36th EPS Conference on Plasma Phys. Sofia, June 29 - July 3, 2009 ECA Vol.**33E**, P-1.133 (2009)

Effects of trapped particle dynamics on the structures of low-frequency shear Alfvén continuous spectrum*

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Introduction.

We analytically derive the structures of the low-frequency shear Alfvén continuous spectrum due to resonant wave-particle interactions with magnetically trapped thermal ions [1]. Our theoretical description asymptotically recovers known results in the relevant limits at both high [2] and low frequencies [3]; furthermore, it is relevant for assessing the accurate kinetic structures that are due to shear Alfvén and acoustic wave spectra in toroidal geometry [1]. Since there is a continuous transition between various shear Alfvén wave and MHD fluctuation branches in many situations of experimental interest [1, 4], the results reported in the present work are of practical relevance for their interpretation when used in the theoretical framework of the general "fishbone-like" dispersion relation [5, 6].

Derivation of the kinetic-inertial layer response.

Since the early observation of Beta induced Alfvén Eigenmodes (BAE), connected with significant redistribution of supra-thermal ions generated by Neutral Beam Ion (NBI) heating [7], significant attention was devoted to exploring low frequency Alfvénic fluctuations in toroidal confinement devices. Here, by low frequency we mean $|\omega| \ll \omega_A \equiv v_A/qR_0$, with $v_A = B/\sqrt{4\pi\rho}$ the local Alfvén speed, ρ the plasma mass density, q the safety factor and R_0 the torus major radius. A variety of experimental observations have recently renewed the interest in the detailed structures of the Alfvén continuum at low frequencies; e.g., the observation of a broad band discrete Alfvén spectrum in DIII-D (with toroidal mode numbers in the range $n \sim 2 \div 40$), excited by both energetic ions (low-n) and thermal ions (high-n) [8] as earlier predicted by theory [2, 9].

A discussion of the connection between low frequency shear Alfvén waves (SAW), MHD fluctuations, Geodesic Acoustic Modes (GAM) [10] and Zonal Flows (ZF) [11] can be found in [1, 4, 6] and references therein, along with a brief summary of experimental evidences, which support the concept that apparently different observations of SAW and MHD modes can be all described within the unified theoretical framework provided by the general "fishbone-like" dispersion relation [5, 6]:

$$i\Lambda(\omega) = \delta\hat{W}_f + \delta\hat{W}_k \quad , \tag{1}$$

which is based on the two scale-length essence of singular (inertial/kinetic) and regular (ideal MHD) structures of the underlying fluctuations. Here, the left hand side (LHS) is the inertial (kinetic) layer contribution due to thermal particles, while the right hand side (RHS) comes from background MHD and energetic particle contributions in the regular ideal regions.

*Work supported by the Euratom Communities under the contract of Association between EURATOM/ENEA, the "Consorzio di Ricerca per l'Energia e le Applicazioni Tecnologiche dell'Elettromagnetismo" (CREATE) and the Italian Ministry of Foreign Affairs.

The scope of this work is to summarize the results of Ref. [1] and to provide a syntetic derivation of the general expression of $\Lambda(\omega)$, which may be used in Eq. (1) in the whole frequency range $0 \leq |\omega| \ll \omega_A$. We show that the general Λ expression asymptotically recovers known results: at low frequency, $|\omega| \ll \omega_{Bi}$, the Graves and Hastie [3] form of the MHD inertia enhancement is reproduced, whereas at high frequencies, $\omega_{Bi} \ll \omega_{Ti} \approx |\omega| \ll \omega_A$, the former kinetic theory result of Ref. [2] is obtained. As corollaries of our derivation, we confirm prior results showing BAE/GAM degeneracy in the long wavelength limit [5, 6, 12] and the identity [12] of the MHD inertia enhancement factor to the ZF polarizability induced by ion temperature gradient (ITG) turbulence [13].

We consider a low $\beta \approx \varepsilon^2$ axisymmetric tokamak plasma equilibrium with shifted circular flux surfaces, where magnetic shear $s = rq'/q$ and $\alpha = -R_0 q^2 \beta'$ define a two-parameter set of plasma equilibria.We also employ straight magnetic field line toroidal coordinates (r, ϑ, ζ), with r the radial-like flux coordinate, ϑ the poloidal angle and ζ the generalized toroidal coordinate chosen such that $q = \mathbf{B}\cdot\nabla\zeta/\mathbf{B}\cdot\nabla\vartheta = q(r)$; meanwhile, prime denotes derivation with respect to r. Furthermore, for the sake of simplicity, we treat all trapped particles as deeply trapped (i.e., characterized by harmonic motion between magnetic mirror points) and consider circulating particles as well circulating (i.e., characterized by constant parallel velocity). Analytic solutions of the coupled system of quasi-neutrality condition and vorticity equation are found for the scalar fields $\delta\phi$ (scalar potential) and $\delta\psi$, defined such as $\boldsymbol{b}\cdot\boldsymbol{\nabla}\delta\psi \equiv (-1/c)\partial_t \delta A_\parallel$ ($\delta A_\parallel$ the parallel vector potential), which fully describe SAW and slow magneto-acoustic wave (SMW) once the fast wave is eliminated assuming perpendicular pressure balance. Solutions at the leading order in an asymptotic expansion in the smallness parameter ω/ω_A show that $\delta\phi$ can be written as $\delta\phi \simeq \delta\phi_0 + \sin\vartheta\,\delta\phi_s$ and $\delta\psi \simeq \delta\psi_0$, with $\delta\phi_0$, $\delta\phi_s$ and $\delta\psi_0$ slowly varying functions along the equilibrium $\boldsymbol{B}$ field (flute-like). One generally has $\delta\phi_0 = I_\Phi(\omega/\overline{\omega}_{Di}, \omega/\overline{\omega}_{De})\delta\psi_0$, with

$$I_\Phi\left(\frac{\omega}{\overline{\omega}_{Di}}, \frac{\omega}{\overline{\omega}_{De}}\right) = 1 + \frac{\sqrt{2\varepsilon}\tau\left(L(\omega/\overline{\omega}_{Di}) + \tau^{-1}L(\omega/\overline{\omega}_{De})\right)}{1 + \tau\omega_{*ni}/\omega + \sqrt{2\varepsilon}\tau\left[1 - \omega_{*ni}/\omega - M(\omega/\overline{\omega}_{Di}) - \tau^{-1}M(\omega/\overline{\omega}_{De})\right]} \quad (2)$$

describing the non-vanishing "flute-like" component of the parallel electric field due to the effect of trapped thermal particle precession resonance, which becomes negligibly small for $|\omega| \gg |\overline{\omega}_{Di}|, |\overline{\omega}_{De}|$. Here, $\overline{\omega}_{Ds} = nq/(rR_0)(cT_s)/(e_s B_0)$ for $s = e, i$ is the deeply trapped particle precession frequency, $\tau = T_e/T_i$, $\varepsilon = r/R_0$,

$$\begin{aligned} M\left(\frac{\omega}{\overline{\omega}_{Ds}}\right) = & -2\frac{\omega}{\overline{\omega}_{Ds}}\left\{\left(1 - \frac{\omega_{*ns}}{\omega} + \frac{3}{2}\frac{\omega_{*Ts}}{\omega}\right)\left[1 + \sqrt{\frac{\omega}{\overline{\omega}_{Ds}}}Z\left(\sqrt{\frac{\omega}{\overline{\omega}_{Ds}}}\right)\right]\right. \\ & \left. - \frac{\omega_{*Ts}}{\omega}\left[\frac{1}{2} + \frac{\omega}{\overline{\omega}_{Ds}} + \left(\frac{\omega}{\overline{\omega}_{Ds}}\right)^{3/2} Z\left(\sqrt{\frac{\omega}{\overline{\omega}_{Ds}}}\right)\right]\right\}, \end{aligned} \quad (3)$$

$$\begin{aligned} L\left(\frac{\omega}{\overline{\omega}_{Ds}}\right) = & -2\left\{\left(1 - \frac{\omega_{*ns}}{\omega} + \frac{3}{2}\frac{\omega_{*Ts}}{\omega}\right)\left[\frac{1}{2} + \frac{\omega}{\overline{\omega}_{Ds}} + \left(\frac{\omega}{\overline{\omega}_{Ds}}\right)^{3/2} Z\left(\sqrt{\frac{\omega}{\overline{\omega}_{Ds}}}\right)\right]\right. \\ & \left. - \frac{\omega_{*Ts}}{\omega}\left[\frac{3}{4} + \frac{1}{2}\frac{\omega}{\overline{\omega}_{Ds}} + \left(\frac{\omega}{\overline{\omega}_{Ds}}\right)^2 + \left(\frac{\omega}{\overline{\omega}_{Ds}}\right)^{5/2} Z\left(\sqrt{\frac{\omega}{\overline{\omega}_{Ds}}}\right)\right]\right\}, \end{aligned} \quad (4)$$

with $Z(x) = 1/\sqrt{\pi}\int_{-\infty}^{\infty} e^{-y^2}/(y-x)\,dy$, $\omega_{*ns} = (T_s c/e_s B)(\boldsymbol{k}\times\boldsymbol{b})\cdot\nabla(n_s)/n_s$ and $\omega_{*Ts} = (T_s c/e_s B)$ $(\boldsymbol{k}\times\boldsymbol{b})\cdot\nabla(T_s)/T_s$. One can also demonstrate $\delta\phi_s = S(\omega, \overline{\omega}_{Di}, \omega_{Bi}, \omega_{Ti})(i/nq)r\partial_r\phi_0$, with $\omega_{Ti} =$

$(2T_i/m_1)^{1/2}/(qR_0)$ the circulating ion transit and $\omega_{Bi} = (\varepsilon/2)^{1/2}\omega_{Ti}$ the deeply trapped ion bounce frequency. Meanwhile, the implicit definition of $S(\omega, \overline{\omega}_{Di}, \omega_{Bi}, \omega_{Ti})$ is given by

$$\delta\phi_s = -\frac{N_1(\frac{\omega}{\omega_{Ti}}) + \Delta N_1(\frac{\omega}{\omega_{Ti}}) + \sqrt{2\varepsilon}P_2(\frac{\omega}{\overline{\omega}_{Di}}, \frac{\omega_{Bi}}{\overline{\omega}_{Di}})}{1 + \frac{1}{\tau} + D_1(\frac{\omega}{\omega_{Ti}}) + \Delta D_1(\frac{\omega}{\omega_{Ti}}) + \sqrt{2\varepsilon}\left[P_1(\frac{\omega}{\overline{\omega}_{Di}}, \frac{\omega_{Bi}}{\overline{\omega}_{Di}}) - P_2(\frac{\omega}{\overline{\omega}_{Di}}, \frac{\omega_{Bi}}{\overline{\omega}_{Di}})\right]} \frac{ir}{nq}\frac{\partial}{\partial r}\phi_0\,, \quad (5)$$

where $P_1(\omega/\overline{\omega}_{Di}, \omega_{Bi}/\overline{\omega}_{Di}) = -2(\omega^2/\overline{\omega}_{Di}^2)[(1-\omega_{*ni}/\omega + 1.5\omega_{*Ti}/\omega)G_2 - (\omega_{*Ti}/\omega)G_4]$ and $P_2(\omega/\overline{\omega}_{Di}, \omega_{Bi}/\overline{\omega}_{Di}) = -2(\omega/\overline{\omega}_{Di})[(1-\omega_{*ni}/\omega + 1.5\omega_{*Ti}/\omega)G_4 - (\omega_{*Ti}/\omega)G_6]$ come from the trapped particles dynamics and, introducing $\Omega_{1,2} = 0.5(\omega_{Bi}/\overline{\omega}_{Di})(\pm 1 + \sqrt{1 + 4\omega\overline{\omega}_{Di}/\omega_{Bi}^2})$, can be calculated from:

$$G_2 = \frac{\overline{\omega}_{Di}/\omega_{Bi}}{\Omega_1+\Omega_2}\left[\Omega_1 Z(\Omega_1) - \Omega_2 Z(\Omega_2)\right] \ , \quad G_4 = \frac{\overline{\omega}_{Di}/\omega_{Bi}}{\Omega_1+\Omega_2}\left[\Omega_1^2 - \Omega_2^2 + \Omega_1^3 Z(\Omega_1) - \Omega_2^3 Z(\Omega_2)\right] \ ,$$

$$G_6 = \frac{\overline{\omega}_{Di}/\omega_{Bi}}{\Omega_1+\Omega_2}\left[(1/2)(\Omega_1^2 - \Omega_2^2) + \Omega_1^4 - \Omega_2^4 + \Omega_1^5 Z(\Omega_1) - \Omega_2^5 Z(\Omega_2)\right] \ .$$

The circulating particle response enters the functions $D_1(\omega/\omega_{Ti})$ and $N_1(\omega/\omega_{Ti})$, with

$$D_1(x) = (1-\omega_{*ni}/\omega)xZ(x) - x[x + (x^2 - 1/2)Z(x)](\omega_{*Ti}/\omega) \ , \quad (6)$$

$$N_1(x) = 2\frac{\overline{\omega_{Di}}}{\omega_{Ti}}\left\{\left(1 - \frac{\omega_{*ni}}{\omega}\right)[x + (1/2 + x^2)Z(x)] - \frac{\omega_{*T}}{\omega}[x(1/2 + x^2) + (1/4 + x^4)Z(x)]\right\} \ , \quad (7)$$

while the modified circulating ion response due to finite trapped particle fraction is given by

$$\Delta D_1(x) = \frac{x}{\pi^{1/2}}\int_0^\infty e^{-y}\ln\left(\frac{x+\sqrt{2\varepsilon y}}{x-\sqrt{2\varepsilon y}}\right)\left[1 - \frac{\omega_{*ni}}{\omega} - \frac{\omega_{*Ti}}{\omega}\left(y - \frac{3}{2}\right)\right]dy \ , \quad (8)$$

$$\Delta N_1(x) = \frac{\overline{\omega}_{Di}/\omega_{Ti}}{\pi^{1/2}}\int_0^\infty y e^{-y}\ln\left(\frac{x+\sqrt{2\varepsilon y}}{x-\sqrt{2\varepsilon y}}\right)\left[1 - \frac{\omega_{*ni}}{\omega} - \frac{\omega_{*Ti}}{\omega}\left(y - \frac{3}{2}\right)\right]dy \ . \quad (9)$$

Given the framework of Eqs. (2) to (9), the general expression of Λ^2 in Eq. (1) is obtained as [1]:

$$\Lambda^2/I_\Phi = \frac{\omega^2}{\omega_A^2}(1-\omega_{*pi}/\omega) + \Lambda_{cir}^2 + \frac{\omega^2\omega_{Bi}^2}{\omega_A^2\overline{\omega}_{Di}^2}\frac{q^2}{\sqrt{2\varepsilon}}\left[P_3 + (P_2 - P_3)S(\omega, \overline{\omega}_{Di}, \omega_{Bi}, \omega_{Ti})\right] \quad (10)$$

where the last term on the RHS represents the trapped particle contribution, with $P_3(\omega/\overline{\omega}_{Di}, \omega_{Bi}/\overline{\omega}_{Di}) = -2[(1-\omega_{*ni}/\omega + 1.5\omega_{*Ti}/\omega)G_6 - (\omega_{*Ti}/\omega)G_8]$ and

$$G_8 = \frac{\overline{\omega}_{Di}/\omega_{Bi}}{\Omega_1+\Omega_2}\left[(3/4)(\Omega_1^2 - \Omega_2^2) + (1/2)(\Omega_1^4 - \Omega_2^4) + \Omega_1^6 - \Omega_2^6 + \Omega_1^7 Z(\Omega_1) - \Omega_2^7 Z(\Omega_2)\right] \ .$$

Meanwhile, the circulating ion response, inclusive of its modification due to finite trapped particle fraction, is accounted for by Λ_{cir}^2, which reduces to the prior result with circulating particles only [2] and, in the more general case analyzed here, is given by [1]

$$\Lambda_{cir}^2 = q^2\frac{\omega\omega_{Ti}}{\omega_A^2}\left[\left(1 - \frac{\omega_{*ni}}{\omega}\right)\left(F\left(\frac{\omega}{\omega_{Ti}}\right) + \Delta F\left(\frac{\omega}{\omega_{Ti}}\right)\right) - \frac{\omega_{*Ti}}{\omega}\left(G\left(\frac{\omega}{\omega_{Ti}}\right)\right.\right.$$
$$\left.\left. + \Delta G\left(\frac{\omega}{\omega_{Ti}}\right)\right) + \frac{\omega\omega_{Ti}}{4\overline{\omega}_{Di}^2}\left(N_1\left(\frac{\omega}{\omega_{Ti}}\right) + \Delta N_1\left(\frac{\omega}{\omega_{Ti}}\right)\right)S(\omega, \overline{\omega}_{Di}, \omega_{Bi}, \omega_{Ti})\right] \ , \quad (11)$$

with the functions $F(x)$, $G(x)$, $\Delta F(x)$ and $\Delta G(x)$ defined as $F(x) = x\left(x^2 + 3/2\right) + \left(x^4 + x^2 + 1/2\right)Z(x)$, $G(x) = x\left(x^4 + x^2 + 2\right) + \left(x^6 + x^4/2 + x^2 + 3/4\right)Z(x)$,

$$\left\{\begin{array}{c}\Delta F(x)\\ \Delta G(x)\end{array}\right\} = \frac{1}{\pi^{1/2}}\int_0^\infty e^{-y}\ln\left(\frac{x+\sqrt{2\varepsilon y}}{x-\sqrt{2\varepsilon y}}\right)\frac{y^2}{4}\left\{\begin{array}{c}1\\ y-3/2\end{array}\right\}dy \ . \quad (12)$$

Relevant limiting cases and discussions.

The general expression for Λ, Eq. (10) obtained in this work [1], can be readily used in connection with Eq. (1) for analyzing a number of kinetic stability problems involving SAW, MHD and Alfvénic drift-wave turbulence (DWT). For very low frequencies $|\omega| \ll \omega_{Bi}$ it is easy to show that the limit of Eq. (10) is [1]

$$\Lambda^2/I_\Phi \simeq (\omega^2/\omega_A^2)(1-\omega_{*pi}/\omega)\left(1+(15/16)\sqrt{2}q^2\varepsilon^{-1/2}+0.5q^2\right) \ . \qquad (13)$$

The result of Eq. (13) can be compared with that by Graves and Hastie [3]:

$$\Lambda^2 = (\omega^2/\omega_A^2)(1-\omega_{*pi}/\omega)(1+1.6q^2\varepsilon^{-1/2}+0.5q^2) \ . \qquad (14)$$

The difference between the factors 1.6 and $(15/16)\sqrt{2} \approx 1.3$ in the two equations is due to our simplified treatment of all particles as deeply trapped/well circulating.

The high frequency (fluid) limit can be obtained for $|\omega| \gg \omega_{Ti}$ in Eq. (10), which gives [1, 2]:

$$\Lambda^2 = (\omega^2/\omega_A^2)\left[1-(7/4+\tau)q^2(\omega_{Ti}^2/\omega^2)\right] \ . \qquad (15)$$

This result on the SAW continuous spectrum accumulation point ($\Lambda = 0$) at low frequency shows that trapped particles do not appreciably alter the dynamics for $|\omega| \gg |\omega_{Bi}|$. In fact, they introduce an $O(\varepsilon)$ frequency shift, whereas they provide the dominant contribution to the plasma inertia at low frequency for $|\omega| \ll |\omega_{Bi}|$ [3]. These considerations are readily extended to GAM, by invoking the degeneracy of BAE and GAM spectra in the long wavelength limit ($\Lambda = 0$ and $\omega_{*pi} = 0$) [5, 6, 12]. Based on our present findings, we may conclude that the GAM frequency shift due to trapped particles is $O(\varepsilon)$.

Our results show that the kinetic layer thermal ion response in the Kinetic Thermal Ion (KTI) gap frequency range [6] is dominated by geodesic curvature, i.e. by transit and/or precession-bounce resonances. Thermal particle precession resonance enters mainly via the non-vanishing "flute-like" component ($|k_\parallel qR_0| \ll 1$) of the parallel electric field, which is negligibly small at $|\omega| \gg |\bar{\omega}_{Di}|, |\bar{\omega}_{De}|$. Moreover, it is evident that kinetic treatments of the thermal plasma components are needed for a realistic description of thermonuclear plasmas, where SAW, MHD and DWT will characterize complex behaviors mediated by their mutual interactions.

Acknowledgments.

Useful discussions with A. Biancalani, C. Di Troia and J.P. Graves are kindly acknowledged.

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