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Large scale magnetic field structures in nonuniform unmagnetized plasma

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Introduction

Since the end of the 1970s, experiments showed that quasi-steady magnetic fields are created in laser-produced plasma, and the physical mechanisms for the self-generated magnetic fields have been examined based on the linear theory of the magnetic electron drift mode involving collisions and also the collisionless regime. Here we focus on the non-linear mechanism of generation of large scale magnetic structures, so called meso-scale magnetic fields, by magnetic electron drift wave turbulence and the further non-linear evolution of the large scale fields.

Basic equations

We use the model of magnetic electron drift mode turbulence with the assumptions of a non-magnetised plasma and a characteristic time scale in-between the inverse electron and ion plasma frequencies, $\omega_{pi} \ll \omega \ll \omega_{pe}$. Hence, the ions can be considered neutralising background and the particle density unperturbed $n = n_0$. The temperature is presented as the sum of an equilibrium and a perturbed part, $T_e = T_0 + T$, and the modes are fed by density and temperature inhomogeneities, $\nabla n_0 = \partial_x n_0 \hat{\mathbf{x}}, \nabla T_0 = \partial_x T_0 \hat{\mathbf{x}}$. The model equations can then be derived from the momentum, Maxwell's and the energy equation [1] and read

$$\frac{\partial}{\partial t}\left(B - \lambda^2 \nabla^2 B\right) + \beta \frac{\partial T}{\partial y} = \frac{e\lambda^4}{m}\left\{B, \nabla^2 B\right\} \tag{1a}$$

$$\frac{\partial}{\partial t} T + \alpha \frac{\partial B}{\partial y} = -\frac{e\lambda^2}{m}\left\{B, T\right\} \tag{1b}$$

with $\alpha \equiv \lambda^2 e T_0 / m \left(2/3\kappa_n - \kappa_T\right), \beta \equiv \kappa_n / e$ and $\lambda \equiv c/\omega_c$ the skin depth. The curly brackets on the RHS denote the Poisson brackets. The linear dispersion relation is then

$$\omega_k = k_y \sqrt{\frac{\alpha\beta}{1 + k^2\lambda^2}}, \tag{2}$$

and the linear coupling between the temperature and the magnetic field is $T_k = \alpha k_y / \omega_k B_k$. A self-consistent description can be achieved with the evolution equations for the large scale structures

$$\frac{\partial T_q}{\partial t} = 0; \quad \frac{\partial B_q}{\partial t} = +\frac{e\lambda^2}{m}\frac{q^2\lambda^2}{1+q^2\lambda^2}\sum_k k_x k_y B_k B_{-k}; \qquad \mathbf{q} = q\hat{\mathbf{x}} \quad \text{(ZF)} \tag{3a}$$

$$\frac{\partial T_q}{\partial t} + i\alpha q B_q = 0; \quad \frac{\partial B_q}{\partial t} + \frac{i\beta q}{1+q^2\lambda^2}T_q = -\frac{e\lambda^2}{m}\frac{q^2\lambda^2}{1+q^2\lambda^2}\sum_k k_x k_y B_k B_{-k}; \qquad \mathbf{q} = q\hat{\mathbf{y}} \quad \text{(MS)} \tag{3b}$$

where (ZF) stands for zonal fields ($\mathbf{q} = q\hat{\mathbf{x}}$) and (MS) for magnetic streamers ($\mathbf{q} = q\hat{\mathbf{y}}$), and we have $k \ll q$, and the wave kinetic equation for the small scale turbulence, represented by the wave-action invariant $N_k = 4\alpha/\beta(1+k^2\lambda^2)|B_k|^2$,

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_k^{NL}}{\partial \mathbf{k}}\frac{\partial N_k}{\partial \mathbf{r}} - \frac{\partial \omega_k^{NL}}{\partial \mathbf{r}}\frac{\partial N_k}{\partial \mathbf{k}} = 2\gamma_k N_k - St(N_k). \tag{4}$$

The linear frequency is not valid anymore, but it is Doppler-shifted by the presence of the large scale "flows" and reads explicitly $\omega_k^{NL} \equiv \omega_k^{Re} + \Delta$, with $\Delta = (1+2k^2\lambda^2)/(1+k^2\lambda^2)\mathbf{k}\cdot\mathbf{v}_B^{(q)} - 1/\sqrt{1+k^2\lambda^2}\mathbf{k}\cdot\mathbf{v}_T^{(q)}$ and $\mathbf{v}_B \equiv -e\lambda^2/(4m)(\nabla B \times \hat{\mathbf{z}}), \mathbf{v}_T \equiv -e\lambda^2/(4m)\sqrt{\beta/\alpha}(\nabla T \times \hat{\mathbf{z}})$.

Excitation of large scale fields

In the previous section, it was stressed out that large scale magnetic fields can be generated by small scale turbulence via magnetic Reynolds stress, as might be seen from Eqs. (3a) and (3b). However, such information is of very general nature and does not give us more detailed knowledge on how and when such large scale fields are generated by underlying turbulence. It is indeed only a necessary condition for the generation and does not permit to determine neither a sufficient condition for the excitation nor the increment. The aim of the present section is to point out these conditions by investigating different regimes of large scale field generation, using the self-consistent model of Eqs. (3) and (4), and to find the corresponding rate of excitation.

As a preparation of what follows, we decompose $N_k = N_0 + \tilde{N}_k, \tilde{N}_k \sim \exp[-i\Omega t + i\mathbf{p}\mathbf{r}]$ and linearise (4) such that we find the relation

$$\tilde{N}_k = \frac{\partial}{\partial \mathbf{r}}(\mathbf{k}\cdot\mathbf{v}_f)\frac{\partial N_0}{\partial \mathbf{k}}R(\Omega, p), \tag{5}$$

where $R(\Omega, p) = i/(\Omega - \mathbf{p}\cdot\mathbf{v}_g)$ is the response function and $\mathbf{v}_f = (1+2k^2\lambda^2)/(1+k^2\lambda^2)\mathbf{v}_B^{(q)} - 1/(\sqrt{1+k^2\lambda^2})\mathbf{v}_T^{(q)}$ the "flow velocity" of the large scale fields.

Kinetic regime

The first considerations use the evolution equations (3), the definition of N_k and Eq. (5) to obtain

$$\frac{\partial \bar{B}}{\partial t} = \mp K_q^2 \int q_{x,y}^2 \frac{k_{y,x}^2 \lambda^2}{1+k^2\lambda^2} k_{x,y} \frac{\partial N_0}{\partial k_{x,y}} R(\Omega, p) d^2\mathbf{k}, \qquad \text{(ZF,MS)} \tag{6}$$

where $K_q^2 \equiv \frac{\beta}{16\alpha} \frac{e^2\lambda^2}{m^2} \frac{q^2\lambda^2}{1+q^2\lambda^2}$ and we assume then $\bar{B} \equiv e\lambda^2/(4m)\left(B_q - \sqrt{\alpha/\beta}T_q\right)e^{i\mathbf{qr}} \sim e^{-i\Omega t+\gamma t}$ and thus get the stability criterion for large scale fields of similar form as for Langmuir turbulence in kinetic theory

$$\gamma^{zf} > 0 \Leftrightarrow k_x \frac{\partial N_0}{\partial k_x} < 0 \quad \text{(ZF)}, \qquad \gamma^{st} > 0 \Leftrightarrow k_y \frac{\partial N_0}{\partial k_y} > 0 \quad \text{(MS)} \tag{7}$$

Hydrodynamic regime

For ZF, it is possible to integrate (6) by parts assuming a monochromatic wave spectrum $N_0^k = N_0\delta(\mathbf{k}-\mathbf{k_0})$ and one obtains a stability criterion similar to the Lighthill criterion for modulational instability,

$$\frac{N_0}{\sqrt{\alpha\beta}k_{0y}} \frac{\partial v_g}{\partial k_x}\bigg|_{k_0} < 0 \qquad \text{for growth of ZF.} \tag{8}$$

Long-term dynamics and coherent structures

We have seen that there are indeed different criteria for the generation of large scale magnetic fields. Now we assume that they have been generated and are looking for a non-linear evolution equation for the "flow velocity" v_f. The basis are the same equations as before. However, we not only expand the wave spectrum in the first order resonant perturbed contribution ($\Omega \sim qv_g$) but we also take into account the first and second order non-resonant components ($\Omega \ll qv_g$), $N_k = N_0 + \tilde{N}_k^r + \tilde{N}_k^{(1)} + \tilde{N}_k^{(2)}$, with $\tilde{N}_k^r$. With that expansion we obtain a non-linear evolution equation for the "flow velocity" v_f

$$\frac{\partial}{\partial t}\frac{\partial}{\partial x} v_f = D_{xx}\frac{\partial^3 v_f}{\partial x^3} + u_x \frac{\partial^2 v_f}{\partial x^2} + b_x \frac{\partial^2}{\partial x^2} v_f^2 \qquad \text{(ZF)} \tag{9a}$$

$$\frac{\partial}{\partial t}\frac{\partial}{\partial y} v_f = -D_{yy}\frac{\partial^3 v_f}{\partial y^3} - u_y \frac{\partial^2 v_f}{\partial y^2} - b_y \frac{\partial^2}{\partial y^2} v_f^2 - \sqrt{\alpha\beta}\frac{\partial^2 v_f}{\partial y^2} \qquad \text{(MS)} \tag{9b}$$

The final step is to look for a stationary solution of the type $v_f(x-u_{0x}t)$ and $v_f(y-u_{0y}t)$ respectively and this yields

$$(u_x + u_{x0})v_f + b_x v_f^2 + D_{xx}\frac{\partial v_f}{\partial x} = C \qquad \text{(ZF)} \tag{10a}$$

$$(u_y + (\sqrt{\alpha\beta} - u_{y0}))v_f + b_y v_f^2 + D_{yy}\frac{\partial v_f}{\partial y} = C \qquad \text{(MS)} \tag{10b}$$

From this non-linear equations it is possible to find a localised solution in-between layers of different velocities v_1 and v_2, drifting along x, y with constant velocity $u_{0x,y}$ in the form of a kink soliton [2]

$$v_f = \frac{1}{2}\left\{v_1 + v_2 + (v_1 - v_2)\tanh\left[\frac{b_{x,y}(v_1 - v_2)}{2D_{xx,yy}}x\right]\right\}, \tag{11}$$

where $b_{x,y} = \xi(k)\int d^2\mathbf{k}(k_{x,y}k_{y,x}^3)/(1+k^2\lambda^2)\left(\partial\omega/\partial k_{x,y}\right)^{-1}\partial/\partial k_{x,y}\left[\left(\partial\omega/\partial k_{x,y}\right)^{-1}\partial N_0/\partial k_{x,y}\right]$, $D_{xx,yy} = \beta/(16\alpha)\left(e\lambda^2/m\right)^2(1+2k^2\lambda^2)/(1+k^2\lambda^2)\int k_{y,x}^2/(1+k^2\lambda^2)k_{x,y}\partial N_0/\partial k_{x,y}R(\Omega, q)d^2\mathbf{k}$ and $\xi(k) = \beta/(4\alpha)\left(e\lambda^2/m\right)^2(1+2k^2\lambda^2)/(1+k^2\lambda^2)q^2\lambda^2/(1+q^2\lambda^2)$.

Conclusions

In this article, we have elucidated some properties of magnetic electron drift mode turbulence. First, two different regimes of instability of zonal magnetic fields (ZF) and magnetic streamers (MS) originating in the interaction with small scale turbulence have been considered. In the kinetic regime, instability criteria concerning the form of the equilibrium wave spectrum have been derived and in the hydrodynamic regime, a criterion similar to the Lighthill criterion has been found for zonal magnetic fields. In the second part of the article, the further nonlinear evolution of both ZF and MS allowed for a stationary solution for the “flow velocity” of the large scale structures and the particular form of a kink or switch soliton was found, i.e. a coherent structure in-between two layers of different “flow velocity”.

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