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# Optimization of Laser Pulses Train for Accelleration of Charged Particles in Plasma by Wake Field using SUR/CA Code

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**The problem statement** To accelerate charge particles in plasma [1] effectively, it is necessary to excite electric field with amplitude close to the 'wave-breaking' through thousands of plasma wave length [2]. A strong laser pulse exiting the field are exposed to non-linear regime [3]. Another approach for proper wake field excite is train of a lot weak pulses [4].

Numerical modeling of the interaction of laser pulses with plasma is based on the Maxwell–Vlasov equatoins ($\alpha = e, H^+$).

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = -\nabla\times\vec{E}, \quad \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \nabla\times\vec{B} - \frac{4\pi}{c}\vec{j}, \qquad \nabla\vec{B} = 0, \quad \nabla\vec{E} = 4\pi\rho, \tag{1}$$

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial \vec{r}_\alpha} + e_\alpha\left(\frac{1}{c}\vec{v}_\alpha\times\vec{B} + \vec{E}\right)\frac{\partial f_\alpha}{\partial \vec{p}_\alpha} = 0, \quad \begin{pmatrix}\rho\\ \vec{j}\end{pmatrix} = \sum_\alpha e_\alpha \int \begin{pmatrix}1\\ \vec{v}_\alpha\end{pmatrix} f_\alpha d\vec{p}. \tag{2}$$

The sistem is the non-linear self-consistent multidimensional. That's why the analitycal solution is difficult without the simplifications. The interaction of the laser pulse with plasma is convenient to describe in terms of the dimensionless values, as following: $[v]{=}c$, $[t]{=}1/\omega_o$, $[m]{=}m_{\rm e}$, $[q]{=}e$, here $\omega_o$ is the laser pulse frequency. Other values can be evaluate through the basic ones.

$$[r] = \frac{c}{\omega_o}, \ [n] = \frac{m_{\rm e}\omega_o^2}{4\pi e^2}, \ [E,B] = \frac{m_{\rm e}\omega_o c}{e}, \ \ [A] = \frac{m_{\rm e}c^2}{e}.$$

The basic parametres of the laser pulse in vacuum is the wave vector $\vec{k}_0 = \vec{e}_z c/\omega_0$, amplitude of the dimensionless vector potential $a_0$, the polarization vector $\vec{g}\perp\vec{k}_0$, and the spatial sizes of the wavetrain: the half-width $L_\perp$ and the length of the growth and the dropping: $L_\parallel^{\rm front}$, $L_\parallel^{\rm end}$ The layer of plasma with the initial density $n_{\rm p}$ is located in the range $0{<}z{<}L^{\rm pl}$, $L^{\rm pl} = 2000$.

| name | VarI | VarII |
|---|---|---|
| $a_0$ | 0.2 | 0.5 |
| $n_p$ | 0.01 | 0.01 |
| $\vec{g}$ | $\vec{e}_x$ | $\vec{e}_x{+}i\vec{e}_y$ |
| $L_\perp$ | 100 | $20\pi$ |
| $L_\parallel^{\rm front}$ | $2\pi$ | $\pi$ |
| $L_\parallel^{\rm end}$ | $10\pi$ | $9\pi$ |

Parametres of cited runs

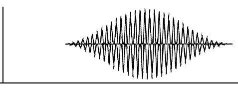
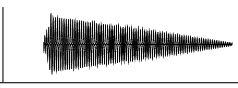
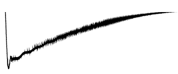
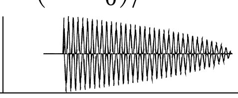
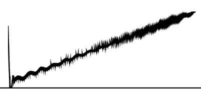
The laser pulse propagates parallel the $z$ axis. The pulses wave vector is normal to the band of the vacuum and the plasma. The electromagnetic field is described by the next relations:

$$\vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \qquad \vec{B} = \nabla\times\vec{A}. \qquad \vec{A} = a_o A_\perp(z) A_\parallel(y) Re\left\{\vec{g}e^{i(\vec{k}_o\vec{r}\,-\,\omega_o t)}\right\}. \tag{3}$$

**Numerical model.** The FDTD (finite-difference time-domain) method in conjunction with local charge density conservative (PIC) particle-in-cell method were used for discretization of the Vlasov-Maxwell equations system. For the model, the phase velocity of the laser pulse in the matter with $\varepsilon,\mu$ differs on the theoretical meaning $c/\sqrt{\varepsilon\mu}$, $V_{\rm ph}^{\rm num}=c\pi\Big/N_\lambda \arcsin\left[\frac{\sqrt{\varepsilon\mu}}{S_c}\sin\frac{\pi S_c}{N_\lambda}\right]$, here $N_\lambda = 2\pi/k_0\Delta^z$ are quantity of the grid nodes per the radiation wave length, and $S_c = c\Delta^t/\Delta^z \le 1/\sqrt{d}$ is the Courant's number, $d$ is the model dimension. The basic calculations were performed with the parametres $S_c = 1/\sqrt{3}$, $N_\lambda = 25$, the control ones were performed with the parametres $N_\lambda = 50$. The error of the wave phase velocity is: 0.17% and 0.04% accordingly.

The phase space is five-dimensional. $(y,z,\vec{p})$, each particle has two spatial coordinates and three projections of the velocity. The electromagnetic fields $\vec{E},\vec{B}$ have three components (2D3V model). The local recursive non-local asynchronous algorithm based on asynchronous cellular automata and recurcive decomposition of dependency graph metodology were used for implementation of above method [5]. The peculiarity of the using the code is the absence of the actual limitations on the modeling range maximal size at the solution preciesion saving during acceptable time. The computer Athlon64x2 4400+/2GB RAM/400GB 2xHD RAID0 is used.

**Wake waves excitation.** The laser pulse propagated through subcritical plasma exite the longitudinal electric wake wave. The phase velocity of the wave in the condition $\omega_o \gg \omega_p$ is:

| $A_{\|\|}$ | Profile $E_x$ | Growth $E_z$ |
|---|---|---|
| $(1+\cos 2\pi(z-z_0)/L)/2$ |  |  |
| $1-(z-z_0)/L$ |  |  |
| $\sqrt{1-(z-z_0)/L}$ |  |  |

$$V_{\rm ph}^{\rm wake}=V_{\rm gr}^{\rm pulse}=c\Bigg/\sqrt{1+\frac{\omega_{\rm p}{}^2}{\omega_0{}^2}} \simeq c\left(1-\frac{\omega_{\rm p}{}^2}{2\omega_0{}^2}\right).$$

The wake wave rise is concerned with the ponderomotive force action. The force depend on the pulse shape: $F \sim \nabla\overline{|E|^2} \sim \overline{|E|^2}/L_{\|}$. The dynamic of the wake wave's growth for the different shapes is shown in the table.

In order to the force was constant, the following pulse's shape choosen: $A_{||}(z) = \sqrt{1-(z-z_0)/L}$. The increasing of the wake field takes place only on the odd half-cycles plasma's wave. Therefore the optimal wave length is $L_{\rm opt} = \lambda_p/2$. For the short pulses the wake wave amplitude depends on the amplitude exiting $\dfrac{eE_{oz}}{m_{\rm e}c\omega_0} = S\frac{\omega_{\rm p}}{\omega_0}\dfrac{a_o{}^2}{\sqrt{1+a_o{}^2}}$, here $S$ is dimensionless coefficient, depended from pulse's shape. In the our calculations $S = 0.53$ for VarI and $S = 0.94$ for Var2, dependence on the amplitude is controlled by variation of $a_0 = 0.1, 0.2, 0.5, 1, 3$.

For the weak pulses the dependence ($E_{ox} \ll 1$) $E_{oz} \sim E_{ox}^2$, and for the relativistic ($E_{ox} \gg 1$) the dependence is linear: $E_{oz} \sim E_{ox}$. Besides, $\dfrac{E_{oz}}{E_{ox}} < \dfrac{\omega_{\rm p}}{\omega_o} \ll 1$. The sequence of the pulses is used to overcome this limitation. For the resonant excitation of the wake field, the distance between leading edges in sequence have to be equal to the plasma wave length.

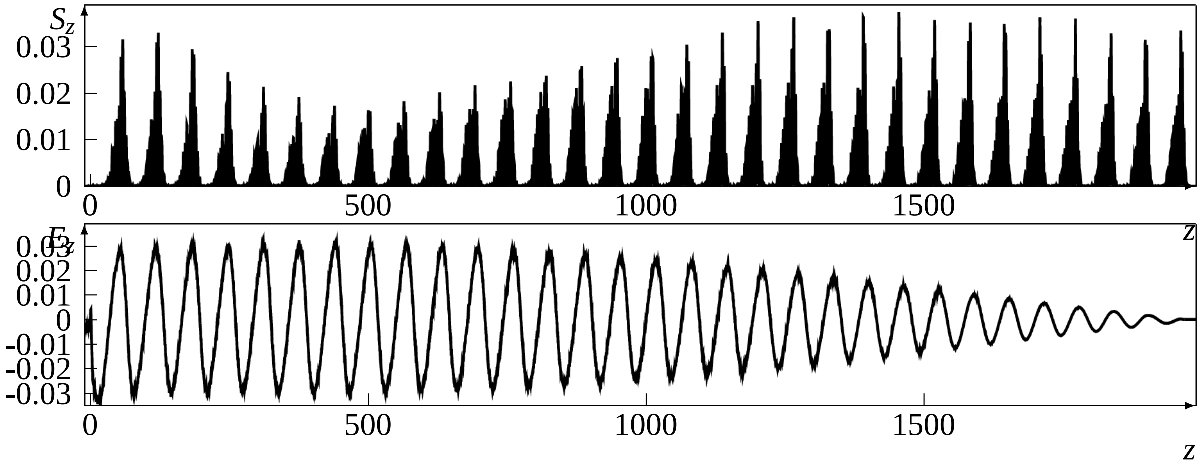



Figure 1: VarI. Pivotal section of of $s_z = E_x B_y - E_y B_x$ (upper) and $E_z$ (below), $t\omega_0 = 1995$

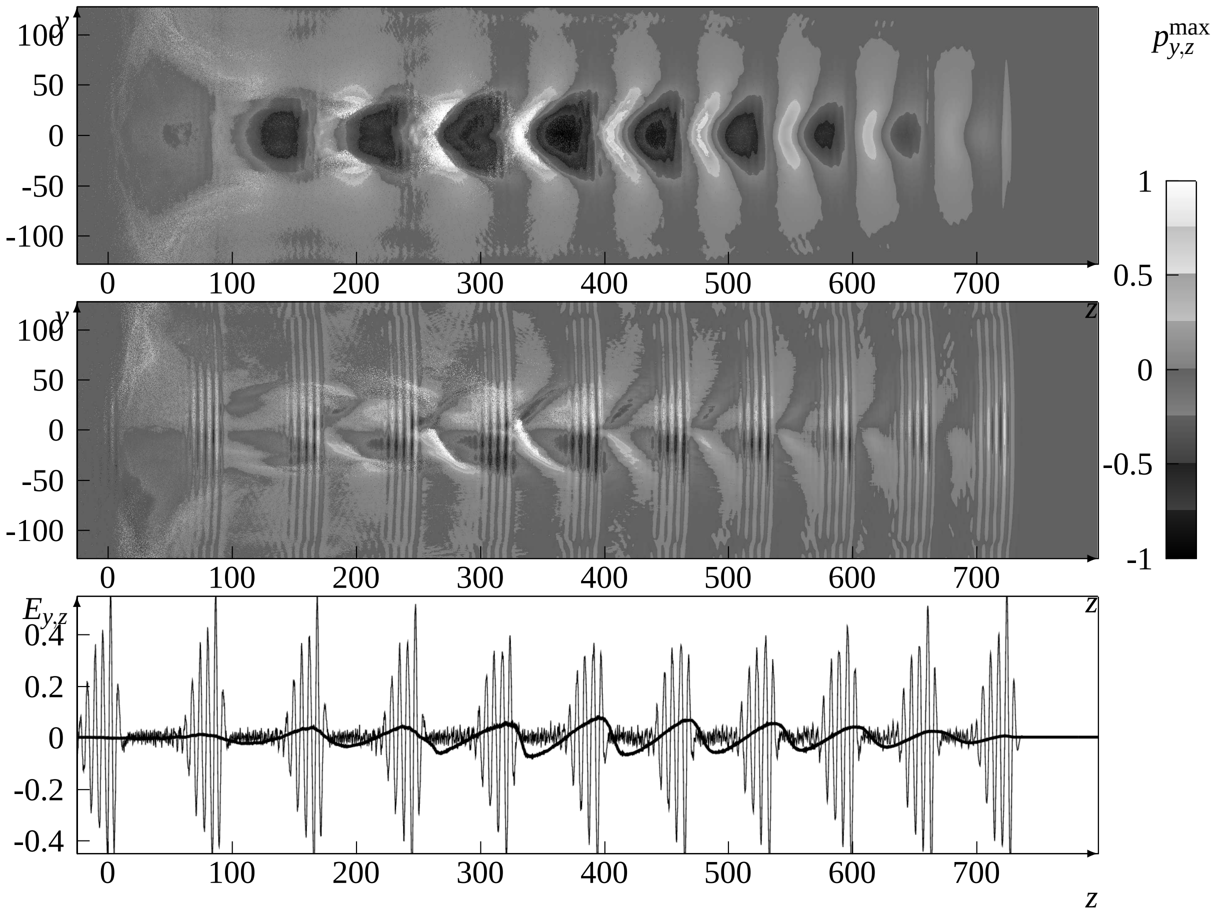



Figure 2: VarII. Spacial distribution of maximal longitudional ($p_z/m_0c$, upper) and transversal ($p_y/m_0c$, in center) momentum of elecrons in the first wavebreaking moment ($t\omega_0 = 739$). Below is the profile of electric fields (thin line for $E_y$, thick line for $E_z$) on the axis ($y = 0$) at the same time moment.

The calculations have shown that the electric field at first really increase linearly but then the field have been constant (fig. 1). Also the decreasing the electromagnetic field of the pulses is shown in fig. 1. The decay of the wake field is concerned with the wavebreaking [6]: $\frac{eE_{WB}}{m_e c\omega_o} = \frac{\sqrt{2}\omega_p}{\omega_o}\sqrt{\frac{\sqrt{2}\omega_o}{\omega_p} - 1}$. We have observed the wavebreaking at the fields of much less then predicted, because the formula is right for the plane waves in the cold plasma. The tranversal dynamics of electrons is shown in fig.2

**Electrons acceleration** The plasma electrons can be cuptured by wake wave after it breaks. The length of acceleration can be approximate from phase sinchronious condition [2]: $L^{acc} \simeq \frac{\lambda_p c}{2(c - V_{ph})} \simeq \frac{2\pi c}{\omega_0}\left(\frac{\omega_0}{\omega_p}\right)^3$.

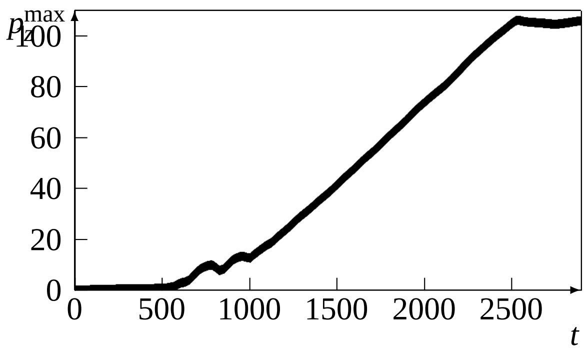



Figure 3: VarII. Maximal momentum $p_z$

The maximal energy growth after the electron acceleration on the length $L^{acc}$ is:

$$\Delta W_{max} \simeq \frac{1}{2} eE_{oz} L^{acc} = \frac{\pi c e E_{oz}}{\omega_0}\left(\frac{\omega_0}{\omega_p}\right)^3. \quad (4)$$

The dynamics of the electron captured by wake wave is shown in Fig. 3. Along the trajectory the averaged accelerated field is $eE_z/c\omega_0 m_e = 0.06$, the total acceleration time $t_{acc}\omega_0 = 1700$ is limited by the distance between the point of the wave breaking and the plasma layer boundary.

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