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Dynamo effect with shear flow in helical MHD turbulence

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Astrophysical magnetic fields are often studied with (single fluid) magnetohydrodynamics equation. In the kinematic limit, the backreaction of the magnetic field on the velocity is neglected. From the physical point of view, this amounts to considering a very weak magnetic field and ignoring the Lorentz Force on the fluid which is quadratic in the magnetic field. For an incompressible conducting fluid, the resulting equations of motion are:

$$\begin{aligned} \partial_t \mathbf{U} + \mathbf{U}\cdot\nabla\mathbf{U} &= -\nabla p + \nu\Delta\mathbf{U} + \mathbf{f}\,, \\ \partial_t \mathbf{B} + \mathbf{U}\cdot\nabla\mathbf{B} &= \mathbf{B}\cdot\nabla\mathbf{U} + \eta\Delta\mathbf{B}\,, \\ \nabla\cdot\mathbf{U} &= \nabla\cdot\mathbf{B} = 0\,. \end{aligned} \tag{1}$$

Here $\mathbf{B}$ is the magnetic field given in units of Alfvén speed, p is the total (hydrodynamical + magnetic) pressure. Here, we studied the turbulence driven by a helical forcing $\mathbf{f}$ with a shear of arbitrary strength. Note that previous studies of the effect of a large-scale shear on the dynamo process were restricted to the limit of a weak shear as the shear was treated perturbatively.

To explain the occurrence of magnetic field on large scales, the prevailing theory is mean-field dynamo [6, 4]. In this framework, the magnetic and velocity fields are decomposed into mean and fluctuating parts: $\mathbf{B} = \langle\mathbf{B}\rangle + \mathbf{b}$ and $\mathbf{U} = \langle\mathbf{U}\rangle + \mathbf{u}$, where the $\langle\bullet\rangle$ stands for an average on the realisation of the small-scale fields. The large-scale magnetic field is then governed by the following equation:

$$\partial_t \langle\mathbf{B}\rangle + \langle\mathbf{U}\rangle\cdot\nabla\langle\mathbf{B}\rangle = \langle\mathbf{B}\rangle\cdot\nabla\langle\mathbf{U}\rangle + \eta\nabla^2\langle\mathbf{B}\rangle + \nabla\times\mathscr{E}\,. \tag{2}$$

The first term on the RHS of Eq. (2) represents the stretching of magnetic field lines by gradient of the mean flow ($\nabla\langle\mathbf{U}\rangle$) and is called the Ω effect. It is an efficient mechanism to create toroidal field from a poloidal field in a system with differential rotation [6]. The term $\mathscr{E} = \langle\mathbf{u}\times\mathbf{b}\rangle$ is the electromotive force, which is often taken to be linear in the mean magnetic field ($\langle\mathbf{B}\rangle$) with the following expansion:

$$\mathscr{E}_i = a_{ij}\langle B_j\rangle + b_{ijk}\frac{\partial\langle B_j\rangle}{\partial x_k} + \dots\,. \tag{3}$$

In the kinematic limit where the magnetic field does not backreact on the velocity field, the tensors a_{ij} and b_{ijk} depend only on the properties of the velocity field. In this contribution, we consider an uniform magnetic field and therefore only the first term on the right-hand side will be of interest. Following [7], this first term can be rewritten as:

$$\mathscr{E}_i = \alpha_{ij}\langle B_j\rangle - (\gamma \times \langle \mathbf{B}\rangle)_i \tag{4}$$

The first term on the right-hand side is the α effect, which is an alternative mechanism to the Ω effect for the amplification of large-scale magnetic fields. The second term on the right-hand side of Eq. (4) describes the transport of mean magnetic flux by turbulence. It was shown by previous authors [4, 7] that anisotropy or inhomogeneity combined with rotation or large-scale shear flow can give rise to the α effect. Thus, this effect is a perfect candidate to explain magnetic fields in systems influenced by Coriolis force (which produces a net helicity) such as in stellar convection zones. On the other hand, an anisotropic or inhomogeneous turbulence alone was shown to generate a magnetic flux transport [2].

To compute these coefficients, we need to solve the equations for the fluctuating parts and compute the electromotive force. To keep the calculation analytically tractable, we prescribe a large scale flow of the form $\langle \mathbf{U}\rangle = -x\mathscr{A}\mathbf{e_y}$ and a uniform large-scale magnetic field $\langle \mathbf{B}\rangle$. To solve the equations for the fluctuating velocity field, $\mathbf{u} = \mathbf{U} - \langle \mathbf{U}\rangle$, and magnetic field, $\mathbf{b} = \mathbf{B} - \langle \mathbf{B}\rangle$, we use the quasi-linear approximation assuming that the interaction between fluctuating fields is negligible compared to the interaction between large and small-scale fields. The equations for the fluctuating fields can then be written as:

$$\begin{aligned}
\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\langle \mathbf{U}\rangle &= -\nabla p + \nu\Delta\mathbf{u} + \mathbf{f}\,, \qquad (5)\\
\partial_t \mathbf{b} + \mathbf{u}\cdot\nabla\langle \mathbf{B}\rangle + \langle \mathbf{U}\rangle\cdot\nabla\mathbf{b} &= \mathbf{b}\cdot\nabla\langle \mathbf{U}\rangle + \langle \mathbf{B}\rangle\cdot\nabla\mathbf{u} + \eta\Delta\mathbf{b}\,,\\
\nabla\cdot\mathbf{u} &= \nabla\cdot\mathbf{b} = 0\,.
\end{aligned}$$

Note that since the first equation of (5) does not involve the magnetic field, the solution for the fluctuating velocity is the same as in the hydrodynamical case given in [3]. Then the magnetic field can be obtained from the second equation of (5) by integrating the velocity field [5].

To compute the electromotive force (4), we consider a short correlated (with correlation time τ_f), homogeneous and isotropic forcing. Specifically, in Fourier space, the correlation function of the forcing is taken as:

$$\langle \tilde{f}_i(\mathbf{k_1},t_1)\,\tilde{f}_j(\mathbf{k_2},t_2)\rangle = \tau_f\,(2\pi)^3\delta(\mathbf{k_1}+\mathbf{k_2})\,\delta(t_1-t_2)\phi_{ij}(\mathbf{k_2})\,, \tag{6}$$

In order to study the α effect, we consider a helical forcing with a spectrum function given by:

$$\phi_{lm}(\mathbf{x},\mathbf{k}) = \frac{E(k)}{8\pi k^4}(k^2\delta_{ij} - k_i k_j) + i\varepsilon_{lmp}k_p\frac{H(k)}{8\pi k^4}\,. \tag{7}$$

The first term on the right-hand side is the energy part of the forcing whereas the second part is the helical part. The energy and the helicity part of the forcing are given respectively by:

$$e_0 = \langle f^2 \rangle = \frac{\tau_f}{(2\pi)^3} \int_0^{+\infty} dk E(k) , \quad (8)$$
$$h_0 = \langle \mathbf{f} \cdot (\nabla \times \mathbf{f}) \rangle = -\frac{\tau_f}{(2\pi)^3} \int_0^{+\infty} dk H(k) ,$$

Turbulence intensity:

to show the effect of a large-scale shear flow on turbulence, we compute the three components of the turbulent kinetic energy $\langle u_x^2 \rangle$, $\langle u_y^2 \rangle$ and $\langle u_z^2 \rangle$. The dependence of these three quantities on the shear is shown on Figure 1 which shows that when the shear is increased, the turbulence is severely reduced. This is due to shear stabilisation: flow shear facilitates the cascade of various quantities such as energy to small scales, enhancing the dissipation rate [1] and thus leading to weak turbulence. Furthermore, turbulence becomes anisotropic with the turbulence in the x direction (the direction of the flow shear) more severely reduced than the one in the y and z direction.

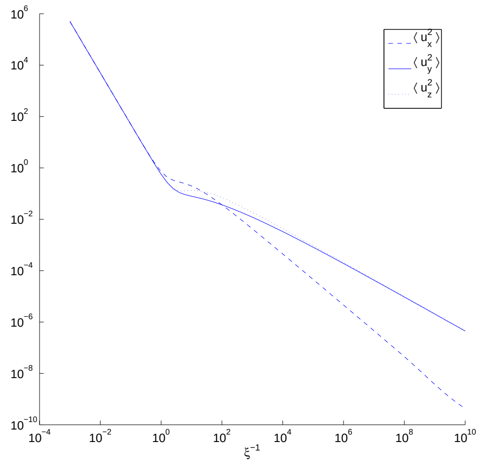
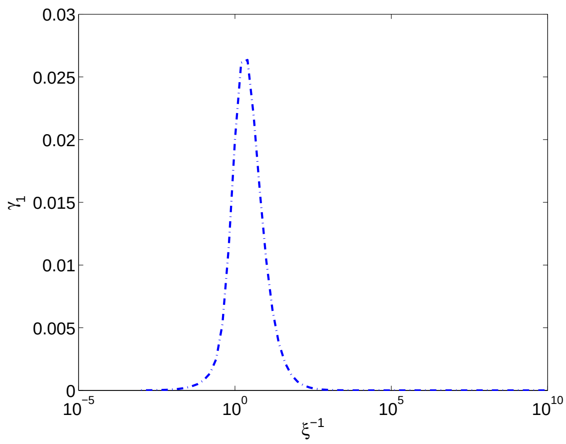


Figure 1: Dependence of the turbulence intensity on the shear. The parameter values are fixed to be: $a = 1$ and $\beta = 0$.

Transport of magnetic flux (γ effect): the vector $\gamma_i = \varepsilon_{ijk} a_{jk}/2$ is obtained as:

$$\gamma = -\frac{\tau_f}{(2\pi)^3} \int d\mathbf{k} \, \frac{H(k)}{8\pi k^2} \begin{pmatrix} 0 \\ 0 \\ \gamma_1 \end{pmatrix} . \quad (9)$$

This shows that the γ effect exists only with an helical forcing (as it is proportional to the helical part of the spectrum $H(k)$). Figure 2 shows the dependence of γ_1 on the shear. It shows that without shear $\gamma_1 = 0$ as in this case, there is no anisotropy in the system and the tensor a_{ij} must therefore be diagonal [7]. For non vanishing but weak shear, we find that the γ coefficient is increasing with shear. This is due to the fact that shear introduces anisotropy in the turbulence: the turbulence fluctuations in the direction of the shear (x) are weaker than in the perpendicular direction (y or z) as shown by [3]. However, for strong shear, we find that the γ effect is decreasing with shear, ultimately vanishing for very strong shear. Using the asymptotic behaviour, the γ effect is behaving as $\mathscr{A}$ and $\mathscr{A}^{-4/3}$ for weak and strong shear, respectively.



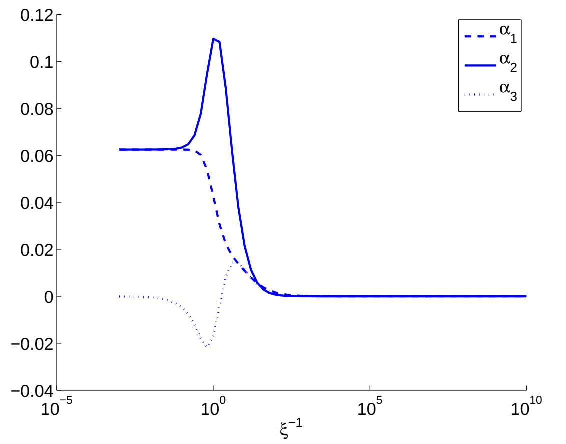


Figure 2: Dependence of the coefficient γ_1 (left) and the coefficient α_i's for the α effect (right) with the shear. The parameter values are fixed to be: $a = 1$ and $\beta = 0$.

α **effect**: the symmetric part of the tensor a_{ij} is the α tensor:

$$\alpha_{ij} = -\frac{\tau_f}{(2\pi)^3}\int d\mathbf{k}\,\frac{H(k)}{8\pi k^2}\begin{pmatrix} \alpha_1 & \alpha_3 & 0 \\ \alpha_3 & \alpha_2 & 0 \\ 0 & 0 & \beta^2\alpha_2 \end{pmatrix}_{ij}. \tag{10}$$

Figure 2 shows that these three coefficients are increasing with shear for weak shear while decreasing with shear for sufficiently strong shear. It can be shown that the diagonal part of the tensor is non-vanishing for $\mathscr{A} = 0$ (and in that case α_1=α_2) whereas the non diagonal part vanishes ($\alpha_3 \sim \mathscr{A}$ for weak shear). Similarly to the γ effect, the presence of a weak shear introduces anisotropy in the system and therefore makes the α tensor non diagonal. On the other hand, all the components of the α effect are reduced for strong shear with the following scalings: $\alpha_1 \sim |\ln\mathscr{A}|\mathscr{A}^{-4/3}$, $\alpha_2 \sim \mathscr{A}^{-5/3}$, and $\alpha_3 \sim \mathscr{A}^{-4/3}$.

In summary, we found that weak shear generates γ effect and non-diagonal components in the α effect and also enhances the diagonal components of the α effect. We also showed that a strong shear quenches turbulence intensity, γ and α effect.

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