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The Effect of Collision Frequency on Neoclassical Polarisation Current

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Introduction

Understanding the evolution of magnetic islands in tearing mode instabilities is important for successful ITER operation. Finite banana orbit width effects give rise to two types of current perturbations, which influence the island evolution: the bootstrap current, which provides a drive for the island growth [1], and the neoclassical polarisation current, which can provide the threshold mechanism [2].

This paper employs drift kinetic theory to study the collision frequency dependency of the neoclassical polarisation current, and its impact on the island evolution. We employ the magnetic geometry, coordinate system and the double expansion method in Ref.[2] to calculate the ion response in both the collisional ($\nu_{ii} \gg \varepsilon\omega$) and collisionless ($\nu_{ii} \ll \varepsilon\omega$) limits. In the low collision frequency limit, the effect of collisions in the vicinity of trapped/passing boundary in the velocity space is considered. This is found to provide the dominant correction to the neoclassical polarisation current.

Ion Response

The ion response to the perturbed magnetic geometry can be described by the drift kinetic equation [3]. The full derivations of electron and ion responses are given in Ref.[2]. The solution for the ion distribution function is written as: $f_i = (1 - q_i\Phi/T_i)F_{\mathrm{M}i} + g_i$, where the perturbation g_i is expanded in terms of two small parameters: $\Delta = w/r$ and $\delta_i = \varepsilon^{1/2}\rho_{\theta i}/w$. Here, w is the island half-width, r is the minor radius, ε is the inverse aspect ratio and $\rho_{\theta i}$ is the poloidal Larmor radius. Thus, we write $g_j = \sum_{m,n} \delta_j^m \Delta^n g_j^{(m,n)}$.

To the first order in δ_i for the ions, the solutions for $g_i^{(0,0)}$ and $g_i^{(1,0)}$ are

$$g_i^{(0,0)} = \frac{F_{\mathrm{M}i}}{n}\frac{dn}{d\chi}\frac{(\omega - \omega_{*i}^T)}{\omega_{*i}}\left[\chi - h(\Omega)\right], \tag{1}$$

$$g_i^{(1,0)} = \frac{v_\parallel F_{\mathrm{M}i}}{v_{\mathrm{th}i}}\frac{\rho_{\theta i}}{L_n}\left[\frac{\omega}{\omega_{*i}} - 2\sqrt{2}\frac{(\omega - \omega_{*i}^T)}{\omega_{*i}}\frac{1}{w_\chi}\frac{dh}{d\Omega}\sqrt{\Omega + \cos\xi}\right] + \bar{h}_i, \tag{2}$$

where χ is the poloidal flux, ξ is the helical angle, Ω labels the perturbed flux surfaces of the island [2], ω_{*i} is the diamagnetic frequency, $\omega_{*i}^T = \omega_{*i}[1 + (v^2/v_{\mathrm{th}i}^2 - 3/2)\eta_i]$, $\eta_i = L_n/L_{Tj}$, and L_n and L_{Ti} are the density and ion temperature gradient length scales respectively. $h(\Omega)$ is an arbitrary flux surface function, which in fact represents the variation in the density gradient in the vicinity of the island, and $\bar{h}_i$ is the constant of integration, independent of the poloidal angle θ. To the same order, the electron responses are identical in form to the ion responses quoted above, with subscript $i \to e$. The crucial difference between the ion and electron responses is that the former is dominated by the $\mathbf{E} \times \mathbf{B}$ drift, caused by the ion's inertia, while the latter is dominated by the fast parallel streaming along magnetic field lines. This difference appears in the constraint equations for $\bar{h}_i$ and $\bar{h}_e$, which are required to fully determine $g_j^{(1,0)}$. However, since the electrons do not make a dominant contribution to the parallel current perturbation, we will not discuss their response in this paper.

We now proceed to calculate $\bar{h}_i$ from the constraint equation, which depends on the collision frequency (the solutions so far (Eqs.(1) and (2)) have been independent of the collision frequency). Assuming $\omega \gg k_\parallel v_\parallel$ for the ions, where $k_\parallel$ is the parallel wavevector, the constraint equation for $\bar{h}_i$ is:

$$-Rqk_\parallel \left\langle \frac{Rq}{v_\parallel} \frac{\omega}{m\tilde{\psi}} \frac{dh}{d\Omega} \left. \frac{\partial g_i^{(1,0)}}{\partial \xi} \right|_\Omega \right\rangle_\theta + \left\langle \frac{Rq}{v_\parallel} C_i \left(g_i^{(1,0)} \right) \right\rangle_\theta = 0 \tag{3}$$

for passing particles, where $\langle ... \rangle_\theta$ denotes an average over a period in θ, and

$$-Rqk_\parallel \frac{\omega}{m\tilde{\psi}} \frac{dh}{d\Omega} \left\langle \frac{Rq}{|v_\parallel|} \right\rangle_\theta \left. \frac{\partial \bar{h}_i}{\partial \xi} \right|_\Omega + \left\langle \frac{Rq}{|v_\parallel|} C(\bar{h}_i) \right\rangle_\theta = 0 \tag{4}$$

for trapped particles, where $\langle ... \rangle_\theta$ now denotes a θ-average between the bounce points.

These equations have analytic solutions for $\bar{h}_i$ in the two collision frequency limits. In the collisionless limit ($\varepsilon\omega \gg \nu_{ii}$), the first term of Eq.(3) is the dominant term. In the collisional limit ($\varepsilon\omega \ll \nu_{ii}$), however, the collision operator term alone determines $\bar{h}_i$. The model collision operator that we employ for determining $\bar{h}_i$, which can correctly describe the neoclassical ion poloidal flows, is [4, 5]:

$$C_{ii}(g) = 2\nu_{ii}(v) \left[\frac{(1-\lambda B)^{1/2}}{B} \frac{\partial}{\partial \lambda} \left(\lambda (1-\lambda B)^{1/2} \frac{\partial g}{\partial \lambda} \right) + \frac{v_\parallel \bar{u}_{\parallel i}}{v_{\mathrm{th}i}^2} F_{\mathrm{M}i} \right], \tag{5}$$

where

$$\bar{u}_{\parallel i} = \frac{1}{n\{\nu_{ii}(v)\}} \int d^3\mathbf{v}\, \nu_{ii}(v) v_\parallel g, \qquad \{F(v)\} = \frac{8}{3\sqrt{\pi}} \int_0^\infty e^{-x^2} x^4 F(x) dx, \tag{6}$$

and $x = v/v_{\mathrm{th}i}$. The solution for $\bar{h}_i$ in the collisionless limit is as given in Ref.[2] (Eq.(58)), and the solution in the collisional limit is:

$$\bar{h}_i = -\frac{\sigma v}{2} \frac{\rho_{\theta i} F_{\mathrm{M}i}}{L_n v_{\mathrm{th}i}} \left(x^2 - \frac{3}{2} - \kappa \right) \eta_i \frac{\partial h}{\partial \chi} B_0 \int_{\lambda_c}^{\lambda} \frac{d\lambda'}{\langle \sqrt{1-\lambda' B} \rangle_\theta} \Theta(\lambda_c - \lambda), \tag{7}$$

where $\kappa = \{(x^2 - 3/2)\nu_{ii}(v)\}/\{\nu_{ii}(v)\}$.

Current Perturbation

From Eq.(7), the parallel current perturbation, $\bar{J}_{\parallel}$, can be calculated using the current conservation equation: $\nabla.\mathbf{J} = 0$. This is constructed from the drift kinetic equation, as outlined in Ref.[2]. The dominant contribution arises from a term involving $g_i^{(1,0)}$ [2]. The result for $\bar{J}_{\parallel}$ in the collisional limit is then:

$$\bar{J}_{\parallel} = -8\left(\frac{\rho_{\theta i}}{w}\right)^3 \frac{w}{L_n}\frac{r}{sL_n} nq_i v_{\mathrm{th}i} \frac{\omega(\omega-\omega_{*pi}-k\eta_i\omega_{*i})}{\omega_{*i}^2} \times \frac{1}{w_\chi^2}\frac{dh}{d\Omega}\frac{d^2h}{d\Omega^2}\left[\cos\xi - \langle\cos\xi\rangle_\Omega\right] + \langle J_{bs}\rangle_\Omega, \tag{8}$$

where $k = -1.17$,

$$\langle J_{bs}\rangle_\Omega = -1.23\varepsilon^{1/2} nq_i v_{\mathrm{th}i}\frac{\rho_{\theta i}}{L_n}\left[(1+\frac{T_e}{T_i}) + 0.522\frac{T_e}{T_i}\eta_e - 0.173\eta_i\right]\left\langle\frac{\partial h}{\partial\chi}\right\rangle_\Omega, \tag{9}$$

and $\langle ...\rangle_\Omega$ represents the flux surface average [2]. Eq.(8) is in agreement with the fluid calculation result in Ref.[6]. The expression for the bootstrap current perturbation (9) is quantitatively a more accurate result compared to the one in Ref.[2]. It turns out that the velocity dependence of the ion collision frequency ν_{ii} is crucial for accurately describing the neoclassical ion poloidal flow. Comparing Eq.(9) with Eq.(76) of Ref.[2], it is clear that the choice of the collision operator has noticeable effects on the numerical coefficients.

Contribution from Dissipation Layer

We now proceed to calculate the contribution to the parallel current perturbation from the dissipation layer, which exists in the vicinity of the trapped/passing boundary, and in which the collisions cannot be neglected. As is discussed in Ref.[2], in the total absence of collisions, $\bar{h}_i$ is discontinuous at the trapped/passing boundary. Hence, the effect of collisions is to smooth out this discontinuity over the width $y \sim \nu_{ii}/\varepsilon\omega$, where y is a shifted pitch angle variable (such that $y = 0$ defines the trapped/passing boundary). In the low collision frequency limit, this region is referred to as the "dissipation layer", which becomes less well-defined in the high collision frequency limit.

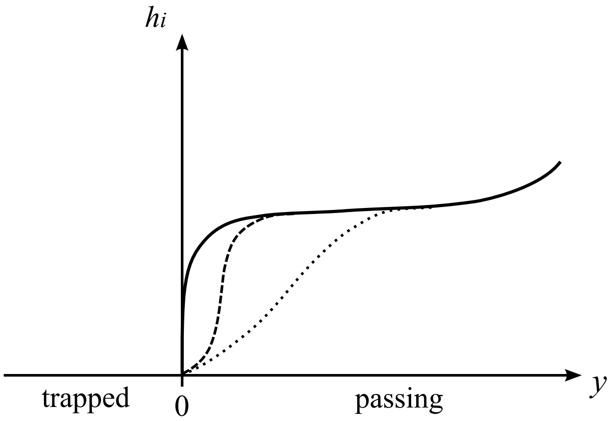

Figure 1: Schematic plot of $\bar{h}_i$ vs. y. Solid line is the plot in total absence of collisions. Dashed line shows the effect of collisions in the dissipation layer. Dotted line is for high collision frequency regime, where the "layer" becomes less well-defined.

Here, we present the contribution from the dissipation layer in the low collision frequency limit (i.e. $\nu_{ii} \ll \varepsilon\omega$). Unlike in the collisionless/collisional limit calculations, both of the terms in the constraint equation (3) are kept. $\bar{h}_i$ is thus calculated, from which the additional contribution to the parallel current perturbation is determined. The result for this $\bar{J}_\parallel$ is:

$$\bar{J}_\parallel = \frac{6}{2^{3/4}}\varepsilon^{1/2} l_\nu \left(\frac{\nu_{ii}}{\varepsilon\omega}\right)^{1/2}\left(\frac{\rho_{\theta i}}{w}\right)^3 \frac{w}{L_n}\frac{r}{sL_n} n q_i v_{\mathrm{th}i}$$
$$\times \frac{\omega(\omega-\omega_{*pi})}{\omega_{*i}^2}\frac{1}{w_\chi^2}\frac{dh}{d\Omega}\frac{d^2h}{d\Omega^2}\left(\frac{w_\chi}{h'}\right)^{1/2}\frac{1}{S^{3/2}}\left(\Xi - \langle\Xi\rangle_\Omega\right), \tag{10}$$

where $l_\nu = [\ln(4\sqrt{\varepsilon\omega/\nu_{ii}})]^{-3/2}$. Consequently, the expression for the NTM threshold island width, w_c, is modified from w_0 in Ref.[2], which now depends on the collision frequency:

$$w_c = w_0\left[1 + 0.37 l_\nu \sqrt{\frac{\nu_{ii}}{\varepsilon\omega}}\right]^{1/2}. \tag{11}$$

Summary

We have determined the expression for the neoclassical polarisation current in the collisional limit ($\nu_{ii}/\varepsilon\omega$), using the drift kinetic theory. The result is in agreement with the fluid calculation of Ref.[6], when the collision operator in Ref.[5] is used. Using the same collision operator, the bootstrap current perturbation in Ref.[2] has been reviewed. The result is in better agreement with the standard expression for J_{bs}, but scaled by $\langle\partial h/\partial\chi\rangle_\Omega$, as expected. The collisional effects in the dissipation layer has been considered. In the low collision frequency limit, the contribution to the parallel current perturbation from the dissipation layer is found to scale as $\sqrt{\nu_{ii}/\varepsilon\omega}$, with an additional weak logarithmic variation in $\sqrt{\varepsilon\omega/\nu_{ii}}$. Consequently, in the low collision frequency limit, the critical island width w_c is found to scale as $l_\nu\sqrt{\nu_{ii}/\varepsilon\omega}$. Work is in progress to determine the collision frequency dependency of w_c for an arbitrary value of $\sqrt{\nu_{ii}/\varepsilon\omega}$.

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