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# The dielectric tensor for a magnetized dusty plasmas - A New Formulation

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A kinetic approach to the problem of wave propagation in dusty plasmas, which takes into account the variation of the charge of the dust particles due to inelastic collisions with electrons and ions, is utilized as a starting point for the development of a new formulation, which writes the components of the dielectric tensor in terms of a finite and an infinite series, containing all effects of harmonics and Larmor radius. The dielectric tensor for a dusty plasma can be divided into two parts, one which is denominated "conventional" and which is formally similar to the dielectric tensor of dustless plasmas, and another which appears due to occurrence of the inelastic collisions between electrons and ions and the dust particles, and which is denominated as the "new" contribution. Both the "conventional" and the "new" contributions can be written in terms of double series, formally containing all harmonics and Larmor radius contributions. These general expressions depend on a small number of integrals which depend on the distribution function. The formulation is quite general and valid for the whole range of frequencies above the plasma frequency of the dust particles, which are assumed motionless.

Using this basic framework, we arrive to expressions for the components of the dielectric tensor which can be separated into two kinds of contributions [1], $\varepsilon_{ij} = \varepsilon_{ij}^{C} + \varepsilon_{ij}^{N}$. Explicit expressions for the components $\varepsilon_{ij}^{C}$ and $\varepsilon_{ij}^{N}$ can be found in Refs. [1, 2]. Particularly, in Ref. [2] the expressions appear according to the formulation and definitions to be used in the present paper. According to this novel formulation, the components of the dielectric tensor can be written in terms of a double summation, one finite and another infinite, in which the contribution of harmonics and Larmor radius terms is shown explicitly. For the 'conventional' contribution, a component $\varepsilon_{ij}^{C}$ can be written as follows

$$\varepsilon_{ij}^{C} = \delta_{ij} + \delta_{iz}\delta_{jz}e_{zz} + N_{\perp}^{\delta_{iz}+\delta_{jz}}\chi_{ij} \ , \tag{1}$$

while a component $\varepsilon_{ij}^{N}$ is written as $\varepsilon_{ij}^{N} = \mathscr{U}_i \mathscr{S}_j$.

We present here only the explicit expressions for the *zz* contributions to the dielectric tensor. The others can be obtained in similar way. For general distributions and arbitrary directions of propagation, the contribution to the ‘conventional’ part appears as follows

$$\chi_{zz} = \frac{v_*^2}{c^2}\sum_\beta \frac{1}{r_\beta^2}\frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=1}^{\infty}\left(\frac{q_\perp}{r_\beta}\right)^{2(m-1)} \tag{2}$$

$$\times \sum_{n=-m}^{m} a(|n|,m-|n|)\left[J(n,m,2;f_{\beta 0}) + iJ_\nu(n,m,1;f_{\beta 0})\right],$$

$$e_{zz} = -\frac{1}{z^2}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\int d^3u\,\frac{u_\parallel}{u_\perp}\mathscr{L}(f_{\beta 0}) \tag{3}$$

$$+\frac{1}{z^2}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}a(0,0)\left[J(0,0,2;f_{\beta 0}) + i\,J_\nu(0,0,1;f_{\beta 0})\right],$$

where

$$J(n,m,h;f_{\beta 0}) \equiv \int d^3u\,\frac{z u_\parallel^h u_\perp^{2(m-1)} u_\perp L(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i\,\tilde{\nu}_{\beta d}^0},$$

$$J_\nu(n,m,h;f_{\beta 0}) = \int d^3u\,\frac{\tilde{\nu}_{\beta d}^0 u_\parallel^h u_\perp^{2(m-1)} u_\perp \mathscr{L}(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i\,\tilde{\nu}_{\beta d}^0}\ ,$$

$$L = \frac{1}{\gamma}\left[\left(\gamma - \frac{q_\parallel}{z}u_\parallel\right)\frac{\partial}{\partial u_\perp} + \frac{q_\parallel}{z}u_\perp\frac{\partial}{\partial u_\parallel}\right],\quad \mathscr{L} = u_\parallel\frac{\partial}{\partial u_\perp} - u_\perp\frac{\partial}{\partial u_\parallel}\ ,$$

with the dimensionless variables

$$z = \frac{\omega}{\Omega_*},\quad q_{\parallel,\perp} = \frac{k_{\parallel,\perp}v_*}{\Omega_*},\quad u_{\parallel,\perp} = \frac{p_{\parallel,\perp}}{m_\beta v_*}\ ,\quad r_\beta = \frac{\Omega_\beta}{\Omega_*},\quad \tilde{\nu}_{\beta d}^0 = \frac{\nu_{\beta d}^0(u)}{\Omega_*},\quad u = \left(u_\parallel^2 + u_\perp^2\right)^{1/2}\ ,$$

where the inelastic collision frequency between plasma particles and dust particles is given by

$$\nu_{\beta d}^0(u) = \frac{\pi a^2 n_{d0} v_*}{u}\left(u^2 - \frac{2q_{d0}q_\beta}{a m_\beta v_*^2}\right) H\left(u^2 - \frac{2q_{d0}q_\beta}{a m_\beta v_*^2}\right)\ .$$

The quantities $\Omega_*$ and $v_*$ are a characteristic frequency and a velocity, respectively, which are considered convenient for normalization in the case of a particular application. For the present application, we use $\Omega_* = \omega_{pe0}^0$ and $v_* = c_s$, where $c_s = (T_e/m_i)^{1/2}$ is the ion-sound velocity and $\omega_{pe0}^0$ is the equilibrium electron plasma angular frequency in the absence of dust. The quantity $q_{d0}$ is the equilibrium value of the charge of the dust particles, which we will denote as $q_{d0} = -Z_{d0}e$.

The contribution of the ‘new’ part, for general distributions and directions of propagation, appears as follows,

$$\mathscr{U}_z = \frac{1}{z}\frac{1}{z + i(\tilde{\nu}_{ch} + \tilde{\nu}_1)}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=0}^{\infty}\sum_{n=-m}^{+m}\left(\frac{q_\perp}{r_\beta}\right)^{2m} a(|n|,m-|n|)J_U(n,m,1,0;f_{\beta 0})\ , \tag{4}$$

$$\mathscr{S}_z = -\frac{a\Omega_*}{2v_*}\frac{1}{z}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=0}^{\infty}\sum_{n=-m}^{+m}\left(\frac{q_\perp}{r_\beta}\right)^{2m} a(|n|, m-|n|)$$

$$\times\left[J_{\nu L}(n,m,1;f_{\beta 0}) + i J_{\nu\nu}(n,m;f_{\beta 0})\right] + \frac{a\Omega_*}{2v_*}\frac{1}{z}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} J_{\nu 0}(f_{\beta 0}) \,, \quad (5)$$

$$\tilde{\nu}_{ch} = \frac{a\Omega_*}{2v_*}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}} J_{ch}(f_{\beta 0}) \,, \quad (6)$$

$$\tilde{\nu}_1 = -i\frac{a\Omega_*}{2v_*}\sum_\beta \frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\sum_{m=0}^{\infty}\sum_{n=-m}^{+m}\left(\frac{q_\perp}{r_\beta}\right)^{2m} a(|n|, m-|n|) J_U(n,m,0,1;f_{\beta 0}) \,, \quad (7)$$

where

$$J_U(n,m,h,l;f_{\beta 0}) = \int d^3u \frac{z(\tilde{\nu}_{\beta d}^0/z)^l f_{\beta 0}}{z - nr_\beta - q_\parallel u_\parallel + i\tilde{\nu}_{\beta d}^0}\frac{u_\parallel^h u_\perp^{2m}}{u} H\left(u^2 + \frac{2Z_{d0}eq_\beta}{am_\beta v_*^2}\right) \,,$$

$$J_{\nu L}(n,m,h;f_{\beta 0}) = \int d^3u \frac{\tilde{\nu}_{\beta d}^0 u_\parallel^h u_\perp^{2m-1} L(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i\tilde{\nu}_{\beta d}^0} \,,$$

$$J_{\nu\nu}(n,m;f_{\beta 0}) = \int d^3u \frac{z(\tilde{\nu}_{\beta d}^0/z)^2 u_\perp^{2m-1} \mathscr{L}(f_{\beta 0})}{z - nr_\beta - q_\parallel u_\parallel + i\tilde{\nu}_{\beta d}^0} \,,$$

$$J_{\nu 0}(f_{\beta 0}) = \int d^3u \frac{\tilde{\nu}_{\beta d}^0}{z}\frac{\mathscr{L}(f_{\beta 0})}{u_\perp} \,, \quad J_{ch}(f_{\beta 0}) = \int d^3u\, f_{\beta 0}\frac{1}{u} H\left(u^2 + \frac{2Z_{d0}eq_\beta}{am_\beta v_*^2}\right) \,,$$

with $\tilde{\nu}_1 = \nu_1/\Omega_*$ and $\tilde{\nu}_{ch} = \nu_{ch}/\Omega_*$ .

Further development can be made in the particular case of bi-Maxwellian distributions for ions and electrons,

$$f_{\beta 0}(u_\parallel, u_\perp) = \frac{n_{\beta 0}}{(2\pi)^{3/2} u_{\beta\perp}^2 u_{\beta\parallel}} e^{-u_\parallel^2/(2u_{\beta\parallel}^2)} e^{-u_\perp^2/(2u_{\beta\perp}^2)} . \quad (8)$$

In this case, and assuming that the collision frequency is replaced by the average value, $\nu_\beta = \int d^3u\, \nu_{\beta d}^0(u) f_{\beta 0}(u)/n_{\beta 0}$, we obtain, for integer $m$ and $h = 1$,

$$J(n,m,1;f_{\beta 0}) = (m!)(\sqrt{2})^{2m+1} n_{\beta 0} u_{\beta\perp}^{2(m-1)} u_{\beta\parallel}$$

$$\times\left\{\zeta_\beta^0\left[1 + \hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right] - (1-\Delta_\beta)\,\hat{\zeta}_\beta^n\left[1 + \hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right]\right\} \,,$$

$$J_\nu(n,m,1;f_{\beta 0}) = (m!)(\sqrt{2})^{2m+1}(1-\Delta_\beta)\, n_{\beta 0} u_{\beta\perp}^{2(m-1)} u_{\beta\parallel}\frac{\tilde{\nu}_\beta}{q_\parallel}\hat{\zeta}_\beta^n\left[1 + \hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right] \,,$$

$$J_\nu(n,m,1;f_{\beta 0}) = (m!)(\sqrt{2})^{2m+1}(1-\Delta_\beta)\, n_{\beta 0} u_{\beta\perp}^{2(m-1)} u_{\beta\parallel}\frac{\tilde{\nu}_\beta}{q_\parallel}\hat{\zeta}_\beta^n\left[1 + \hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right] \,,$$

$$J_U(n,m,1,l;f_{\beta 0}) \simeq -\Gamma\left(m + \frac{1}{2}\right)(\sqrt{2})^{2m-1+h}\left(\frac{\tilde{\nu}_\beta}{z}\right)^l n_{\beta 0}\, u_{\beta\perp}^{2m-1} u_{\beta\parallel}^h\, \zeta_\beta^0\left[1 + \hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right] \,,$$

$$J_{\nu L}(n,m,h;f_{\beta 0})=\frac{\tilde{\nu}_\beta}{z}J(n,m,h;f_{\beta 0})\;.$$

The cases of $h=0$ and 2, also necessary, are obtained in similar way [2]. Moreover, we obtain

$$J_{\nu\nu}(n,m;f_{\beta 0})=\left(\frac{\tilde{\nu}_\beta}{z}\right)^2(m!)(\sqrt{2})^{2m+1}\left(1-\Delta_\beta\right)n_{\beta 0}u_{\beta\perp}^{2(m-1)}u_{\beta\parallel}\,\zeta_\beta^0\left[1+\hat{\zeta}_\beta^n Z(\hat{\zeta}_\beta^n)\right]\;,$$

$$J_{\nu 0}(f_{\beta 0})=0\;,$$

$$J_{ch}(f_{e0})=\frac{2n_{e0}}{(2\pi)^{1/2}u_{e\parallel}\Delta_e}\int_0^1 d\mu\,\frac{\Delta_e}{1+\mu^2(\Delta_e-1)}e^{-|\chi_\parallel^e|[1+\mu^2(\Delta_e-1)]/\Delta_e}\;,$$

$$J_{ch}(f_{i0})=\frac{2n_{i0}}{(2\pi)^{1/2}u_{i\parallel}\Delta_i}\int_0^1 d\mu\,\frac{\Delta_i}{1+\mu^2(\Delta_i-1)}\;,$$

$$e_{zz}=\frac{1}{z^2}\sum_\beta\frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{\left(1-\Delta_\beta\right)}{\Delta_\beta}+\frac{1}{z^2}\sum_\beta\frac{\omega_{p\beta}^2}{\Omega_*^2}\frac{1}{n_{\beta 0}}\left[J(0,0,2;f_{\beta 0})+iJ_\nu(0,0,1;f_{\beta 0})\right]\;.$$

The normalized average collision frequencies, are given by

$$\tilde{\nu}_\beta=\frac{\nu_\beta}{\Omega_*}=2(\sqrt{2\pi})(\varepsilon n_{i0})\frac{c^3}{\Omega_*^3}\frac{a^2\Omega_*^2}{c^2}\frac{v_*}{c}\frac{u_{\beta\parallel}}{\Delta_\beta}I^\beta\;, \tag{9}$$

where

$$I^i=\int_0^1 d\mu\,\frac{\Delta_i}{1+\mu^2(\Delta_i-1)}\left[\frac{\Delta_i}{1+\mu^2(\Delta_i-1)}+\chi_\parallel^i\right]\;,$$

$$I^e=\int_0^1 d\mu\left[\frac{\Delta_e}{1+\mu^2(\Delta_e-1)}\right]^2 e^{-|\chi_\parallel^e|[1+\mu^2(\Delta_e-1)]/\Delta_e}\;,$$

and where $\Delta_\beta=u_{\beta\perp}^2/u_{\beta\parallel}^2=T_{\beta\perp}/T_{\beta\parallel}$, and where $u_{\beta\perp}=v_{\beta\perp}/v_*$ and $u_{\beta\parallel}=v_{\beta\parallel}/v_*$, with $v_{\beta\perp}=\sqrt{T_{\beta\perp}/m_\beta}$ and $v_{\beta\parallel}=\sqrt{T_{\beta\parallel}/m_\beta}$.

The equilibrium charge of the dust particles, for a given value of the dust density $n_{d0}=\varepsilon n_{i0}$, can be obtained from the condition of equilibrium for the collisional charging of the dust particles, which can be expressed as $\sum_\beta q_\beta n_{\beta 0}\tilde{\nu}_\beta=0$. Using the average collision frequencies, which are given by Eq. (9), the equilibrium condition is written as follows,

$$\sum_\beta q_\beta n_{\beta 0}\frac{u_{\beta\parallel}}{\Delta_\beta}I^\beta=Z\frac{u_{i\parallel}}{\Delta_i}I^i-(Z-\varepsilon Z_{d0})\frac{u_{e\parallel}}{\Delta_e}I^e=0\;, \tag{10}$$

where we have used the ion charge $q_i=Ze$, and the electron density is obtained from the neutrality condition $n_{e0}e=n_{i0}Ze-n_{d0}Z_{d0}e$.

## References


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