Relativistic laser piston model: ponderomotive ion acceleration in dense plasmas using ultra-intense laser pulses

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The paper is dedicated to a detailed analysis of the quasi-stationary plasma acceleration by the laser ponderomotive force applying analytical methods and one-dimensional (1D) particle-in-cell (PIC) simulations. In the analytical model we describe the internal structure of the laser piston assuming complete plasma evacuation in the vicinity of the traveling laser wave. PIC simulations confirm the principal results of the analytical model concerning the quasi-stationary character of the piston motion, its velocity, the energy of accelerated ions and the laser energy absorption. Moreover, the simulations reveal two complementary effects: (i) the piston velocity oscillates around a stationary value that leads to broadening in the energy spectrum of the accelerated ions and (ii) electrons that escape the piston and move towards the incoming ultra-intense laser wave are cooled due to high frequency radiation losses. This latter effect reduces the plasma density behind the piston and stabilizes the laser pulse propagation.

There are two qualitatively different regimes of the relativistic laser pulse propagation through a dense plasma. The regime of induced transparency is commonly related to plasma densities below or slightly above the critical density. For higher densities, such a strong laser pulse experiences the stimulated Raman scattering, self-focusing and self-modulation. The laser pulse propagation becomes unstable and is accompanied by high energy losses. Another regime is realized in a strongly overdense plasma. Here, the laser pulse ponderomotive force pushes electrons ahead producing a cavitation zone. The electron density peak reflects the laser pulse and the charge separation field pulls the ions forward. As a result, the interaction of an ultra-intense laser pulse with a high-density plasma forms a double layer structure (often called a laser piston) that separates the propagation path of the laser pulse, where there is no plasma, from a shocked plasma with no laser pulse. The piston velocity has been found in Refs. [1, 2] from the condition of the momentum conservation in the piston reference frame. Figure 1 displays the
structure of the piston obtained from the analytical model and particle-in-cell (PIC) modeling.

In the reference frame of the double layer, the dimensionless vector potential for a circularly polarized laser pulse represents a standing wave with the amplitude $2a_0$ in the downstream region. The unperturbed ion density $n_{0i}$ is upstream, $x' > x'_s$. At position $x' = x'_s$, the ion density undergoes a jump from $n_{0i}$ to $2n_{0i}$. In the interval between, $0 < x' < x'_s$, there is an overlap of the particles moving towards the piston and reflected from it. The ions are decelerated passing from $x' = 0$ to $x' = x'_f$. Then they reverse their motion and are accelerated from the piston. These two steps correspond in the laboratory frame to acceleration from a zero velocity to the velocity $v_f$ and then to $v_i$.

The conservation of the ion particle flux provides a relation between the ion density and ion velocity for incoming and reflected ions. The ion energy conservation, $\varepsilon'_i + Z e \Phi' = m_i c^2 (\gamma_f - 1)$, provides the value of the potential jump in this layer, $Ze \Phi'(x'_f) = m_i c^2 (\gamma_f - 1)$, needed to stop and reflect the ions. The distribution of the electrostatic potential follows from the Poisson equation rewritten as an equation for the ion relativistic factor in the piston reference frame

$$d^2 \gamma'_i / dx'^2 = 2\omega_{pi}^2 \gamma_f \beta_f / \beta'_i c^2;$$

where $\beta'_i = (1 - \gamma'_i^{-2})^{1/2}$ and $\omega_{pi} = (Z^2 e^2 n_{0i} / m_i \varepsilon_0)^{1/2}$ is the upstream ion plasma frequency. Then the first integral of Eq. (1) provides a relation between the electric field and the ion energy in the charge separation layer. In particular, at the position of the electron peak the electrostatic field reads

$$E_x(0) \simeq (2/Z \varepsilon_0) m_i c \omega_{pi} \gamma_f \beta_f = 2\sqrt{\alpha} E_0 \sqrt{(1 - \beta_f) / (1 + \beta_f)}.$$  

Within a numerical factor, it agrees with the estimate obtained in Ref. [2]. The expression (2) shows that the maximum electrostatic field in the piston is approximately equal to twice the electric field of the incident laser pulse at small piston velocities. By integrating Eq. (1) once more, we define the position of the ion turning point and the thickness of the ion sheath.
layer, \( \Delta_i' = -x_i' \). In the non-relativistic piston regime, \( \beta_f \ll 1 \), the width of the ion separation layer is \( \Delta_i \approx v_f/3\omega_{pi} \). One can also find the ion circulation time in the charge separation layer \( t_i \approx 2\gamma_f/\omega_{pi} \). In the non-relativistic piston regime, \( \beta_f \ll 1 \), it depends only on the ion plasma frequency. Within a numerical factor, the formula for \( t_i \) agrees with the estimate for the ion acceleration time in Ref. [2, 3].

The solution for the laser piston velocity is stable in the sense that increasing \( v_f \) would induce stronger plasma momentum deposition and slowing down the piston. In the opposite, a decrease of the piston velocity below the stationary level would induce a stronger photon momentum deposition and increase the acceleration of the piston. Nevertheless, PIC simulations [2, 3] show that at super-high laser intensities the laser piston exhibits regular oscillations around the stationary solution. A detailed analysis of our PIC simulations shows that \( t_i \) defines the characteristic period of these oscillations.

The analytical results on the piston structure were compared with the PIC simulations taking into account electron radiation losses [4], that could be important for super-high laser intensities. Simulations were carried out in 1D geometry. The particle motion is relativistic with three components of the electron and ion momenta and the correspondent components of electric and magnetic fields. The radiation losses of electrons in the electromagnetic field of the laser wave are calculated for each electron on each time step. The model was tested against several known exact solutions and demonstrated its robustness and accuracy.

In the run with \( a_0 = 20 \) and an initial deuterium plasma density \( n_{0i} = 20n_c \) we verified the predictions of the piston model described above and elucidate the deviations from the model related to the dynamics of individual particles. The simulation results are in reasonable agreement with the model. In this particular case the reflection coefficient \( R = 0.87 \) agrees with the Doppler shift of the reflected light \( 0.13 \omega_0 \). The expected value of the ion momentum, \( p_{x,i} = 0.138 m_i c \), is in agreement with the simulation, where it varies in the range \( (0.12 \div 0.15) m_i c \). The simulation demonstrates periodic oscillations in the reflected light, in the ion momentum, and in other characteristics. The period of these oscillations is very close to the characteristic time of ion acceleration \( t_i \), which is calculated within the stationary model.

As a result of oscillations of the laser piston velocity and of the electric field, the ion momentum distribution is broadened and the correspondent energy distribution acquires a width \( \sim 10\% \) around the estimated value of 18 MeV. The major part of electrons is ejected upstream from the piston. At the final instant of the simulation at \( t = 100T_0 \), 13% of the laser energy is absorbed, 12.8% is converted to ions and only 0.2% – to electrons. This is in good agreement with the expected laser energy transformation to ions estimated as 12.9%.
In the case of a lower plasma density, \( n_0 = 10 n_c \), a laser pulse with the amplitude \( a_0 = 100 \) and a circular polarization, we observe a more turbulent interaction. A small part of ions breaks out and approaches the energy of 600 MeV, which is in the expected range of 500 MeV. However, the major part of ions is traveling with the pulse, and these ions are gaining a much lower energy \( \sim 110 \) MeV. The overall performance of the laser piston is less than the expected 51\% at the end of simulation. The ions are acquiring 31.4\% of the laser pulse energy, while the electron part increases to 5.7\%. This reduced performance is also supported by increasing radiation losses in the range above 100 keV that consume 43\% of the laser energy, while in the previous case their role was insignificant, less than 0.1\%.

Comparing both examples, strong electron heating can be observed in the latter case. We evaluate the effective electron energy \( \langle \varepsilon_{x,e} \rangle \approx 90 \) MeV and the piston Mach number \( M \approx 1.5 \). Such a low value explains the fact that the double layer cannot stop all the particles and some electrons and ions are observed in the downstream region. The energy conversion into electrons and into high-frequency radiation increases significantly. The energy conversion into electrons is 14.4\% and the radiation losses amount to 48.4\%. The radiation losses depend very strongly on the laser intensity. For the low intensity case, \( 1.6 \times 10^{21} \) W/cm\(^2\) the radiation losses are relatively small, less than 1\% even at linear polarization. However, for the higher intensity case, \( 4 \times 10^{22} \) W/cm\(^2\) and for the lower density, they consume almost half of the laser pulse energy. It is necessary to note, that electrons escaping the piston, can gain and lose a significant part of their energy in interaction with incident and reflected light and with the electrostatic field. For this reason, the evaluation of the energy losses in this case is not straightforward.

**Acknowledgements**

This work is supported by the ANR under the contract No. BLAN07-3-186728, by the Region Aquitaine under project No. 34293, by the European support program Marie Curie IRSES project # 230777, and by the HiPER and ELI European projects.

**References**


