Current Drive Calculations: Benchmarking Momentum Correction and Field-Line Integration Techniques

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The adjoint approach is a standard tool for calculating the current drive (CD) efficiency in electron cyclotron ray-tracing codes. In the most common version of this approach, the first Legendre harmonic of the solution of the (*first order*) linearized drift-kinetic equation (DKE) with a parallel momentum conserving collision operator is used. This treatment is equivalent to a generalized Spitzer function for arbitrary collisionalities [1, 2] and would require the solution of the DKE in 4D-phase space for stellarators (3D for tokamaks). Momentum correction techniques are based on *mono-energetic* transport coefficients calculated from the solution of the equivalent DKE with the simple Lorentz form of the pitch-angle collision term without momentum conservation, where the radius, r, and the velocity, v, are only parameters. Only the flux-surface-averaged momentum-corrected parallel flows are then estimated without again solving the DKE. With precalculated databases of mono-energetic transport coefficients (from DKES [3] or NEO-MC [4] code), this technique is well suited for calculating the electric conductivity and the bootstrap current (also for NBCD) in arbitrary magnetic configurations [2].

The ECCD source function, i.e., the quasi-linear diffusion term with the Maxwellian in linear theory, however, is highly localized in 4D-phase space. In principle, this requires also the 4D-solution of the (adjoint) DKE which may only be obtained analytically in the collisional (*classical* Spitzer problem) and in the collisionless limits. The collisionless solution, $g(x, \lambda)$, given in Ref. [5] with the normalized magnetic moment, λ, is constant on the flux-surface

$$g(\lambda \leq 1) = \frac{3}{4}\,\frac{\langle b^2\rangle}{f_c}\,K(x)\int\limits_{\lambda}^{1}\frac{d\lambda}{\langle\sqrt{1-\lambda\, b}\rangle} \quad , \quad g(\lambda \geq 1) = 0 \qquad \text{and} \tag{1}$$

$$C_{l=1}(K) - \frac{f_t}{f_c}\,\nu(x)\,K = -\nu_0\, x\, F_M \qquad \text{with} \qquad f_c = 1 - f_t = \frac{3}{4}\,\langle b^2\rangle\int\limits_{0}^{1}\frac{\lambda\, d\lambda}{\langle\sqrt{1-\lambda\, b}\rangle}, \tag{2}$$

where $x = v/v_{th}$, $b = B/B_{\max}$, $C_{l=1}$ is the first Legendre term of the full linearized collision operator, and f_t is the trapped particle fraction. The collisionless Spitzer function, $S(x)$, is defined by $K = S(x) \cdot \exp(-x^2)$ (the collisional limit is obtained for $f_t = 0$). The integro-differential equation (2) is solved numerically. A fast and accurate approximation to the collisionless Spitzer function is obtained from a generalization of the variational principle [6] introduced in Ref. [7]. The simple test function used (polynomial of 5th degree in x) leads to an overestimate for $x > 4$.

In the *high-speed-limit* (HSL) approach widely used for ECCD calculations (see, e.g., Ref. [8]), the energy diffusion as well as the integral contributions from the first-order Rosenbluth potentials are omitted in the collision operator at high velocities, i.e., parallel momentum conservation is violated. This simple-minded approach leads to a significant underestimation of ECCD for $x < 3.5$ (see Figs. 1 and 2).

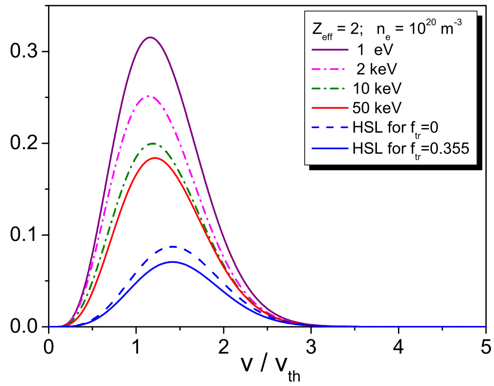

Figure 1: Spitzer function calculated for different temperatures.

The second term in the Spitzer Eq. (2) represents the parallel mono-energetic viscosity describing the friction of circulating particles with the trapped ones, and is generalized [2] for arbitrary collisionalities by introducing the *effective* trapped particle fraction, $f_{\text{tr}}^{\text{eff}}(x)$, which is defined using the mono-energetic conductivity coefficient, D_{33}, by $f_{\text{tr}}^{\text{eff}} = 1 - f_c^{\text{eff}} = 1 - \frac{3}{2}\left(\nu(x)/v v_{th}\right) D_{33}(\nu^*)$ with the mono-energetic "collisionality", $\nu^* = \nu(x) R / v \iota$ (R is the major radius). This definition guarantees the limits $f_{\text{tr}}^{\text{eff}}(x \to 0) = 0$ and $f_{\text{tr}}^{\text{eff}}(x \to \infty) = f_t$ given in Eq. (2). Both collisional (at very low T_e) and collisionless (at very high T_e) limits are confirmed; see Fig. 1.

For ECRH/ECCD scenarios with high T_e (in ITER up to 30 keV), electrons interacting with the RF-field are very close to the collisionless limit. However, relativistic effects must be taken into account. This solution is obtained analytically for the HSL approach [8], but the problem is not trivial if momentum conservation is needed. The simplified weakly relativistic solver [1] based on the variational principle[6, 7] is fast enough for ray-tracing calculations. Here, the fully relativistic collision operator [10] is expanded in a power series in (T_e/mc^2). The fully relativistic solver SYNCH [11], which solves the Spitzer problem in the collisionless limit, can also be used for ray-tracing. The present version, being oriented originally only to tokamaks, is already applicable for an arbitrary configuration. In Fig. 2, solutions of the Spitzer problem from the different approaches are shown: i) weakly relativistic solver implemented in the code TRAVIS, ii) fully relativistic solver SYNCH, iii) HSL-solution (for reference). Calculations are performed for $T_e = 25$ keV, where relativistic effects become significant. In the range $u < 3$, corresponding to energies 200 keV, the weakly relativistic and the fully relativistic solutions coincide well. The HSL-solution converges to the fully relativistic solution with momentum conservation only for ultra-relativistic electrons, $\gamma \gtrsim 1.9$, i.e., $E \gtrsim 0.5$ MeV. Currently, SYNCH is being implemented in the code TRAVIS.

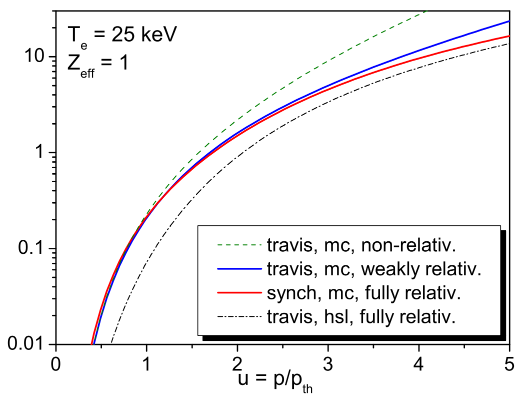

Figure 2: Spitzer function, $S(u)$, calculated with different approaches (mc denotes momentum conservation).

With the field-line integration technique, the NEO-2 code [9] solves the linearized DKE for arbitrary magnetic configurations. Equivalent to the moment equation approach, a Sonine polynomial expansion with respect to x^2 is applied to the distribution function and the full linearized collision operator (which conserves momentum and energy). The resulting set of coupled partial differential equations (with respect to the normalized magnetic moment, λ, and spatial coordinates) is solved using an adaptive grid over λ. Contrary to the momentum correction technique, the complete *local* solution (along the field line) can be calculated for arbitrary collisionalities. So far, NEO-2 with the full energy dependence (with momentum conservation) is restricted to tokamak (3D-phase space) since the field-line integration is very time-consuming for stellarators, where this technique is efficient enough only for mono-energetic calculations. Consequently, only tokamak configurations are used for benchmarking.

For the highly localized ECCD, the collisionless solution of Eqs. (1,2) must be extended to very small, but finite collisionalities using momentum correction techniques. Except in the close vicinity of the maximum of B on the flux-surface, the current diffuses from the passing particle region ($\lambda < 1$) into a narrow sheath of barely trapped particles (the width of this sheath scales with the square-root of the collisionality). This feature is modelled by adding a constant to the integral of Eq. (1) for $\lambda < 1$ with an exponential decay for $\lambda > 1$ (with continuous derivative at $\lambda = 1$) with the normalization to $f_c^{\rm eff}(x)$. In the close vicinity of the maximum of B, this approach fails and g is simply scaled by the factor $f_c^{\rm eff}(x)/f_c$.

In Fig. 3, preliminary results of benchmarking for the circular tokamak with aspect ratio equal to 4 are shown for $T_e = 1$ keV, $n_e = 6.65 \cdot 10^{19}$ m^{-3}, $Z_{\rm eff} = 1$ and $\iota = 0.52$. In order to estimate the effect of barely trapped electrons, the function g was calculated for such sufficiently collisional plasma by the code NEO-2 and the approximation described above. As reference, the collisionless solution from SYNCH in the non-relativistic limit is added. At the minimum-B

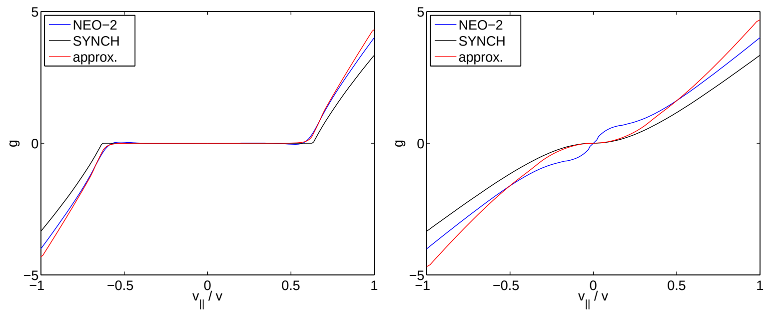

Figure 3: Pitch-dependence of the Spitzer function calculated for $v/v_{\rm th} = 2$ by different codes.

point (left) both calculations with finite collisionality show the effect of current diffusion from the trapped region: the Spitzer function remains finite in the trapped-passing boundary layer. At the maximum-B point (right) an additional effect of finite collisionality can be seen from the NEO-2 curve: very slow particles starting from this point produce a current higher than that expected from the collisionless approach (this is seen by higher g values). This is due to the combined effect of acceleration of these particles by the magnetic mirroring force and their collisional diffusion further into the passing region within a single transit time.

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