DETERMINATION OF PAIR INTERACTION POTENTIAL FOR PARTICLES IN NON-IDEAL DISSIPATIVE SYSTEMS

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Information on the potentials of inter-particle interaction is necessary for an analysis of various physical characteristics of systems (pressure, energy, compressibility, viscosity, thermal conduction, etc.) [1]. Laboratory dusty plasma is a good experimental model for studying of non-ideal systems. At present time the problem of the form of the interaction potential between dust particles in plasma has no satisfactory solution.

Here we present a new technique for the determining of dust parameters in laboratory plasma, such as the interaction potential between dust grains and the external confining potential. The technique is based on a solution of the inverse problem describing movement of dust particles by a system of Langevin equations and allows taking into account the friction factors. A special feature of the solved problem consists in "irreversibility" of the Langevin equations. Nevertheless, in special cases we can reduce the random errors, associated with a stochastic (thermal) motion of particles; and the inverse problem may be solved by fitting the required parameters of a direct problem on particle movement to the information on displacement of particles that can be easily found both in numerical and in real experiments.

In contrast to the methods used in statistical theory of liquids [2], the technique proposed in this paper requires no additional assumptions on the relationship between the pair correlation function and the pair interaction potential and can be applied to analyzing strongly correlated systems of interacting particles.

The solution of direct problems was carried out by the Langevin molecular dynamics method based on the solving of differential equations with the stochastic force F_{ran}, which takes into account processes leading to the established equilibrium temperature T of particles that characterizes kinetic energy of their stochastic motion [3]. The system of N_p motion equations (N_p is a number of grains) include also the forces of pair interaction $\mathbf{F}_{int}$ and external forces $\mathbf{F}_{ext}$:

$$M\frac{d^2\vec{l}_k}{dt^2} = \sum_j F_{int}(l_{kj})\frac{\vec{l}_k - \vec{l}_j}{l_{kj}} + \vec{F}_{ext} - M\nu_{fr}\frac{d\vec{l}_k}{dt} + \vec{F}_{ran}\,. \tag{1}$$

Here $F_{int}(l) = -\frac{\partial U}{\partial l}$, and $l = \left|\vec{l}_k - \vec{l}_j\right|$ is the interparticle distance, ν_{fr} is the friction coefficient, and M is the particle mass. The calculations were carried out for various types of pair isotropic potentials U(l) that represented different combinations of power-law and exponential functions, commonly used for simulation of repulsion in kinetics of interacting particles:

$$U = U_c\,[c_1 \exp(-\kappa_1\, l/l_p) + c_2\,(l_p/l)^n \exp(-\kappa_2\, l/l_p)], \qquad (2)$$

Here $c_{1(2)}$, $\kappa_{1(2)} = l_p/\lambda_{1(2)}$ and n are variable parameters, l_p is the mean interparticle distance, λ_i is the screening length , and $U_c = (eZ)^2/l$ is the Coulomb potential. In the context of investigation of dusty plasma the particular interest are presented the screened Coulomb potential, $U = (eZ)^2 \exp(-l/\lambda_1)/l$ (c_1=1, c_2=0, $\kappa_1 = l_p/\lambda_1$), and the potentials with the power-law asymptotic of the screening weakens for large distances $l >> \lambda_1$: $U \propto l^{-2}$, or $U \propto l^{-3}$ ($\kappa_1 = l_p/\lambda_1$; κ_2 =0; n =1-2; $c_1 >> c_2$) [4-7].

Simulation were carried out in two-dimensional case, for monolayer of grains trapped in the linear electrical field, $\mathbf{F}_{ext} = eZ\mathbf{E}(r)$, $\mathbf{E}(r) = \alpha\mathbf{r}$, with radial symmetry (where r is the distance from a grain to the center of a trap, and α is the magnitude of gradient of electrical field $\mathbf{E}$). Motion equations (1) were solved for various values of effective parameters, namely the effective coupling parameter $\Gamma^* = 1.5 l_{pm}^2 U''(l_{pm})/(2T)$, and the scaling parameter $\xi = |U''(l_{pm})|^{1/2}(\pi M)^{-1/2}\,\nu_{fr}^{-1}$. Here l_{pm} is the most probable distance between the particles in the crystalline structure (with □ □* □ 110), which can be obtained from the pair correlation functions g(l). The value of effective parameter Γ^* was varied from 5 to 180, and the scaling factor ξ was changed in the range from 0.2 to 5, typical for the conditions of dusty plasma experiments. The number of interacting particles was varied from 2 to N_p = 500.

The solution of a direct problem (1) was recorded in the form of coordinates of separated particles for the different moments of time t_m. The velocity $\mathbf{V}_k$ and the acceleration $\mathbf{a}_k$ of a particle at the instant t_m were calculated as

$$\mathbf{V}_k(t_m) = \frac{d\vec{l}_k}{dt} \cong \{\mathbf{l}_k(t_{m+1}) - \mathbf{l}_k(t_m)\}/\Delta t; \quad \mathbf{a}_k(t_m) = \frac{d^2\vec{l}_k}{dt^2} \cong \{\mathbf{V}_k(t_{m+1}) - \mathbf{V}_k(t_m)\}/\Delta t. \qquad (3)$$

Here □t = (t_{m+1} - t_m) is the time step of the solution of direct problem. Thus, the data of numerical experiments used for the solving of the inverse problem was similar to the data, which is usually registered by a video camera in real laboratory experiments.

To restore the force of pair interaction $F \equiv F_{int}$ and, respectively, the pair potential U we used the power expansion of the type

$$F = \sum_{i=1}^{I_p} a_i l^{-i-1}; \quad U = \sum_{i=1}^{I_p} \frac{a_i l^{-i}}{i}. \qquad (4)$$

Here a_i are the expansion coefficients, and I_p is the number of expansion terms. The total force F_{pp} acting on the k- particle from the other particles of a dust cloud, can be written as

$$\vec{F}_{pp}{}^{k} = \sum_{j=1,\, j\neq k}^{N_d-1} \sum_{i=1}^{I_p} a_i \frac{\vec{l}_k - \vec{l}_j}{l_{kj}{}^{i+2}}. \qquad (5)$$

The force F_{pt} acting on the k- particle by an external field was approximated by polynomial

$$\vec{F}_{pt}{}^{k} = \vec{r}_k \sum_{i=1}^{I_t} b_i r_k^{i-1}. \qquad (6)$$

Here b_i are the expansion coefficients, r_k is the distance from the k -particle to the trap center, and I_t is the number of expansion terms. Thus, the inverse problem is reduced to finding unknown coefficients a_i , b_i and v_{fr} of the linear system of motion equations for each analyzed particle for various moments of t_m during the whole experiment

$$M\vec{a}_k = -\nu_{fr} M\vec{V}_k + \vec{F}_{pp}{}^{k} + \vec{F}_{pt}{}^{k}. \qquad (7)$$

The inverse problem was solved by fitting the parameters of Eq. (7) to the numerical data by the least-squares method. To avoid the influence of random forces, the number of equations in the analyzed system was much larger than the number of unknown parameters of the inverse problem.

The solutions of the inverse problem are presented in Fig.1 for different parameters (N_p, Γ^* and ξ). The obtained interaction forces F (and, respectively, U) correspond to the initial functions in the range of analyzed particle motion trajectories for the distances $l \leq L_c$. Here L_c is the characteristic size of the dust cloud (its radius). Errors in determining friction coefficient and gradient of external field in case of solving the inverse problem for $\Delta_c t = \Delta t$ were less than 5 %. ($\Delta_c t$ is the step of recording of particle coordinates in numerical experiment.)

A first experimental approbation of proposed technique was carried out for dust clusters consisting of N_p =11 (at the pressure P = 0.03 Torr) and N_p = 29 (P = 0.06 Torr) grains and for part of dust cloud (~550 registered grains, P = 0.045 Torr). The video-record was processed by the special software, which allowed the identification of positions for each particle in the field of video-system view.

The pair forces F(l) measured by the technique proposed in this work are plotted in Fig.2 with an approximation of experimental data by the curves $f(l) \propto l^{-2}$. Characteristic frequencies of dust oscillations ($\omega_c = [2.7|F'(l_p)|/(\pi M)]^{-1/2}$), calculated from measured F(l) functions and obtained by the independent technique [8], are in a good accordance (in the limits of experimental errors ~10-15%). The obtained power law $F(l) \propto l^{-2}$ may be related to a weak screening of dust under experimental conditions, or can provides a proof of validity of the Wigner-Seitz-cell model in the ordered systems with $\Gamma = (eZ)^2/(l_p T) >> 1$ [6, 9].

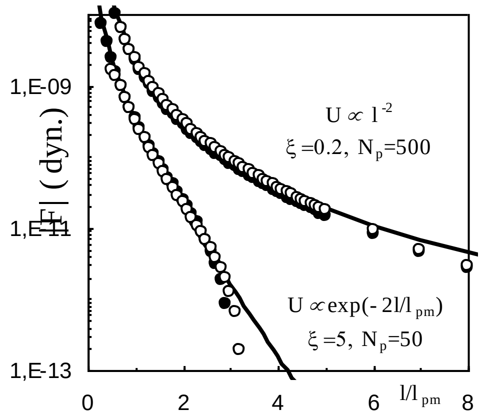

Fig.1. The □F(l/l_{pm})□ functions for dust clouds, consisted of N_p particles, for different U and □ . Lines are the given potentials; symbols are the results of □F□ reconstruction for various □*: (□)- 7.5; (□) - 180.

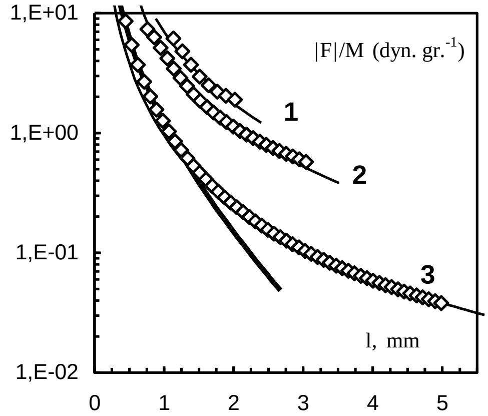

Fig.2. The |F|/M – functions (symbols) for various experiments: **1** – cluster N_p=11, **2** – cluster N_p=29, **3** – part of dust cloud N_p~550. Fine lines are the approximations of experimental data by the $f \propto l^{-2}$; the heavy line – by the $f \propto \exp(-l/l_{pm})(1+ l/l_{pm})l^{-2}$.

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