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Quasi-Classical Theory of the Radiative–Collisional Cascade in Rydberg Atoms

M.B. Kadomtsev, M.G. Levashova, V.S. Lisitsa, N.N. Nagel

NFI RRC "Kurchatov Institute", Moscow, Russia

Highly excited (Rydberg) atomic states are observed both in laboratory and astrophysical plasmas. We can mention recent observations of radiorecombination lines, caused by radiative transitions between hydrogen levels with principal quantum numbers $n \sim 100$ in astrophysical plasmas [1,2], the recombined electrons with n as large as ~ 1000 - 2000 in the experiments with highly charged ions in the storage rings [3], highly excited atomic states produced by direct laser excitation [4]. For $n >> 1$, the system of quantum-mechanical kinetic equations for atomic level populations in n and l (orbital quantum number) becomes too complicated for numerical solution. On the other hand, for large n the transition probabilities are described quite well by the quasi-classical approximation, and the problem is reduced to a single kinetic differential equation in energy and angular momentum space that gives transparent picture of evolution and makes the quasi-classical approach much more preferable.

We developed a two-dimensional (in n and l) quasi-classical model of the radiative–collisional cascade for hydrogen- like systems in plasmas [5-7]. The structure of the quantum kinetic equation for the radiative-collisional cascade and its iterative solution is as following:

$$\left[\hat{L}_C + \hat{L}_r^q\right] f(n,l) + q(n,l) = 0; \quad \hat{L}_c f_k(n,l) - A(n,l) f_k(n,l) + q_k = 0, \tag{1}$$

$$q_k \equiv \sum_{n'=n+1} \sum_{l'=l\pm 1} f_{k-1}(n',l') A(n',l' \to n,l) \tag{2}$$

where $\hat{L}_c$, $\hat{L}_r^q$, are collisional and radiative transition operators respectively, $f(n,l)$ is population distribution function in nl-space, and $q(n,l)$ is an external source of atomic state population, $A(n,l)$ is radiative decay rate. The distribution f is presented in a form of series $f = f_0 + f_1 + f_2 + \ldots + f_k + \ldots$, that corresponds to a successive emission of quanta. Each term f_k of this series can be determined from the Eqs.(1, 2) using f_{k-1} calculated at the previous step, while f_0 can be found from Eq.(1) by substituting the original source.

The collisional diffusion coefficients in nl-space were determined by calculating the transfer of energy and angular momentum during a classical collision of the atomic electron with charged plasma particles [5-7]. We took account of both the collisions with plasma electrons, which are mostly inelastic, and collisions with ions, which are mostly elastic for low ion velocities with respect to those of bound atomic electrons. Following [8] we introduce the parameter $n_0 = [3z^6\sqrt{T_e}\,/\,4\sqrt{2\pi}Le^6c^3N_e]^{1/8}$ (in atomic units), which is the principal quantum number for which the inelastic collisional and radiative rates are equal. N_e, T_e are the electron density and temperature respectively, L is the Coulomb logarithm.

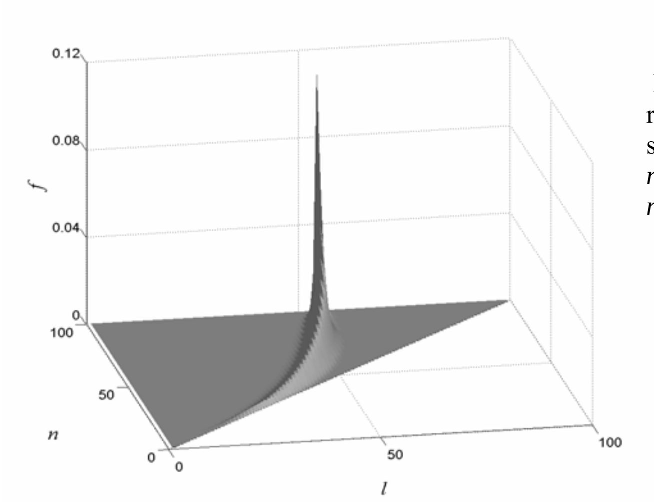

Fig. 1. Electron distribution function for quantum radiative cascade originating from a selective source which maintains f = 1 at the state with n_{s1}=100 and l_{s1}=55. (In the source point n,l = 100,55 one has f =1, not shown here)

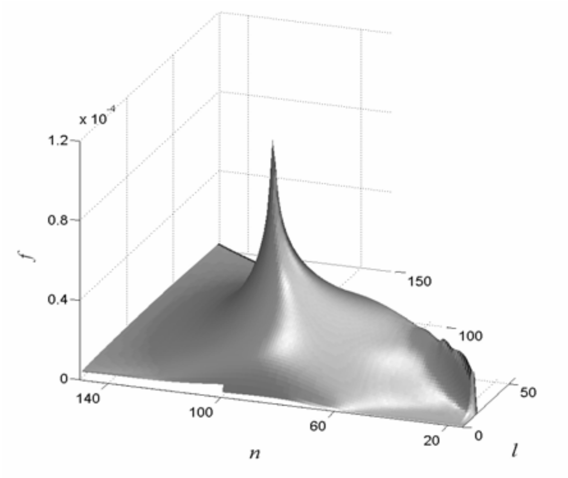

Fig. 2. Radiative cascade distorted by collisions with plasma electrons. Source is the same as in Fig.1.

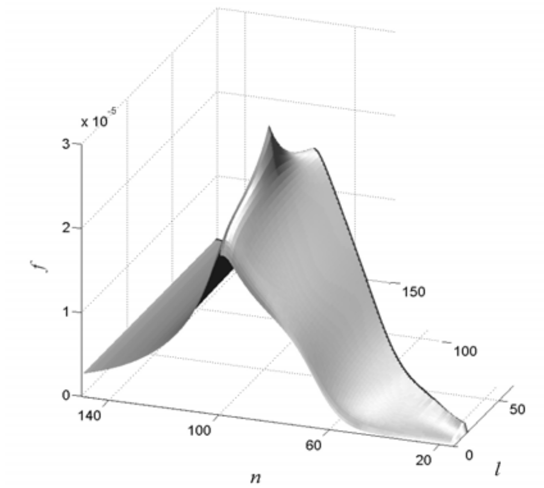

Fig. 3. Distribution function for radiative-collisional kinetics with collisions of bound electrons with plasma protons and electrons. Purely radiative cascade picture is almost completely distorted by collisions. Source is the same as in Fig.1.

We calculated populations of atomic states for a selective (delta-function) source (Fig.1-3). Figure 1 corresponds to purely radiative cascade (i.e. without collisions), when the radiative cascade flows along the respective classical characteristic. Including the collisions with electrons (Fig. 2) leads to a strong mixing of populations over principal and orbital quantum numbers. Including the collisions with ions (Fig. 3) leads to additional l-mixing of the level populations. On the whole, Fig. 1-3 allows us to judge the relative role of the radiation and collisions of bound electrons with various plasma particles. We perform calculations for n_0~50. This corresponds, in particular, to the conditions of an astrophysical plasma: electron density $N_e \sim 10^3$ cm^{-3} and electron temperature $T_e \sim 1$ eV. This may also correspond to the conditions of tokamak plasmas if one considers the cascade in the impurity ions with charge $Z = 26$ (i.e. iron), $N_e \sim 10^{13}$ cm^{-3} and $T_e \sim 2$ keV.

We took account of three-body and photorecombination sources of population for a plasma of low density and moderate temperature corresponding to the conditions for recombination line observation in astrophysical plasmas [1,2]. Our calculations reveal a noticeable nonequilibrium in the population in orbital quantum number l (Fig.4).

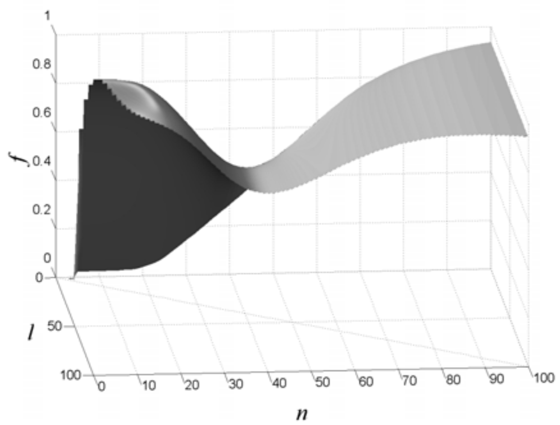

Fig. 4. Total two-dimensional distribution function when the three-body and photorecombination sources of population act simultaneously; $N_e = 2.5 \cdot 10^3$ cm^{-3} and $T = 1$ eV.

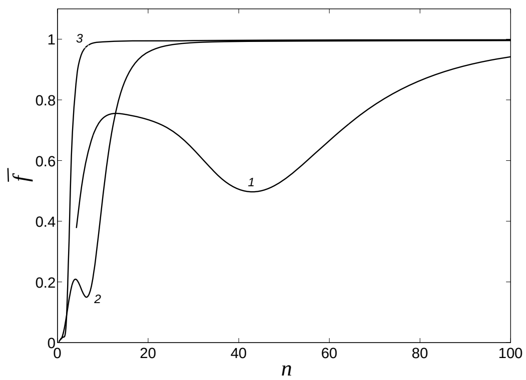

Fig. 5. Total (including the photorecombination and threebody sources of population) one-dimensional distribution function obtained by integrating the two-dimensional function over the variable l: $\bar{f} \equiv \langle f \rangle_l = 2\int f\, l\, dl / n^2$ at $T = 1$ eV and $N_e = 2.5 \cdot 10^3$ (1), 10^9 (2), and 10^{13} (3) cm^{-3}.

Comparison was made with one-dimensional calculations by averaging over the quantum numbers l (Fig.5). This comparison reveals a characteristic population minimum attributable to the competition between the collisional and radiative state populations. This minimum in the two-dimensional model is appreciably deeper than that in the one-dimensional models (see [1],.Fig.2.23).

Based on the populations found, we calculated the intensities of spectral lines in the range of transition frequencies corresponding to a certain spectral interval of observations (Fig.6). The dependences of the transition intensities on the principal quantum number of the upper level at fixed frequency ω of the observed transitions determined by the conditions for recombination line observation (see [1,2]). Figure 6 presents the line intensities for the transitions from the $n = 50$–100 levels at a frequency of the observed spectra near $\omega = 8 \cdot 10^{-6}$ at. units. Comparison of the intensities for statistically equilibrium and nonequilibrium (based on the above scheme) populations of the upper levels reveals a noticeable difference between them, which is important in interpreting the Rydberg spectra. The results allow us to judge the degree of nonequilibrium of the populations of Rydberg atoms in astrophysical plasmas.

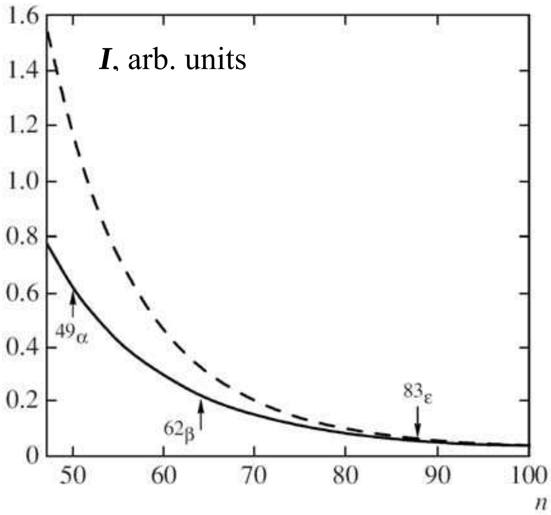

Fig. 6. Rydberg line intensities for the transitions from the n = 50–100 levels at fixed observation frequency $\omega = (1/n^2 - 1/n^2) = 8 \times 10^{-6}$ at. units corresponding to the transition with $\Delta n = 1$ between the 50 and 49 levels (the population minimum in Fig. 6) at $N_e = 2.5{\cdot}10^3$ cm^{-3} and T = 1 eV; the arrows indicate the positions of some Rydberg lines.

In summary, our two-dimensional quasi-classical model enabled us to calculate the radiative–collisional kinetics of electrons in Rydberg atoms in a transparent and simple form. This approach is effective for large quantum numbers since it allows us to avoid solving a set of quantum-mechanical kinetic equations for atomic level populations and improves the solution step by step by using the iterative procedure. The universality of Rydberg atomic states kinetics allows the use of the model for describing the atomic spectra in a wide range of plasmas.

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