Simulations of long time behaviour of internal kink modes with the XTOR code.

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Introduction

Although there exists a widely-used model for the scaling of sawteeth (presented by Porcelli, Boucher and Rosenbluth, see [1]), the behaviour of sawteeth cycles has never been simulated over a long time (typically, several tens of sawteeth) to our knowledge. We are developing a non-linear two-fluids code to address this, but in order to get a reference framework, it seemed useful to begin by running simulations with a simpler resistive MHD model with thermal transport.

Sawteeth were first detected on the ST tokamak in Princeton (see [2]) and on similar machines. For such small, ohmically-heated tokamaks, resistive MHD with thermal transport should be a relevant model (except for the reconnection dynamics of the crash, which are not the object of this study). A parametric study of internal kink oscillations using the XTOR code (see [3]) with this model is shown, in order to investigate whether an oscillatory regime with an $(m,n) = (1,1)$ internal kink exists within our range of parameters. This range of parameters corresponds to small, ohmically-heated tokamaks.

Physical model and key parameters

The XTOR code implements the resistive MHD equations :

$$\partial_t \mathbf{v} = -(\mathbf{v}\cdot\nabla)\mathbf{v} + \mathbf{j}\times\mathbf{B} - \nabla p + \nabla\nu\nabla\cdot\mathbf{v} \tag{1}$$

$$\partial_t \mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B}) - \nabla\times\eta\mathbf{j} \tag{2}$$

$$\partial_t p = -\Gamma p\nabla\cdot\mathbf{v} - \mathbf{v}\cdot\nabla p + \nabla\chi_\perp\nabla p + \nabla\cdot\left[\mathbf{B}\left(\frac{\chi_\parallel}{B^2}(\mathbf{B}\cdot\nabla)\,p\right)\right] + H \tag{3}$$

where $\mathbf{v}$ is the fluid velocity, $\mathbf{B}$ is the magnetic field, $\mathbf{j}$ the current, η the resistivity and ν the viscosity. The density is considered constant.

For the source term H in eq. (3) , we choose $H = \nabla\chi_\perp\nabla p_{ini}$, so that H basically tends to return the pressure to its initial state. Two key parameters of the system are $\chi_\perp$, and the Lundquist number $S = \eta^{-1}$ (in XTOR units). On the one hand, $\chi_\perp$ controls the build-up of the pressure via the source term. On the other hand, since we keep the η profile constant throughout the simulation, η controls the current profile evolution. The current profile is initially fixed by

$\eta j_\phi = E_{wall}$. Another significant parameter is the poloidal beta on the $q = 1$ surface, β_p. This parameter is important because the ideal $(1,1)$ internal kink growth rate depends directly on it (see [4]).

For the sake of pertinence, we restrict our study to values of β_p below or around the ideal threshold ($\beta_{plim} = 0,21$ in our reference case), as observed in ohmically-heated tokamak plasmas.

In our simulations, we use a fixed value for central S, namely $S_c = 10^6$, which corresponds to our domain of interest. The $\chi_\perp$ parameter takes the values $10^{-6}, 3.10^{-6}, 10^{-5}, 3.10^{-5}, 10^{-4}, 3.10^{-4}$; the ratio $\chi_\perp/\eta$ thus varies from 1 to 300, which includes the experimental range for these coefficients. Our β_p values are 0,07, 0,14, 0,21, and 0,23. The $\chi_\parallel$ value is 1, 10, or 100. The viscosity ν is taken to be 5.10^{-6}. Our starting equilibrium is given by the CHEASE code (see [5]) ; the initial q-profile is shown in fig. 1.

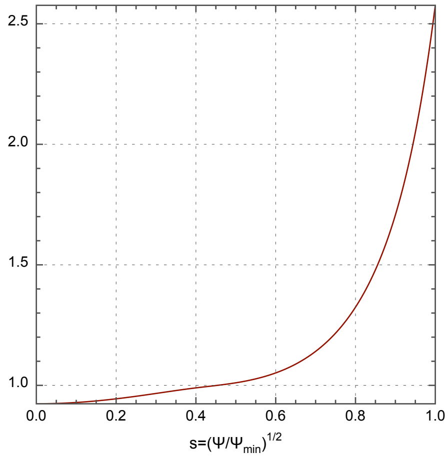

FIG. 1 – Initial q profile

Distribution of regimes in parameter space

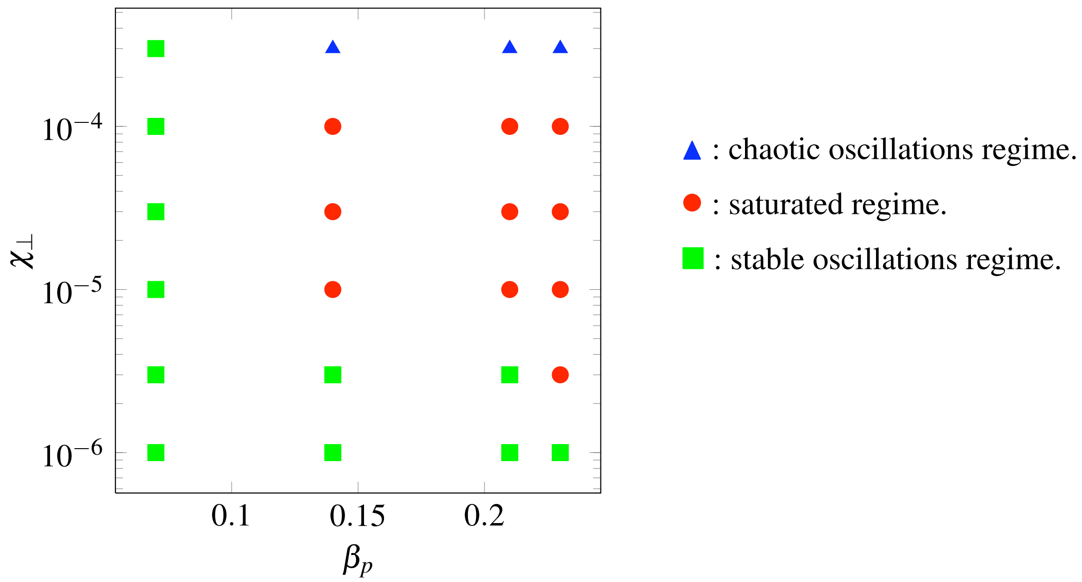

FIG. 2 – Distribution of the different oscillations regimes in $(\chi_\perp, \beta_p)$ space.

We see three different regimes in our $(\chi_\perp, \beta_p)$ parameter space : one of stable oscillations, one of damped oscillations (called saturated), and one of chaotic oscillations. Their distribution is shown on fig. 2 for $\chi_{\|} = 100$. It is worth remembering that experimental β_p for ohmic sawteeth are usually around $0,1$. Around this value, the stable and saturated regime can both be observed, so they are both relevant.

Stable oscillations regime

For small values of β_p (e.g. $0,07$), we observe stable internal kink oscillations over long-time simulations (5.10^5 Alfven times). Each time the $(1,1)$ kink develops, we see a crash and a flattening of the core pressure that is sawteeth-like (see fig. 3). However, our model is not meant to describe accurately the reconnection dynamics of the crash. This is evidenced by the too large ratio between crash and ramp-up time.

The oscillations are actually caused by a balance between the perpendicular transport time and the instability growth time. It is important to note that such a balance will be altered outside our specific range of parameters.

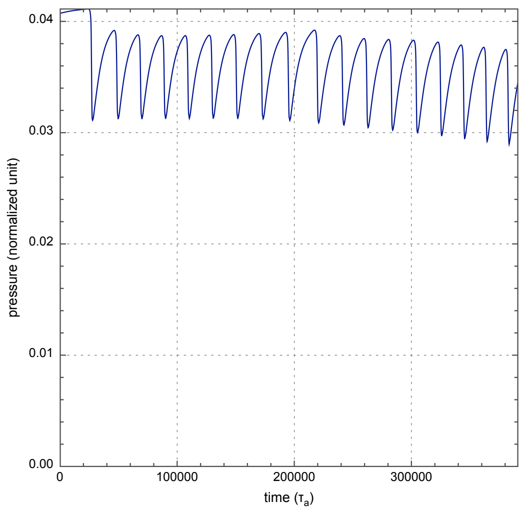

FIG. 3 – Central pressure evolution for $\beta_p = 0,07, \chi_{\perp} = 3.10^{-6}, \chi_{\|} = 100$.

Saturated regime

For higher values of β_p (from $\beta_p = 0,14$), which are typical of ohmic sawteeth in tokamaks such as JET and for $\chi_{\perp} \geq 10^{-5}$, another situation appears. In this new regime, the kink oscillations are quickly damped (in less than 10^5 Alfven times). The larger $\chi_{\perp}$ is, the faster the damping. After that, the plasma core remains stuck in a three-dimensional kinked configuration (see fig. 4). This is consistent with the time scales argument presented above. As β_p increases, γ grows steeply, and there is no more time for the transport source term to rebuild the starting configuration.

In this state, we observe the presence of a permanent reconnection layer, associated with a convection cell. They are both located around the initial $q = 1$ surface (q always stays very close to 1 inside this surface). The system reaches a saturated state. It is an equilibrium, since the pressure and magnetic flux introduced through the source term are constantly evacuated by the reconnection and convection structure.

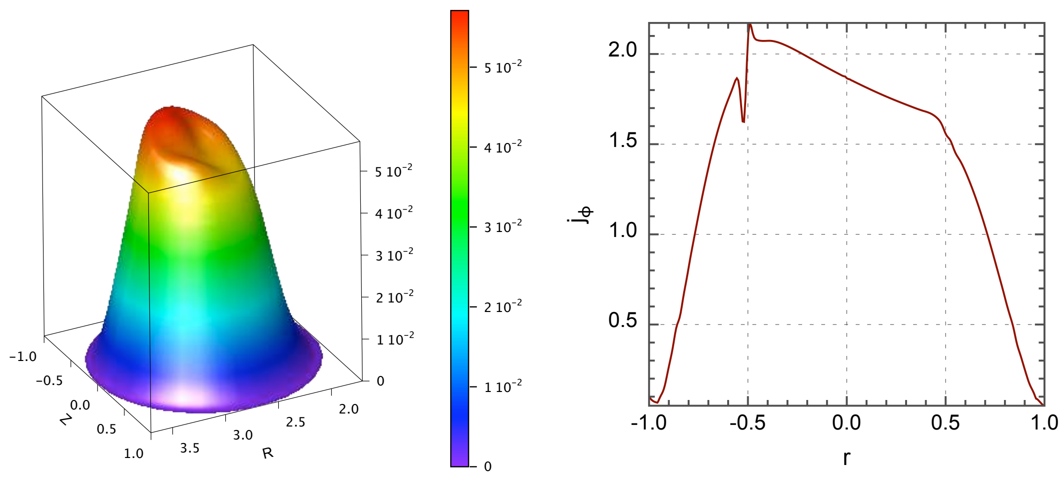

FIG. 4 – Left : 3-D pressure plot and right : j_ϕ versus r in saturated regime for $\beta_p = 0,14, \chi_\perp = 3.10^{-5}, \chi_{||} = 10$.

We note that at higher $\chi_\perp$ ($\chi_\perp = 3.10^{-4}$), chaotic oscillations and coupling with higher order modes (such as (2,1) or (3,2)) can also happen.

Conclusion

We have presented a parametric study of a plasma corresponding to the one of a small, ohmically-heated tokamak : low β_p , low Lundquist number. For the lowest β_p cases, oscillations of the internal $(1,1)$ kink are observed. We believe that they appear because at low β_p there is no separation of time scales between the kink and the perpendicular transport. It is not guaranteed that they will remain if S is increased, since these times scales would then become separated. For higher β_p, we have found that the plasma sets itself in a three-dimensional kinked configuration with a permanent reconnection layer and a convection cell around the largest $q = 1$ surface. This seems to confirm that MHD alone cannot reproduce the hysteresis necessary to observe sawteeth.

Références

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