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Slab ITG modes in the presence of a sheared ion velocity

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Abstract

We have developed a one-dimensional theoretical model for slab ITG modes in the presence of an inhomogeneous equilibrium fluid velocity of the ions along the main magnetic field direction. Additional physical effects like magnetic field gradient and curvature are included *via* a gravitational drift. The effects of magnetic shear is also taken into account. An extended analysis of the relevant dispersion relation and of the associated quasilinear fluxes is presented.

Introduction

A theory of spontaneous momentum generation due to the toroidally asymmetric excitation of radially localized ion temperature gradient (ITG) modes [1], in the presence of a radial profile of the toroidal ion velocity, has been formulated by B. Coppi [2]. According to this theory, the toroidal rotation of the plasma column is determined by the transport properties of the plasma, which in turn are governed by the modes made unstable by the plasma configuration: typically, electrostatic drift modes excited by the ion pressure gradients. The model developed in [2] can be also used to investigate whether an externally induced plasma rotation (for example, with NBI) can interfere with the growth rate of ITG modes, thus affecting the relevant turbulent spectra and ultimately the plasma transport properties [3]. A preliminary study of the slab ITG mode stability has been performed in [4]. Moreover, linear gyrokinetic calculations of toroidal momentum transport in tokamak geometry have been developed in [5]. Here, we have extended the slab model of Ref.4 to include a spatially non uniform magnetic field, its curvature, the presence of a background transverse (radial) electric field, and magnetic shear. We present a detailed analysis of the relevant dispersion relation and of the associated quasilinear (QL) fluxes.

The electrostatic dispersion relation

The linearized two-fluid equations in the guiding-center approximation, in a plasma slab non uniform in the *x* direction, where the electrostatic potential perturbation is assumed to be localized around $x=x_0$, are written as:

$$\varpi\tilde{n}_k = n_0 k_y \tilde{v}_{y,k} + n_0 k_z \tilde{v}_{z,k} - in_0'\tilde{v}_{x,k} - in_0 \frac{\partial \tilde{v}_{x,k}}{\partial x} \tag{1}$$

$$Mn_0\varpi\tilde{v}_{z,k} = +k_z\tilde{p}_k + \left(\nabla\cdot\underline{\underline{\tilde{\pi}}}\right)_{z,k} + q_i n_0 k_z \tilde{\varphi}_k - iMn_0 U_{0z}'\tilde{v}_{x,k} \tag{2}$$

$$\varpi\tilde{p}_k = -ip_0'\tilde{v}_{x,k} + \gamma p_0 k_z \tilde{v}_{z,k} + \gamma p_0 k_y \tilde{v}_{y,k} - i\gamma p_0 \frac{\partial \tilde{v}_{x,k}}{\partial x} \tag{3}$$

$$\frac{\tilde{n}_k}{n_0} = \frac{e\tilde{\varphi}_k}{T_e} \tag{4}$$

$$n_0 = n_{0i} = \frac{n_{0e}}{Z} \tag{5}$$

where the *x*- and *y*-components of the perturbed drift velocity are

$$\tilde{v}_{x,k} = -\frac{ik_y\tilde{p}_k}{n_0 M\Omega_i} - \frac{iq_i k_y \tilde{\varphi}_k}{M\Omega_i} \tag{6}$$

$$\tilde{v}_{y,k} = -\frac{3\tilde{p}_k}{2n_0 MR\Omega_i} - \left(\frac{dp_0}{dx} - \frac{3p_0}{2R}\right)\frac{e\tilde{\varphi}_k}{n_0 M\Omega_i T_e} \tag{7}$$

respectively. Here, the 2D space Fourier transform $\tilde{A}(x,y,z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk_y \int_{-\infty}^{+\infty} dk_z \tilde{A}_k(x,t) e^{i(k_y y + k_z z)}$ has been applied, where the time dependence of each Fourier coefficient is in the form of $\tilde{A}_k(x,t) = \tilde{A}_k(x)\exp(-i\omega_k t)$, with the complex frequency $\omega_k = \omega_k^R + i\gamma_k$, $-_{\omega k} = -_k^{c.c.}$, and $\tilde{A}_{-k}(x) = \tilde{A}_k^{c.c.}$. From now on, it is understood that $x=x_0$. In Eqs.(1-3), M, $q_i = Ze$, $\Omega_i(x) = q_i B_0(x)/Mc$, $\gamma = 5/3$ are the ion mass, electric charge, cyclotron frequency and adiabatic index, respectively. Moreover, the Doppler-shifted frequency $\varpi \equiv \omega_k - k_y U_{0y} - k_z U_{0z}$, the perpendicular $k_\perp = k_y$ and the parallel $k_\parallel = k_z$ components of the wave-vector, have been introduced. Finally, $n_0(x)$, $T_0(x)$, $p_0(x)=n_0(x)T_0(x)$, $U_{0y}(x)$, $U_{0z}(x)$, are the unperturbed (*0*-index) density, temperature, pressure of the ions, *y*- and *z*-components of the ion velocity, respectively. The effect of a non-uniform background magnetic field and of the associated curvature is included by means of a "gravitational" drift. The *0th* and *1st* order accelerations in Eq.(7) write $g_{0x} = \frac{1}{R}\left(v_{\parallel micro}^2 + \frac{1}{2}v_{\perp micro}^2\right) = \frac{3T_0}{2MR}$, $\tilde{g}_x = \frac{3\tilde{T}_k}{2MR}$, respectively, where the microscopic velocities have been evaluated *via* the ion thermal speed, $v_{\parallel(\perp)micro}^2 = \frac{T_0 + \tilde{T}_k}{M}$, where $\tilde{T}_k = \left(\tilde{p}_k - T_0\tilde{n}_k\right)/n_0$. Finally, $R = B_0/B_0'$. The apex indicates the total differentiation with respect to *x*. Since the dispersion relation is quite cumbersome in the most general case, we report here its form resulting from the inclusion of the magnetic field non-uniformity and curvature in the gravitational drift, only. The relevant dispersion equation for the dimensionless frequency $w = \varpi/k_y c_s$ (where $c_s = \sqrt{T_e/M}$, $T_e(x)$

being the electron temperature) writes: $w^3 + a_2 w^2 + a_1 w + a_0 = 0$. The dimensionless coefficients are

$$a_2 = -\frac{Z r_n}{\tau^{1/2}}\left(1 - \frac{\gamma\tau}{Z}\right) + a\tau^{1/2} r_B (2 + \gamma)$$

$$a_1 = Z\chi\left[\frac{U'_{0z}}{\Omega_i} - \chi\left(1 + \frac{\gamma\tau}{Z}\right)\left(1 + \sigma^2\right)\right] - \gamma Z r_n^2 + a r_B\left\{\tau a r_B (1 + 2\gamma) - Z r_n\left[\eta_i\left(1 + \frac{\gamma\tau}{Z}\right) + \frac{2\gamma\tau}{Z} - \gamma - 1\right]\right\}$$

$$a_0 = Z\chi^{1/2\prime}\ r_n\left[\Omega\frac{U_{\overline{0}z}}{\gamma\ _i} +'\ (1 + \tau_i)\left(1 + \frac{\Omega\chi}{Z}\right)\left(1 + \sigma^2\right)\right]\eta\chi^{1/2} a r_B\left\{a^2\chi\Omega r_B^2 + a Z r_n r_B\left[1\,\eta\left(1 + \frac{\Omega\chi}{Z}\right)(1 + \tau_i) + \Omega\right]\eta\right.$$

$$\left.\eta Z\Omega r_n^2 + Z'\ \left[\frac{U_{\overline{0}z}}{\gamma\ _i}\eta'\ \left(1 + \frac{\Omega\chi}{Z}\right)\left(1 + \sigma^2\right)\right]\right\}$$

where we have defined $L_n = n_0/|dn_0/dx|$, $L_T = T_0/|dT_0/dx|$, $\eta_i = L_n/L_T$, $\rho_{Li} = v_{Ti}/\Omega_i$, $r_n = -_{Li}/L_n$, $r_B = -_{Li}/R$, $\chi = k_z/k_y$, $\sigma = k_y\rho_{Li}$, $a = 3/2$.

ITG growth rate and quasilinear fluxes with $U'_{0z}(x) \neq 0$

The dispersion relation in the presence of a transverse electric field and of magnetic shear, that is an *x*-dependent k_y, retains the form of a third-degree equation. The inclusion of $U'_{0z}(x) \neq 0$ introduces an odd dependence of the coefficients a_i (i=0,1,2) on $k_{||}$, leading to an asymmetry in the fluctuation spectrum, and then to a transverse momentum flux [2,4]. The dispersion equation has been solved for a number of cases with the aim of elucidating the role of a non-uniform ion flow in ITG stability. The magnetic shear is introduced giving a q(x) profile and then calculating the corresponding $B_y(x)$. In Fig.1 contour plots of the normalized growth rate $\frac{\Omega_k}{c_s/R}$ of the unstable mode are shown in the plane $R/L_U - R/L_T$ (*i*) and in the plane $R/L_U - \tau$ (*ii*), for U_{0z}-profiles peaked on-axis and $k_{||}/k_\perp < 0$, corresponding to the most unstable case. White regions are stable, while the darker the levels, the higher the corresponding growth rates. The levels are 0.023 (a), 0.076 (b), 0.13 (c), 0.18 (d), 0.23 (e), 0.29 (f), 0.34 (g), 0.4 (h), in plot *i*; 0.025 (a), 0.065 (b), 0.1 (c), in plot *ii*. Generally speaking the presence of a non-zero U'_{0z} makes the system more unstable. However, if the ion temperature profile is steep enough, the presence of a sheared ion flow does not affect any more the stability (*i*). Moreover, it turns out that the temperature ratio is stabilizing above a value ~ 1.5, while it is destabilizing below that value (*ii*). Notice that for flatter density profiles, the curves shift downwards. Here, $R/L_n \approx 4.8$, $\varpi = 1.4$, $T_i \approx 3.5\,\mathrm{keV}$, $B = 1.7\,\mathrm{T}$ have been considered. In Fig.2 the QL momentum flux Γ_U (in Kg/m·s^2, plot *i*) and the QL ion energy flux Γ_p (in J/m^2·s, plot *ii*), both normalized to $e^2\langle|\tilde{\varphi}|^2\rangle/8\pi T_e^2$, are shown *vs x* (m) for typical AUG parameters [3]. At every *x*, an integration over all unstable modes with $|k_\perp| \leq 0.1/\rho_{Li}$ and $|k_\parallel| \perp 10^{\leq 3}|k_\rho|$ has been performed. Large numbers on the vertical axis are due to the smallness of the normalization constant $e^2\langle\tilde{\phi}^2\rangle/8\pi T_e^2$.

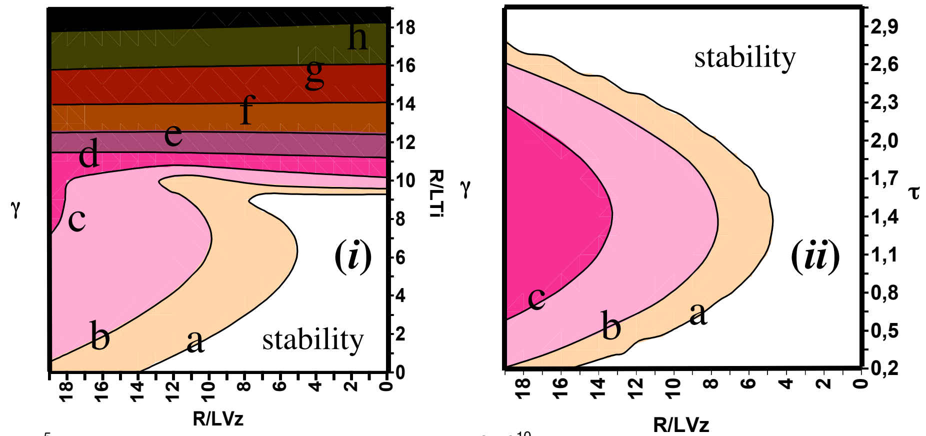

Fig.1

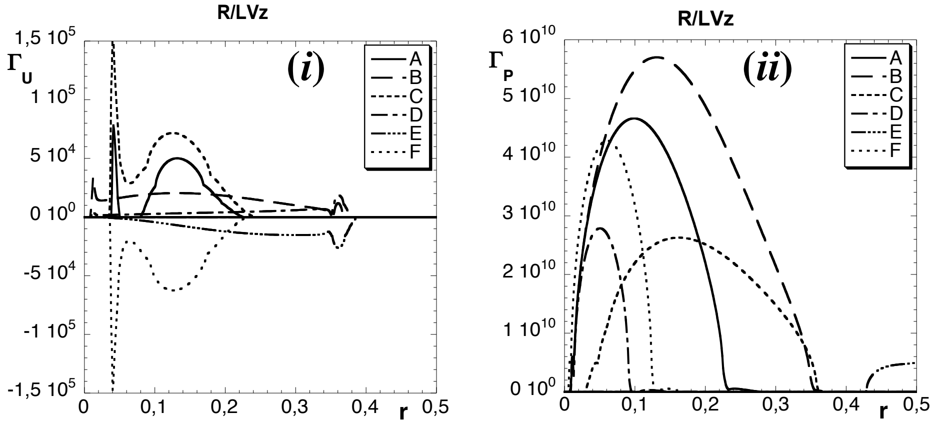

Fig.2

The lines correspond to different *x* (radial) profiles of ion density, temperature and momentum, which are assumed as parabolas in *x/a*, raised to the power α, β, and δ, respectively. In Fig.2.*i*, curve *A* corresponds to α=1.5, β=1, δ=2; *B* to α=0.5, β=4, δ=4; *C* to α=1.5, β=1, δ=4; *D* to α=0.5, β=4, δ=0.5. Curves *E* and *F* correspond to momentum profiles peaked at the plasma edge, with α=0.5, β=4, δ=4, and α=1.5, β=1, δ=2, respectively. The ion energy flux turns out to be independent of U'_{0z}. In Fig.2.*ii*, *A* corresponds to α=1.5, β=4, *B* to α=0.5, β=4; *C* to α=3, β=1; *D* to α=0.5, β=1; *E* to α=3, β=4; *F* to α=3, β=6.

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