

Magnetic Flux Loop Design for NCSX

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The NCSX stellarator is a 3-field period quasi-axisymmetric device under construction at PPPL. First plasma is scheduled for 2009. A substantial effort has gone into designing a comprehensive set of magnetic flux loops, mounted on the vacuum vessel (VV), whose main purpose is the diagnosis of plasma equilibrium magnetic fields. Signals are expected to be predominantly stellarator symmetric (SS) with toroidal modenumbers, n , per torus equal to a multiple of 3. However plasma instabilities and coil imperfections will generate non-SS fields with $n = \pm 1, \pm 2, \pm 4, \pm 5, \dots$. Novel concepts introduced in the design work include -- the use of SVD techniques applied to magnetic signals from a database of equilibria to select optimally placed flux loops, and -- randomly distributing flux loops among adjacent symmetry locations to better discriminate between asymmetric perturbations.

Goals for the NCSX flux loop design include: (a) Loops should be effective in reconstructing both SS and non-SS equilibrium fields; (b) Loops should be concentrated in regions where the plasma contribution to the total signal is significant (easily detected), and distinguishable from the measured total signal; (c) Loops should be sufficiently numerous to provide adequate spatial resolution of the modes; (d) Loops should be sufficiently large in area to provide a useful signal, yet be consistent with goal (c); (e) A subset of the loops should have the capability of resolving $n=3, m=5$ or 6 resonant field perturbations. These goals were folded in to the NCSX flux loop design, leading to a final design with 225 flux loops at specific locations on the VV torus.

Essential to the design analysis was the generation of a large database of 2500 SS free-boundary VMEC equilibria incorporating random combinations of a variety of current and pressure profiles. Equilibria span the range of plasma parameters and shapes that can be achieved in NCSX. Using codes V3RFUN and V3FIT [1], the flux distribution over one half-period of the VV surface was calculated using a trial set of 100 flux loops that completely tile the surface. A least-squares regression of the flux loop signals was performed on magnetic field values from the plasma and coils at a mesh of points on the surface of a close-fitting vacuum toroidal surface (TS) surrounding all of the equilibria. Two Singular Value

Decomposition (SVD) algorithms were developed for ranking the effectiveness of the flux loops in reconstructing the equilibrium magnetic fields on the TS. The first algorithm (essentially Jolliffe's B2 method [2]) is a "rejection" based procedure where examination of the eigenfunction corresponding to the smallest SVD eigenvalue leads to rejection of a single diagnostic from the trial set. The rank of the signal matrix is then reduced by eliminating the row corresponding to the rejected diagnostic and an SVD of the reduced matrix is performed. The procedure is repeated, rejecting one diagnostic after each SVD until the Total Least Squares (TLS) regression fitting error exceeds a tolerable minimum, leaving a minimal set of acceptable diagnostics. The second method is a "retention" based algorithm. A truncated SVD regression analysis identifies the top few (4-8) "most important" diagnostics in the trial set. These diagnostics are placed in a reserved pool and SVD regression is performed on the reduced matrix formed by eliminating columns of the diagnostics signal matrix corresponding to the reserved diagnostics. The procedure is repeated until the TLS regression fitting error exceeds the tolerable minimum. Highly ranked diagnostics identify regions of the VV surface that are important for placing actual flux loops.

Normalizing toroidal fields in the equilibrium database to 1.5T, the average (over the equilibria) magnitude of the normal component of the magnetic field, B_n^{plas} , at the VV, due to the plasma currents is calculated to be $< 0.064\text{T}$. Plots of the ratio $B_n^{\text{plas}}/B_n^{\text{total}}$ subject to the condition $B_n^{\text{plas}} > 0.01\text{T}$ provide guidance for placement of flux loops that meet design goal (b). A comforting correlation is found between regions of the VV surface that satisfy this condition and regions where trial flux loops are calculated to have high rank by the SVD analysis.

The plasma-to-VV separation distance, d_{sep} , in NCSX is a non-uniform function of poloidal and toroidal angle. For example, at the $\phi = 0$ (bean) symmetry plane d_{sep} varies between approximately 35cm at the outboard midplane and 5cm at the inboard midplane. At the $\phi = \pi/3$ (bullet) symmetry plane, the d_{sep} variation is 6 – 11cm. An analytic estimate of plasma-to-VV signal attenuation based on a slab model and rectangular flux loops implies that the minimum loop dimension should be of order d_{sep} for reasonable detection of the signal.

A dedicated sub-array of loops was designed to optimize the possibility of detecting resonant field perturbations, particularly those responsible for generating $n=3$, $m=5$ or 6 islands. To

this end, a 128x128 array of dipoles was uniformly distributed on the VV and a response matrix calculated between individual dipole currents and the radial magnetic field generated on resonant surfaces of a baseline VMEC full-current, full-beta equilibrium. SVD inversion of the response matrix and appropriate projection onto individual resonant harmonics provides patterns on the VV showing regions where the dipoles are effective in generating magnetic islands. By reciprocity, these effective dipole regions also define the location and size of flux loops that are optimally suited for detecting resonant fields. These are regions along the inside wall of the vertically elongated cross-section of the VV.

To provide a poloidally continuous array of flux loops 16 equally spaced flux loops will be wound on one of the three spacers that connect adjacent periods of the VV. Two of these loops surround symmetry points of the VV torus and will be useful for detecting departures from SS. Loops will also be wound around each of the two symmetry points on the remaining two spacers. Additional sub-arrays have been chosen to provide continuous flux loop coverage in the toroidal direction on the inboard and outboard mid-planes. Of the loops on these toroidal arrays, six surround symmetry points.

Combining the results from the described analyses, 225 two-turn saddle flux loops will be installed on the NCSX vacuum vessel and spacers. Of this number, 151 have distinct shapes and have the primary function of resolving SS modes. These loops can be placed in one of the six available half-periods of the VV, and essentially tile the half-period. Since the signal from a loop that is mirror-reflected and/or translated toroidally by ± 1 period is unchanged, loops for detecting SS modes can be distributed to equivalent locations over the full torus without compromising the ability to detect these modes. From the point of view of resolving non-SS (NSS) modes, however, there is no such equivalence. Distributing loops over the full torus is, in fact, essential for NSS mode detection. Calculations of the condition number (ratio of largest to smallest singular values) of the matrix of NSS mode signals for various spatial distributions of the flux loops show that scattering all 225 loops over all 6 half periods but sampling each of the 151 SS positions at least once is the preferred choice, and leads to the most robust inversion of the diagnostic signals. This is illustrated in Figure 1 where the condition number is compared for a model calculation where flux loops are arrayed in a single half-period of the VV (curves c2, c4), and where flux loops are distributed randomly over the full torus (curves c1, c3). NSS modes with $m \leq 3$, $|n| \leq 3$ are included in the calculation. In the case of the curves c1 and c2, the NSS modes are ordered according to the computed value of

the “relevance measure” $1/[m^2 + n^2]$, whereas for curves c3 and c4, a “natural” ordering is determined by iteration such that the local slope of the condition number curve is minimized.

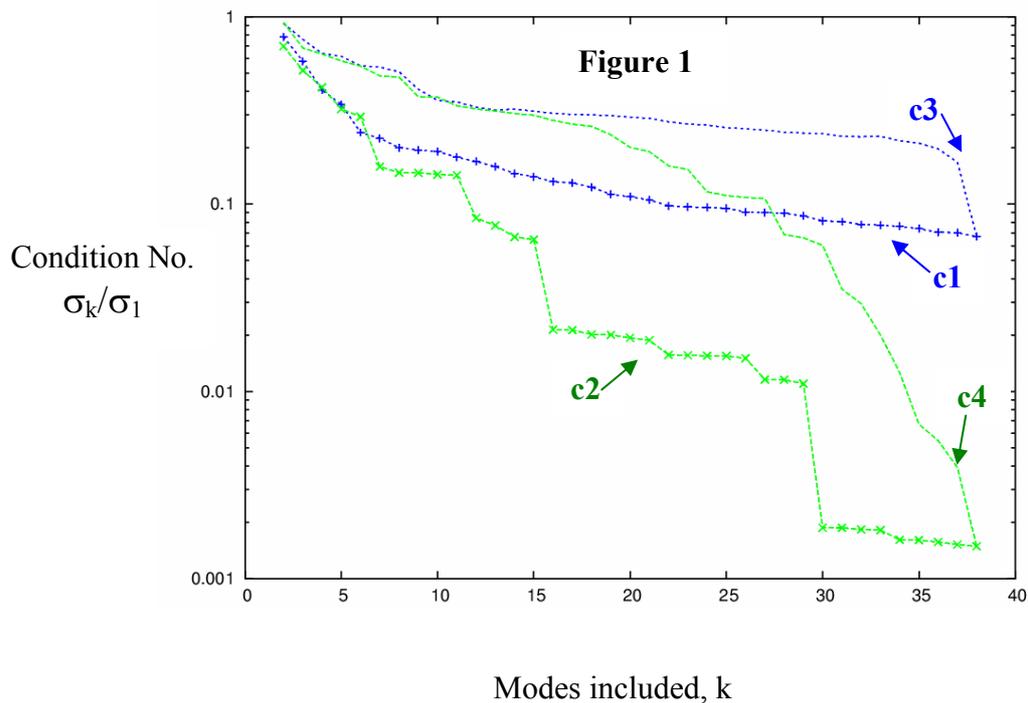
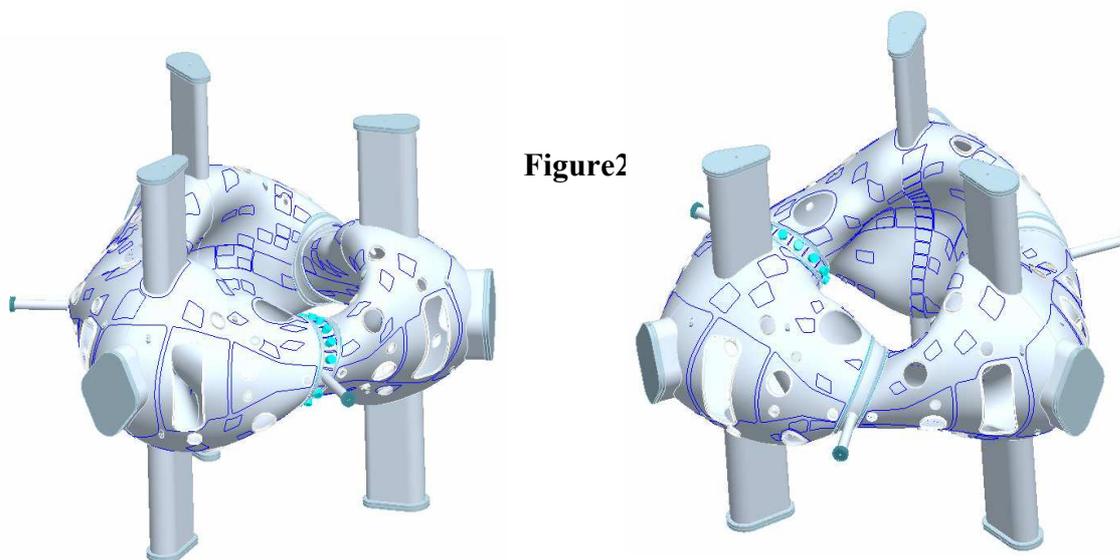


Figure 2 shows the final design choice for VV flux loops for NCSX. A more complete discussion the physics design of the flux loop system will be presented elsewhere.



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References

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