

The effect of particle size on the floating potential of dust in a collisionless plasma.

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Abstract

The most widely used theory for calculating the floating potential of a plasma immersed dust grain is the Orbit Motion Limited (OML) model. The accuracy of OML is explored by comparing with results from a numerical simulation, and its applicability is characterised by two dimensionless parameters.

Introduction.

The floating potential of a dust grain in a plasma determines the details of the dust-plasma interactions. It is therefore important to be able to predict the value of the floating potential (ϕ_g) accurately. The most basic charging mechanism is the collection of ions and electrons from the plasma. If other charging mechanisms (e.g., photo- or thermionic emission of electrons) are neglected then the orthodox theoretical approach to calculating ϕ_g is via the OML model [1] (review by Allen [2]). This assumes an unmagnetised, steady state, collisionless plasma in which both ions and electrons have a Maxwellian distribution far from the dust grain. OML also assumes a spherically symmetric potential, this will be discussed briefly later with respect to flow. When deriving the ion and electron currents due to OML a “grazing” limit is specified as the closest orbit before touching the grain. For a given energy, any attracted particle with an impact parameter less than this grazing limit will be collected by the dust grain and any with a larger impact parameter will not. However, the form of the potential profile may be such that this assumption is not valid [3], hence OML has come under much scrutiny (e.g. [4, 5]).

A more accurate model is the kinetic approach of full Orbit Motion (OM [6]). In this paper we use Hutchinson’s PIC code SCEPTIC [7, 8] to test the validity of OML and OM.

Stationary Plasma:

We define the following dimensionless parameters:

$$\rho = \frac{a}{\lambda_D}, \beta = \frac{T_i}{T_e}$$

where a is the dust radius, λ_D the electron Debye length and T_i , T_e the ion and electron temperatures respectively.

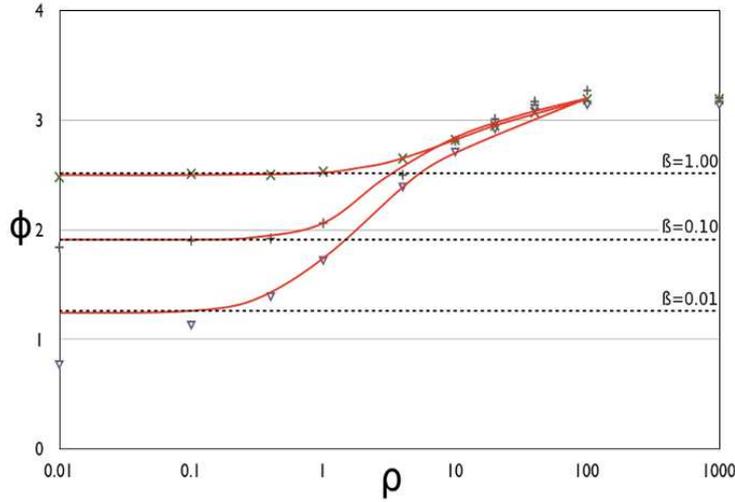


Figure 1: SCEPTIC values of the floating potential for $\beta = 0.01(\nabla)$, $0.1(+)$ and $1(\times)$. The red lines are the corresponding values from full orbit motion theory (OM). The three dashed lines are the OML values (independent of radius).

The floating potential, normalised by $\frac{k_B T_e}{e}$, from SCEPTIC is compared with that of OM in Figure 1. It can be seen that the SCEPTIC generated points agree very well with the red OM line over the whole range of ρ , except for $\rho = 0.01$, $\beta = 0.01$. We will explore this low β region in a later publication. In the limit of large ρ we see the SCEPTIC results approaching an asymptote. OM has different asymptotes depending on β . We would expect to see different asymptotes [4, 9] for varying β but SCEPTIC does not seem to reproduce this. Simulations become difficult for $\rho > 1000$ as it becomes increasingly difficult to resolve to the Debye length scale and still populate the mesh cells sufficiently.

The applicability of OML as determined from the data given in Fig. 1, and additional runs for intermediate values of β , is shown as the shaded area in Fig. 2. For a given β and ρ within the shaded area OML will be within 10% of the floating potential according to SCEPTIC. The curves in Fig.2 are conditions for when OML may be used reliably, there forms are given below:

$$\rho_{max} = (2\beta + 0.6)^2 \quad \text{for } \beta > 0.05 \quad (1)$$

$$\rho_{max} = (-0.02 + 22\beta - 110\beta^2) \quad \text{for } \beta \leq 0.05 \quad (2)$$

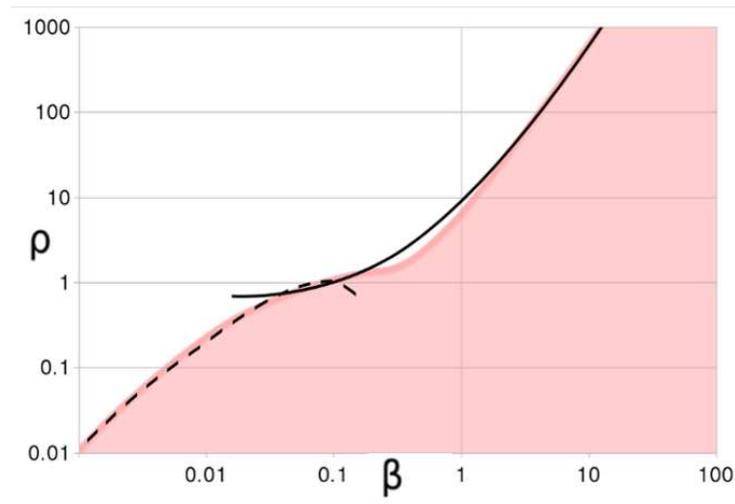


Figure 2: The shaded area indicates where the floating potential returned by SCEPTIC is within 10% of OML. The solid curve is eq. (1), dashed is eq. (2).

Non-stationary plasma:

Whilst we have defined a region where OML is acceptably accurate, it is only for the case of a stationary plasma. Many plasma scenarios we may wish to consider are not stationary. For the charging of a spacecraft in the solar wind, flow speeds on the order of 100s of kilometers per second are encountered (Thermal Mach numbers $\gg 1$). In tokamaks, plasma thermal Mach speeds in excess of unity are also frequently observed. These large flow velocities would be expected to have some effect on the floating potential. Provided there is a spherically symmetric potential, OML may be extended to include drifting ions by using a shifted Maxwellian for the ion velocity distribution [11], herein we shall call this approach the shifted OML. The assumption of a spherically symmetric potential is a very dubious one, as it is the best theory we have, we shall compare it with SCEPTIC. Hutchinson [12] has already compared SCEPTIC with shifted OML for $\beta = 0.1$ and 1.0 , we extend the area investigated. Three examples with thermal Mach numbers (v) of 0.6 , 1.0 and 1.5 are shown in Fig. 3. The shaded areas again indicate regions where SCEPTIC is in agreement (within 10%) of the shifted OML value. Fig. 3 (a), $v = 0.6$, resembles Figure 2 for $\beta > 0.8$. Even at this relatively low flow velocity we see that the shifted OML theory is inadequate for most plasmas. Increasing the Mach number further leaves the lower limit of shifted OML applicability relatively unchanged but disagrees for larger β . For the majority of dust in tokamaks however, the shifted OML approach seems to generate sufficiently accurate values, according to SCEPTIC, to warrant its continued use. As can be seen from Fig. 3 (c), even for flow velocities of $v = 1.5$ the shifted OML approach is correct for β between $1 \rightarrow 4$ and for ρ of $0.01 \rightarrow 10$.

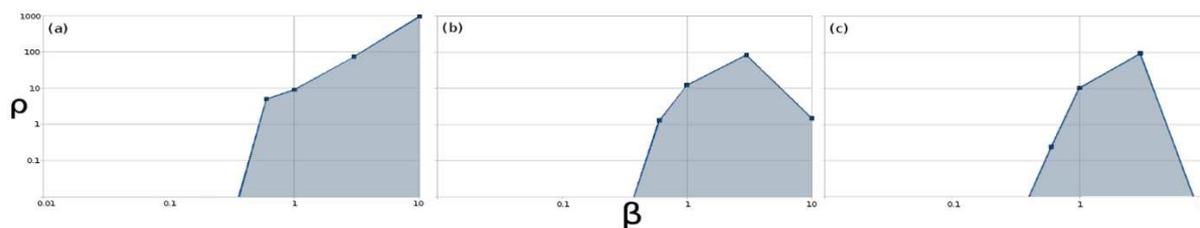


Figure 3: The floating potential according to SCEPTIC is within 10% of the shifted OML value in the shaded areas. (a) $\nu = 0.6$, (b) $\nu = 1.0$ and (c) $\nu = 1.5$.

Conclusion:

We have demonstrated that OML is a good approximation in certain regimes and provided expressions to indicate when OML may be used in confidence. For plasmas of $\beta \sim 1$ then OML is a good approximation providing $\rho < 10$. For plasma of $\beta \gg 1$ SCEPTIC and OML are in close agreement. The SCEPTIC results indicate that, whilst OML may not be strictly correct, ultimately its error is only small. OM provides a much closer fit to the SCEPTIC results over the whole range of ρ . The OM theory for large ρ has different asymptotes depending on β , this is not reproduced by SCEPTIC.

Investigation of the effects of a flowing plasma on the floating potential indicate that shifted OML may be appropriate for hot ion plasma, conditions like this may be found in the scrape off layer of tokamaks. For cold plasmas SCEPTIC disagrees with the shifted OML approach suggesting a new theory is required. Floating potentials found via the shifted OML theory are therefore unlikely to be reliable for the fields of dust in the interstellar medium or space craft charging in the solar wind. Further investigation is needed into the limiting flow speed at which the area displayed in Fig.2 is significantly in error. This limit is expected to be low as even relatively low flows will distort the spherical symmetry of the problem.

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