R\zeta$, $2\pi R$ is the length of the system, $B_m^{ext} = b_{mn}^{ext}(r_w)$ with b_{mn}^{ext} being the part of b_{mn} created by the currents behind the wall (the error field and the control field), and $\Gamma_m^0 = -2M$ with $M = |m| \neq 0$ or $\Gamma_m^0 = -2$ for $m = 0$. See details in [1, 5]. Experiments [6, 7] have shown that the linear model based on (1) is good in describing the observed RFA effects. Its derivation from the first-principle equations in toroidal geometry is given in [5].

Assuming the time dependence (with real γ and Ω)

$$B_m = B_m^0 \exp(\gamma t + in\Omega t) \tag{2}$$

we obtain from (1) for $B_m^{ext} = \text{const}$: $\Gamma_m = \tau_w(\gamma_0 + in\Omega_0)$ with γ_0 and Ω_0, the "natural" instantaneous growth/decay rate of the mode and the angular frequency of its toroidal rotation. These are the measurable quantities, see [1, 6, 7]. In the cylindrical model, Γ_m is determined by rb_m' / b_m at the plasma surface S_{pl}, since in the plasma-wall vacuum gap [1, 5]

$$\frac{rb'_{mn}}{b_{mn}} = -(M+1) - \frac{2M\Gamma_m x^{2M}}{2M + \Gamma_m(1 - x^{2M})}\,, \tag{3}$$

the prime means the radial derivative and $x = r/r_w$. The plasma properties enter the problem when rb'_{mn}/b_{mn} at S_{pl} is given in terms of the plasma parameters.

If the perturbation in the plasma is described by the linearized equations and the inertia (dependence of rb'_{mn}/b_{mn} on the growth rate) can be neglected, the left hand side of (3) must be determined by the plasma equilibrium only. Accordingly, with Γ_m a constant in a fixed equilibrium state independent of B_m, B_m^{ext} and the (complex) growth rate, equation (1) becomes linear, which is the main assumption here.

3. Excitation of the rotating mode in the RFX-mod [2]. In these experiments the mode with $(m,n) = (1,-6)$ was sustained by the feedback system producing such a harmonic with amplitude at the wall prescribed by [2]

$$B_m^f + \tau_f \frac{\partial B_m^f}{\partial t} = -KB_m\,. \tag{4}$$

Here K is a complex quantity and τ_f is the characteristic time of the feedback circuit. Replacing B_m^{ext} in (1) by B_m^f from (4) we obtain

$$\tau_w \tau_f \frac{\partial^2 B_m}{\partial t^2} + (\tau_w - \Gamma_m \tau_f)\frac{\partial B_m}{\partial t} - (\Gamma_m + \Gamma_m^0 K + \tau_f \frac{\partial \Gamma_m}{\partial t})B_m = 0\,. \tag{5}$$

With $\Gamma_m = \text{const}$ and B_m in the form (2), this reduces to a dispersion relation similar to that in [2], though expressed in other parameters. In the thin-wall approximation, we have to disregard the terms with τ_f since $\tau_f << \tau_w$ in the RFX-mod [2]. Then equation (5) becomes

$$\tau_w \frac{\partial B_m}{\partial t} = (\Gamma_m + \Gamma_m^0 K)B_m\,. \tag{6}$$

For $B_m(t)$ in the form (2) this yields

$$n\Omega\tau_w = \text{Im}(\Gamma_m + \Gamma_m^0 K)\,, \tag{7}$$

which shows (since Γ_m^0 is real by definition) that the mode rotation frequency Ω can be controlled by the imaginary component of K. Similar dependence of Ω on $\text{Im}\,K$ in the RFX-mod experiments [2] proves that the model discussed here is adequate.

In [2] this frequency was presented as a function of the phase shift $\Delta\varphi$ between the mode and the external perturbations. The feedback algorithm (4) implies that $K = |K|e^{i\Delta\varphi}$, if the term with τ_f can be disregarded. Then (7) gives

$$\Omega = \Omega_0 + \frac{\Gamma_m^0}{n\tau_w}|K|\sin\Delta\varphi . \quad (8)$$

To obtain an observable rotating mode, the feedback gain K must be selected so that γ would be close to zero. According to (6) and (2), $\gamma\tau_w = \mathrm{Re}(\Gamma_m + \Gamma_m^0 K)$. With Γ_m^0, Γ_m and K defined above, we obtain $\gamma = 0$ at $\Gamma_m^0|K|\cos\Delta\varphi = -\gamma_0\tau_w$. Then (8) reduces to

$$n(\Omega - \Omega_0) = -\gamma_0 \mathrm{tg}\Delta\varphi . \quad (9)$$

This well agrees with the RFX-mod results [2]. In [2] the model was more complicated, though it was based on an equation equivalent to (5). In [2] the model included the Newcomb's equation solving by a stability code, while we obtain the result analytically using equation (6) only, assuming the plasma response linear or Γ_m independent of $\mathbf{b}$.

4. Dependence of the RWM growth rate on the plasma-wall separation. In the experiments on JT-60U the growth rates of the current-driven RWMs have been measured in similar discharges, but with different plasma-wall separation [3]. From shot to shot the plasma-wall gap at the high-field side was changed from 40 to 30 and 20 cm. All this is equivalent to changing $x_p = r_p / r_w$ in (3) while keeping rb'_{mn}/b_{mn} at S_{pl} unchanged (r_p is the plasma radius). With smaller plasma-wall separation the growth times of the $m/n = 3/1$ instabilities (being of the order of τ_w) became longer, which means smaller γ [3].

If we assume linear plasma response, rb'_{mn}/b_{mn} in the plasma must be determined by the equilibrium only. With rb'_{mn}/b_{mn} at $x = x_p$ the same in discharges with different plasma radius, but similar equilibrium profiles, the quantities Γ_m must satisfy the equation

$$\frac{\Gamma_m x_p^{2M}}{2M + \Gamma_m(1 - x_p^{2M})} = \mathrm{const}, \quad (10)$$

where the constant depends on the unchanged equilibrium pressure and current profiles. Then

$$\Gamma_m = \frac{2M}{D(r_p / a_0)^{2M} - 1}, \quad (11)$$

where $D \equiv 2M/\Gamma + 1$ and $\Gamma = \Gamma_m(\alpha = 1)$ is a reference quantity for the plasma with minor radius $r_p = a_0$. Since D must be a constant determined by the equilibrium profiles only, equation (11) gives an explicit dependence of Γ_m on the plasma radius r_p.

The data in [3] correspond to real $\Gamma_m = \gamma\tau_w$. Then $D = 2M/(\gamma_0\tau_w) + 1$, where γ_0 is the growth rate of the mode in the plasma with $r_p = a_0$. In this case equation (11) shows that the plasma unstable with $r_p = a_0$, with $\gamma_0 > 0$, becomes less unstable with larger r_p. In other words, the growth rate γ becomes smaller than γ_0 when the plasma-wall gap is narrowed. This is exactly the effect observed in similar discharges with different r_p in the JT-60U [3].

5. RWM precursors. Recently an $n=1$ slowly growing kink-like rotating mode was observed just before the RWM onset in the JT-60U [4]. Let us try to explain the observations.

With static $B_m^{ext} \neq 0$ and constant Γ_m a general solution to (1) is

$$B_m = B_m^0 \exp(\Gamma_m \tau) - \Gamma_m^0 B_m^{ext} \frac{\exp(\Gamma_m \tau) - 1}{\Gamma_m}, \qquad (12)$$

where $\tau = t/\tau_w$ and $B_m^0 = B_m(t=0)$. Consider (12) at the stability boundary, $\gamma = 0$. At $\Gamma_m \to 0$ it gives $B_m = B_m^0 - \Gamma_m^0 B_m^{ext} \tau$, which is a nonrotating precursor of RWM. This solution was discussed in [1] and earlier papers cited there. Such linear growth of the weakly stable low-n modes was indeed found in the EXTRAP T2R reversed field pinch [7].

With $\Omega_0 \neq 0$, we have $\Gamma_m = in\Omega_0 \tau_w$ at $\gamma = 0$, and Eq. (12) gives us oscillating solution with the same time dependence and appearance at the same conditions (at $\gamma \to 0$) as the RWM precursor observed in the JT-60U [4]. With $\gamma \neq 0$ solution (12) is either decaying or growing. For a stable plasma, $\gamma < 0$, it gives a stationary state $B_m^{st} = B_m^{ext} \Gamma_m^0 / \Gamma_m$.

In JT-60U the precursor appeared spontaneously, without deliberate external excitation. A natural trigger must be the error field, which can be seen from our equations. By definition, B_m^0 is the amplitude of b_m at $t = 0$. If $t = 0$ is the start of the discharge, when the plasma does not contribute to B_m, we have $\Gamma_m = \Gamma_m^0$ and $B_m^0 = B_m^{ext}$. Then for $t > 0$ we obtain from (1) $B_m = B_m^{ext} A(t)$. Therefore, $B_m^0 \propto B_m^{ext}$ for arbitrary choice of $t = 0$, so that the both terms in (12) contain B_m^{ext}. This means that the oscillating part of (12) is proportional to the static error field amplitude B_m^{ext}. This can be called error-field induced rotating precursor of RWM.

[1] V.D. Pustovitov, Nucl. Fusion **47**, 563 (2007).

[2] T. Bolzonella, et al., Phys. Rev. Lett. **101**, 165003 (2008).

[3] G. Matsunaga, et al., Plasma Phys. Control. Fusion **49**, 95 (2007).

[4] G. Matsunaga, et al., *Proc. 22nd Int. Conf. on Fusion Energy (Geneva, 2008)*, EX/5-2.

[5] V.D. Pustovitov, Phys. Plasmas **15**, 072501 (2008).

[6] H. Reimerdes, et al., Nucl. Fusion **45**, 368 (2005).

[7] D. Gregoratto, et al., Phys. Plasmas **12**, 092510 (2005).