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Cyclotron maser radiation from inhomogeneous plasmas

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Cyclotron maser instabilities are of importance in laboratory devices like gyrotrons and in a variety of astrophysical processes. Our objective here is to describe some properties of these instabilities in an inhomogeneous system. We consider a simple electron distribution, a monoenergetic ring with all the electrons rotating with the same velocity around the magnetic field lines. In a homogeneous system, dispersion curves are as shown below, for propagation perpendicular to the magnetic field. The dispersion relation takes the form $k^2 = F(\omega)$ and in an inhomogeneous plasma the corresponding differential equation for the wave amplitude is

$$\frac{d^2\phi}{dx^2} = -F(\omega, x)\phi$$

where the *x* dependence in *F* comes from the spatial dependence of either the magnetic field or the density, both of which are involved in the full expression. We

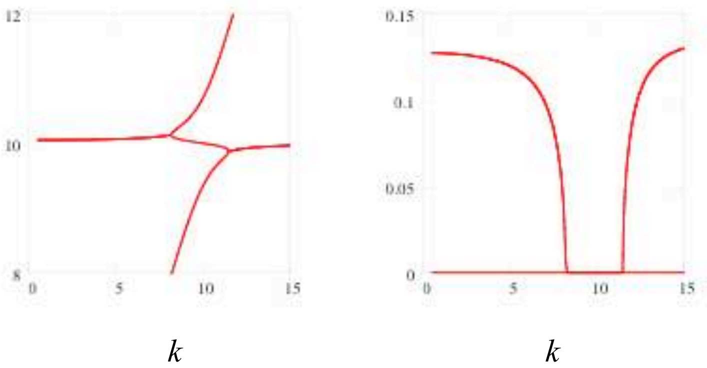

Fig 1. Real (left) and imaginary (right) parts of the frequency for a ring with energy corresponding to γ=1.02 and $\omega_{pe} / \omega_{ce} = 10$. The non-zero imaginary parts belong to the branches with ω around 10.

concentrate on magnetic field gradient, since we look at low density plasma with the cyclotron frequency well above the plasma frequency, where the density dependence is relatively unimportant. With ω real *F* is real and its variation through the resonance region is as in Fig.2

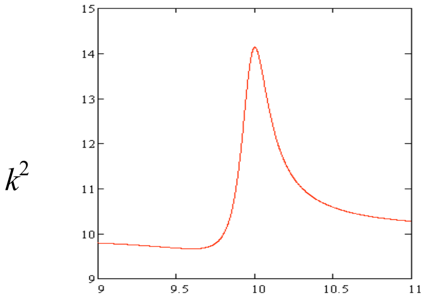

Fig.2 Variation of k^2 through resonance with $\omega = 10$.

With this type of variation we would expect the wave to propagate through this region with no reflection or absorption and this is borne out by numerical solution of the equation. So, even though the wave is unstable in a uniform system, this does not seem to go over into simple spatial growth in an inhomogeneous system. What we need to do is look for solutions in which there is growth in a localized region from which energy propagates away.

To find such solutions we note that, as pointed out in [1], a suitable choice of ω with a complex value can give the sort of spatial variation of k shown in Fig. 3 in which the

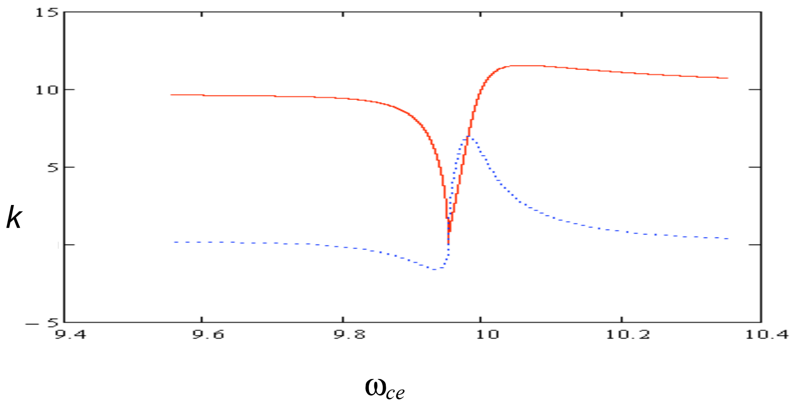

Fig.3 The real (full) and imaginary (dotted) parts of k when $\omega = 10 + 0.1287i$. The real and imaginary parts both pass through zero when ω_{ce} is a little below 10. The root with $\mathrm{Re}(k) \geq 0$ is plotted and there is of course a root with opposite sign.

wavenumber passes through zero. This confluence of the forward and backward propagating waves allows them to interact and for us to obtain, for suitable magnetic field profiles, a solution of the required form. We first note that the behaviour around where k passes through zero is similar to that at a normal cut-off, though in the latter case k^2 just goes from positive to negative real values while here it goes along a line in the complex plane. Nevertheless, the way in which the WKB solutions connect on either side of the zero can be found by taking the local variation to be linear so that the system is described by Airy's equation, $\phi'' + ax\phi = 0$ with a complex. The normal situation at a cut off is that an evanescent mode on one side, representing just one of the two possible WKB solutions (the other growing exponentially), goes to an equal superposition of incoming and outgoing modes on the other side. This is just the well known result that the wave is totally reflected. In the more general case the behaviour is still to connect one of the WKB modes on one side to an equal superposition of the two modes on the other. In our present case if we take a solution with only an outgoing wave to the left then it connects to equal incoming and outgoing waves to the right. In our previous paper [1], we concluded, on the basis of numerical evidence, that the incoming wave to the right was absent and that we could obtain a solution of the type we need in a simple linear gradient. A typical numerical solution is shown in Fig. 4. The boundary condition is that there be only an outgoing wave on the left.

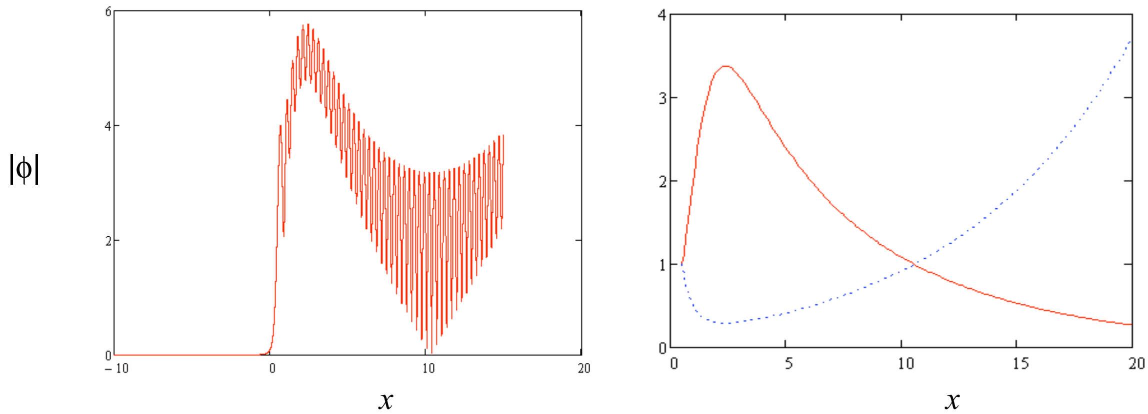

Fig.4 Numerical (left) and WKB (right) solutions for the wave amplitude. The outgoing wave in the right hand plot is the full line. In both cases outgoing and incoming waves have the same amplitude at around x=10.

Outgoing waves decay away from the resonance simply because of the complex value of ω, not because of any damping. It can be seen that the incoming wave amplitude only becomes large at some distance from the resonance. However, if we assume that the incoming and outgoing waves have equal amplitude at the zero of k^2, and follow the amplitudes from there using the WKB approximation we get the result of the right hand graph in Fig. 4. It is clear that there is good agreement in the way that the amplitudes of the two waves cross over, verifying our conclusion about the connection across the zero being described by Airy's equation.

The question now is how we can obtain a physically realistic solution, with only outgoing waves from a localised region of instability. The answer is to take, not a linear field profile, but one going through a minimum. There will then be two points at which k^2 goes to zero and if the parameters are chosen correctly then the solution can match onto outgoing waves on both sides, connecting to waves propagating in both directions in the central region between the zeros. This gives the type of eigenvalue problem to be expected in this type of situation and the behaviour is qualitatively similar to the solution in slab geometry we found previously [1]. While the exact growth rate will need detailed calculation, it can be seen that a condition for the existence of such an instability is that k^2 passes through zero, or at least a value very close to zero, to give the coupling between forward and backward waves. If ω is significantly different from the value described above then each WKB mode just passes through the resonance region independently, as in the case when ω is real. As can be seen from our example, the growth rate in the inhomogeneous system will be somewhat less than the maximum growth rate in the homogeneous system, but of a similar order of magnitude.

To conclude, cyclotron maser instabilities are important in a number of contexts, both astrophysical and laboratory. We have looked at a simple ring distribution, whose stability properties in a homogeneous system are well known but which turns out to have interesting properties in an inhomogeneous system. In many practical applications, of course, more complex electron velocity distributions will be appropriate, such as the ring distribution we have analysed previously. In future work we intend to consider in more detail the behaviour of cyclotron maser instabilities in inhomogeneous geometry, bringing together analytic results and computer simulations.

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References

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